COMPLETE ARITHMETIC

WENTWORTH-SMITH

1909

WENTWORTH-SMITH MATHEMATICAL SERIES

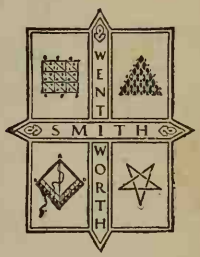
COMPLETE ARITHMETIC

BY

GEORGE WENTWORTH

AND

DAVID EUGENE SMITH



GINN AND COMPANY

BOSTON · NEW YORK · CHICAGO · LONDON

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WENTWORTH-SMITH
MATHEMATICAL SERIES

BY

GEORGE WENTWORTH
AND
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PREFACE

In deciding the form of arrangement for this book the authors accept the fact that each of the two well-defined types of text-books in arithmetic has advantages. A teacher may prefer either one or the other, according to circumstances or training; and the same teacher might conceivably, in the exercise of the best pedagogic judgment, use under one set of conditions a text-book of one type, and when conditions changed employ a book of the other kind.

With books of the type arranged for the recurrent treatment of topics the market is well supplied. Some of the extreme books of this recurrent form are, it is true, too fragmentary in arrangement to give the pupil that feeling of mastery which is his right and his reward. But excellent books of this type, which avoid its perils and provide all its good features, are now available. There is, however, great need for arithmetics having the topical arrangement but thoroughly modern in spirit and in material. That need this book is written to supply. The leading principle that has made the Wentworth books the standard in America for a generation has been followed: Present the reason briefly but clearly, then furnish such an amount of practice that the pupil cannot forget the principle. Theory is reduced to a minimum, and practice is abundant, in both abstract and concrete work, since the experience of a century proves beyond question that each of these two lines must be treated fully to prepare the pupils for life.

The concrete problems are modern in the best sense. Those that seem to be real are real; modern business

customs are followed, and the needs of the future citizen are always kept in mind. But in this book is found no trace of the unfortunate tendency of some of our recent writers to consider an example concrete when it treats of topics or contains technicalities that no pupil understands and few teachers are expected to know.

The book covers all of the topics ordinarily studied after a primary arithmetic has been completed, omitting such subjects as have become obsolete or are too technical for general study. Under each topic there are found an unusually large number of well-graded examples, and at frequent intervals collections of problems without numbers have been inserted as exercises in terse mathematical statement. The topics properly close with sets of exercises that relate to the vocational interests of our people, to the end that pupils may leave school with the real applications of arithmetic to common life clearly in their minds. A few recreations are also given, that the lighter side of arithmetic may be seen along with its more serious aspect.

Any corrections or suggestions relating to the work will be thankfully received.

The authors hope that teachers who recognize the strong features of the topical arrangement, who wish for an abundance of well-graded problems requiring thought in their solution, who are sympathetic with the movement to replace the obsolete by the genuine applications of arithmetic to the American life of to-day, who are opposed to certain of the extreme attempts of the present time that are sure to result disastrously to scholarship,—that such teachers will find in this work a sane, modern, and helpful treatment of the subject.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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COMPLETE ARITHMETIC

CHAPTER I

WRITING AND READING NUMBERS

1. Units. The standards by which we count or measure are called *units.*

Thus in speaking of \$4 the unit is \$1; in speaking of 5 eggs the unit is 1 egg, but in speaking of 10 dozen eggs the unit is 1 dozen eggs; in speaking of 40 thousand bricks the unit is a thousand bricks; in counting 1, 2, 3, 4, and so on, the unit is 1; in measuring long distances the unit may be 1 mile.

2. Numbers. That which shows how many times a unit is taken, or what part of a unit is taken, is called a *number.*

Thus 3 indicates that 1 has been taken 3 times; \$6 indicates that \$1 has been taken 6 times; $\frac{4}{5}$ indicates that four fifths of 1 is taken; hence 3, \$6, and $\frac{4}{5}$ are all numbers.

3. Abstract and Concrete Numbers. A number used without reference to any particular unit is called an *abstract number;* if the unit is named, the whole expression is called a *concrete number.*

For example, 5 is an abstract number, but \$5 and 5 bushels are concrete numbers. It must be understood, however, that the true number part is always abstract. That is, in \$5 the true number part is 5. It is convenient, however, in explaining the solution of problems to make the distinction between abstract numbers and concrete numbers.

4. Integers. Numbers applied to whole units are called *whole numbers, integral numbers,* or *integers.*

Fractional numbers, already familiar to the class from work in the earlier grades, and mentioned in § 2, are treated later.

5. Notation and Numeration. The writing of numbers is called *notation;* the reading of numbers is called *numeration.*

6. Kinds of Notation. The class is already familiar with the common or Arabic notation (so called because it came to Europe through the Arabs), and the Roman notation.

The Arabic characters, sometimes called *figures* or *ciphers*, originated in India. They were employed without the zero more than 2000 years ago, and the zero or naught has been in use over 1200 years. These figures have been known in Europe more than 900 years, but did not become generally used until about 400 years ago, when they finally displaced the clumsy notation of the ancient Romans.

7. Numerals. The characters used in any system of notation are called *numerals.*

Thus we speak of Arabic numerals and of Roman numerals. The Arabic numerals are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The Roman numerals are I, V, X, L, C, D, M. The Arabic characters, excluding 0, are sometimes called *digits.*

8. Place Value. The value of a figure in a number written in Arabic numerals depends upon its position in the number and is called the *place value* of the figure.

Thus, in the number 53, the 3 occupies units' place and means 3 units; the 5 occupies tens' place and means 5 tens. In the Roman system VIII, although made up of V (five) and III (three), like 53, means only eight, because the numerals have no such place value.

9. Orders of Units. Each successive place in a number that may be occupied by a figure is called an *order of units.*

These orders increase from right to left in a tenfold ratio. Hence the Arabic system is called a *decimal system* (from the Latin *decem*, ten). The ones (or units) are called *units of the first order;* the tens, *units of the second order;* and so on.

10. Names of the Orders. As already learned in school, the number names of the first few orders are as follows:

Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units
1	4	3	2	6	5	7	1

11. Periods. When the figures of a number are five or more, we separate them into groups of three figures each by commas or by spaces, beginning at the right. These groups are called *periods.*

Thus we write thirty-two thousand, seven hundred fifty-six, 32,756; it is often printed with a space instead of the comma, thus, 32 756.

12. Separatrix. The comma separating two periods of a number is called the *separatrix.*

13. Names of the Periods. Beginning to name the periods at the right, the first period is called the *units' period;* the second, the *thousands' period;* the third, the *millions' period;* and the fourth, the *billions' period.*

We rarely use number names above millions. The names of the periods above billions are trillions, quadrillions, quintillions, sextillions, septillions, and so on; but these need not be learned.

14. Reading Numbers. In reading numbers we read each period by itself and add the name of the period. Avoid all useless words.

Thus we read 2,341,406 "two million, three hundred forty-one thousand, four hundred six." It is customary to omit the name of the units' period.

EXERCISE 1

Read aloud the following numbers:

1. 46.
2. 85.
3. 70.
4. 24.
5. 132.
6. 201.
7. 357.
8. 777.
9. 1234.
10. 1001.
11. 2052.
12. 3030.
13. 52,176.
14. 31,427.
15. 50,050.
16. 22,222.
17. 215,215.
18. 600,600.
19. 303,303.
20. 123,456.
21. 1,250,000.
22. 2,500,000.
23. 3,333,333.
24. 10,010,010.
25. 12,000,112.
26. 25,150,150.
27. $25,000,000.
28. $32,125,000.
29. $57,000,250.
30. $92,350,206.

Read aloud the following sentences:

31. The area of Alabama is 52,250 square miles.
32. The area of Arkansas is 53,850 square miles.
33. The area of Florida is 58,680 square miles.
34. The area of Georgia is 59,475 square miles.
35. The area of Indiana is 36,350 square miles.
36. The area of Iowa is 56,025 square miles.
37. The area of Kansas is 82,080 square miles.
38. The area of Kentucky is 40,400 square miles.
39. The area of Minnesota is 83,365 square miles.
40. The area of Alaska is 590,844 square miles.
41. The area of Nevada is 110,700 square miles.
42. The area of Texas is 265,780 square miles.
43. The area of the United States is 3,616,484 square miles.

15. Writing Numbers in Arabic Figures. The class has already learned to write numbers. Beginning at the left, we write each period by itself, followed by a comma, putting a zero in every vacant place.

Thus, to write thirty million, forty thousand, seven hundred six, we first write the order of millions, 30, followed by a comma; then 40 thousand, placing a zero in the place of hundred-thousands; then, after the comma, 706. The number then stands as follows:

30,040,706.

EXERCISE 2

Write in figures, arranged in periods, the following:

1. Five thousand, eight.
2. Nine thousand, fifty-seven.
3. Seven thousand, two hundred.
4. Eight thousand, one hundred one.
5. Twelve thousand, four hundred forty.
6. Fifteen thousand, six hundred fifty-six.
7. Eighteen thousand, seven hundred forty-nine.
8. Twenty-five thousand, eight hundred eleven.
9. One hundred fifty-two thousand, eighty-eight.
10. Two hundred six thousand, four hundred one.
11. Four hundred forty thousand, one hundred six.
12. Eight hundred seventy-five thousand, twenty.
13. Nine hundred ninety-nine thousand, eighty-four.
14. One million, one thousand, one hundred one.
15. Three million, three hundred fifty-five thousand, six.
16. Twenty-three million, two hundred forty-eight thousand, six hundred forty-four.

Copy the following as neatly as possible, writing the numbers in Arabic figures:

17. California is three hundred seventy-five miles wide and seven hundred seventy miles long.

18. Montana has an area of one hundred forty-six thousand, eighty square miles.

19. Colorado has an area of one hundred three thousand, nine hundred twenty-five square miles.

20. Louisiana is two hundred seventy-five miles in length and has an area of forty-eight thousand, seven hundred twenty square miles.

21. The extreme length of Oklahoma, east and west, is five hundred eighty-five miles, and it has an area of seventy thousand, fifty-seven square miles.

22. Pennsylvania has an area of forty-five thousand, two hundred fifteen square miles and is a little larger than Tennessee which has an area of forty-two thousand, fifty square miles.

23. Virginia and New York do not differ greatly in area, the former having forty-two thousand, four hundred fifty square miles and the latter forty-nine thousand, one hundred seventy square miles

24. China has an area of four million, two hundred seventy-seven thousand, one hundred seventy square miles, and the British Empire has an area of eleven million, four hundred thirty-five thousand, two hundred eighty-three square miles.

25. The Russian Empire has an area of eight million, six hundred sixty thousand, three hundred ninety-five square miles, and the United States has, with the islands owned by it, an area of three million, seven hundred fifty-six thousand, eight hundred eighty-four square miles.

Write in figures, arranged in periods, the following:

26. Eighty-four million, seven hundred six.

27. Forty-two million, one thousand, one hundred.

28. Seventy-eight million, two hundred forty thousand.

29. Sixty-eight million, eight hundred seventy-three thousand.

30. Ninety-one million, two hundred thousand, one hundred fifty-one.

31. One hundred forty million, seven hundred forty-six.

32. Two hundred thirty-three million, one hundred seventeen thousand, eight.

33. Two hundred twenty-two million, two thousand, two hundred two.

34. Three hundred twenty-four million, five hundred sixty-seven thousand, eight hundred ninety.

35. Three hundred ninety-two million, six hundred forty thousand, five hundred seventy-nine.

36. Four hundred forty-four million, two hundred sixty-seven thousand, three hundred eighty-seven.

37. Five hundred twenty-eight million, six hundred twenty-six thousand, eight hundred forty-seven.

38. Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one.

39. Seven hundred seventy-two million, four hundred seventy-three thousand, five hundred eighty-nine.

40. Eight hundred eighty-eight million, eight hundred eighty-eight thousand, eight hundred eighty-eight.

41. One hundred ninety-eight billion, seven hundred sixty-five million, four hundred thirty-two thousand, one hundred fifty-seven.

Copy the following as neatly as possible, writing the numbers in Arabic figures:

42. The number of inhabitants of Asia is estimated as eight hundred fifty million.

43. It is supposed that there are about one hundred twenty-seven million inhabitants in Africa.

44. It is estimated that the diameter of the earth at the equator is forty-one million, eight hundred fifty-two thousand, four hundred four feet.

45. In a certain year New York manufactured goods to the value of two billion, one hundred seventy-five million, seven hundred sixty-six thousand, nine hundred dollars.

46. In a certain year Pennsylvania manufactured goods to the value of one billion, eight hundred thirty-five million, one hundred four thousand, four hundred thirty-one dollars.

47. In a certain year the value of the agricultural implements manufactured in the United States was estimated as one hundred one million, two hundred seven thousand, four hundred twenty-eight dollars.

48. Some authorities estimate the area of the continents and islands of the earth as fifty-one million, two hundred thirty-eight thousand, eight hundred square miles; others put the area at fifty-two million, eight hundred twenty-one thousand, six hundred eighty-four square miles.

49. English is now spoken by over one hundred thirty million, three hundred thousand persons; German by over eighty-four million, two hundred thousand; Russian by nearly a million more than German; French by fifty-two million, one hundred thousand; Spanish by about forty-six million, five hundred thousand; and Italian by more than thirty-four million.

16. Writing United States Money. In writing United States money the *dollar sign* ($) is written before the number. A period, called a *decimal point*, is placed between the dollars and the dimes.

For example, 125 dollars is written $125; 10 dollars and 48 cents (that is, 10 dollars, 4 dimes, 8 cents) is written $10.48; 5 dollars and 8 cents is written $5.08.

EXERCISE 3

Read aloud the following:

1. $15.	9. $2.25.	17. $21.00.	25. $4825.25.
2. $24.	10. $3.75.	18. $35.25.	26. $7275.05.
3. $40.	11. $6.42.	19. $42.75.	27. $427.82.
4. $90.	12. $7.28.	20. $33.03.	28. $5207.60.
5. $2.70.	13. $5.05.	21. $125.50.	29. $4270.07.
6. $1.30.	14. $7.09.	22. $325.75.	30. $500.05.
7. $4.60.	15. $3.04.	23. $2450.25.	31. $5000.05.
8. $5.20.	16. $9.08.	24. $3575.50.	32. $9999.99.

33. In 1800 the public debt of our country was $15.63 for each inhabitant; in 1900 it was $14.52.

34. The average amount for each depositor in the savings banks of the United States recently was $429.64.

35. Recently the public debt of the United States was $2,492,231,518.54.

36. The average pay of locomotive engineers on the railroads in our country is about $4.35 a day; conductors average about $3.75 a day.

37. In a certain year the United States produced iron and steel to the value of $804,034,918.75, the value of that produced by the Middle Atlantic States being $478,687,519.80.

Write the following, using the dollar sign and Arabic figures:

38. Ten dollars and fifty cents.

39. Eighteen dollars and eight cents.

40. Four dollars and seventy-five cents.

41. Sixteen dollars and fifteen cents.

42. Twenty dollars and seventy-two cents.

43. Twenty-five dollars and thirty cents.

44. Thirty-seven dollars and fifty cents.

45. Forty-eight dollars and seventeen cents.

46. Seventy-seven dollars and seven cents.

47. Eighty-five dollars and eighty-five cents.

48. One hundred dollars and fifty cents.

49. Two hundred four dollars and ten cents.

50. Two hundred two dollars and two cents.

51. Five hundred thirty dollars and six cents.

52. Six hundred six dollars and sixty-six cents.

53. Four hundred forty-two dollars and seventy cents.

54. Three hundred thirty-three dollars and thirty cents.

55. Six hundred seventy-five dollars and eighteen cents.

56. Seven hundred forty-two dollars and ninety-five cents.

57. One thousand, four hundred seven dollars and five cents.

58. Two thousand, nine hundred forty-eight dollars and twenty-five cents.

59. Twenty-five thousand, seven hundred sixteen dollars and seventy-five cents.

60. Eighty-six thousand, nine hundred seventy-seven dollars and thirty-seven cents.

61. Forty-eight thousand, five hundred sixty-three dollars and seventy-five cents.

62. Seventy-six thousand, two hundred thirty-six dollars and ninety-three cents.

63. Sixty-four thousand, nine hundred twenty-seven dollars and five cents.

64. Ninety-five thousand, seven hundred forty-nine dollars and forty-four cents.

65. Seventy-nine thousand, six hundred seventeen dollars and fifty cents.

66. One hundred sixty thousand, forty-nine dollars and eighty-four cents.

67. Two hundred fifty-three thousand, four dollars and seventy-three cents.

68. Seven hundred sixty-five thousand, eight hundred twenty-five dollars and eleven cents.

69. Four hundred forty thousand, five hundred sixty-three dollars and nine cents.

70. Three hundred seven thousand, two hundred fifty dollars and thirty-seven cents.

71. Nine hundred eighty-four thousand, five hundred thirty-one dollars and sixty-five cents.

72. Six hundred fifty-seven thousand, seven hundred twenty-six dollars and thirty-five cents.

73. Seven hundred thousand, three hundred ninety-three dollars and five cents.

74. Eight hundred sixty-eight thousand, fifty-two dollars and seven cents.

75. Four hundred seventy-three thousand, eight hundred twenty-seven dollars and ninety-five cents.

17. The Roman Numerals. The Roman notation is used chiefly for numbering chapters and for dates. It employs seven capital letters, as follows:

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

The first nine numbers are written thus:

I, II, III, IV or IIII, V, VI, VII, VIII, IX.

The tens are written thus:

X, XX, XXX, XL, L, LX, LXX, LXXX, XC.

The hundreds are written thus:

C, CC, CCC, CD, D, DC, DCC, DCCC, DCCCC or CM.

The numbers from eleven to nineteen are written thus:

XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX.

The following are examples of other numbers:

XXII = 22	XCVIII = 98	CLXV = 165
XXXIV = 34	LXXV = 75	CCLIX = 259
XLIX = 49	LXXXI = 81	MDCCXC = 1790

MDCCCCXI or MCMXI = 1911

EXERCISE 4

Write in Arabic figures:

1. XIV.	4. LIX.	7. LXXX.	10. MDCC.
2. XXIX.	5. LXVI.	8. XCVI.	11. MDCXX.
3. XXXI.	6. LXXV.	9. DCCXI.	12. MDCCCCX.

Write in Roman numerals:

13. 26.	16. 69.	19. 135.	22. 525.
14. 32.	17. 81.	20. 241.	23. 1892.
15. 50.	18. 99.	21. 363.	24. 1928

CHAPTER II

ADDITION OF INTEGERS

18. Addition. You have already learned how to add numbers and are now ready to read the definition of addition. The operation of finding a number equal to two or more numbers taken together is called *addition.*

Thus, to add 7 and 8 is to find a number, 15, that is equal to these numbers taken together.

$$\begin{array}{r} 7 \\ 8 \\ \hline 15 \end{array}$$

19. Sum. The result obtained by adding is called the *sum.*

Thus 15 is the sum of 7 and 8.

20. Symbol. The symbol of addition is this form of a cross, +. It is read *plus*, a word meaning *more.*

$$\begin{array}{r} 7 \\ +\,8 \\ \hline 15 \end{array}$$

The expression $7 + 8 = 15$, as you have long ago learned, is read "7 plus 8 equals 15." Sometimes in school the plus sign is used to indicate addition in a column, as here shown.

21. Like Numbers. Numbers applied to the same unit are called *like numbers.*

Thus \$3 and \$5 have the same unit, \$1; 3 feet and 2 feet have the same unit, 1 foot; 7 and 15 have the same unit, 1.

22. Addends. The numbers added are called *addends. Only like numbers can be added and the sum is like the addends.*

That is, we cannot add \$2 and 3 yards, or 7 pounds and 4 books. If we add \$12 and \$13, the sum must be like the addends; that is, it must be \$25, not 25 feet. It is true that we can put 4 books with 7 crayons, but we cannot add them so as to get 11 books or 11 crayons; we have to think of them only as *things*, and say that we have 11 things.

23. Addition of Digits. You have already learned how to add numbers, but the following directions will help you:

(1) *First add from the bottom upwards and write the sum.*

(2) *Then add from the top downwards, to see that you have made no mistake.*

This is called *checking* the result, and is very important in all calculations.

(3) *Try to group digits in adding, thus allowing you to work more rapidly.*

For example, in this case we may begin with 6 and add each digit in order, thinking thus: "6, 15, 18, 25, 29." It is better, however, to look at the 6 and 9 and think "15"; then look at the 7 and 3 and think of 10 as added to the 15, saying to ourselves "25"; then to look at 4 and think "29."

4
7
3
9
6
29

EXERCISE 5

Add the following, checking your work and timing yourself:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
2	3	4	3	8	9	9	8	4	2	4	2
3	4	2	9	2	8	2	8	6	1	2	8
5	7	8	7	8	7	3	8	5	4	6	7
7	6	9	7	3	6	1	2	5	7	3	6
9	2	3	4	9	5	4	2	3	3	4	3

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
3	7	6	4	4	7	5	6	7	3	8	4
4	2	8	1	6	7	2	8	8	4	2	7
2	4	3	2	4	8	8	3	2	2	9	8
6	1	4	8	5	8	7	9	8	8	1	3
4	1	9	7	3	2	2	4	9	7	7	9
3	2	2	3	2	3	4	5	3	9	3	2

Copy the numbers in lines 25 to 35 inclusive, and add; then add the columns numbered 36 to 43 inclusive:

	36.	37.	38.	39.	40.	41.	42.	43.
25.	2	3	2	2	3	9	8	1
26.	1	4	1	9	8	1	4	3
27.	3	7	3	3	4	2	9	5
28.	4	7	4	9	4	8	2	6
29.	2	8	3	4	1	3	7	6
30.	8	2	2	1	9	4	3	6
31.	7	3	4	2	8	1	6	9
32.	6	4	8	4	3	9	5	2
33.	4	6	6	6	2	7	8	3
34.	1	2	1	3	7	6	5	4
35.	2	3	4	2	6	2	7	5

Proceed in the same way for lines numbered 44 to 57 inclusive, and for columns numbered 58 to 65 inclusive:

	58.	59.	60.	61.	62.	63.	64.	65.
44.	1	8	6	2	4	7	3	5
45.	2	5	9	3	8	4	7	6
46.	3	4	1	5	8	6	9	7
47.	2	7	4	1	7	6	3	5
48.	7	1	5	2	8	3	9	4
49.	1	3	5	6	2	8	4	7
50.	2	4	6	3	9	5	8	7
51.	4	2	3	8	5	7	9	6
52.	3	1	2	8	6	4	7	5
53.	2	6	6	4	7	8	3	9
54.	7	9	1	6	2	4	3	5
55.	8	2	3	6	9	5	7	4
56.	1	4	5	2	8	7	3	6
57.	9	1	4	6	2	5	7	3

24. Addition of Two-Figure Numbers. You have also learned how to add two-figure numbers, but the following directions may help you to understand the work:

(1) *Write the addends so that units of the same order are in the same column.*

(2) *Add each column by itself, beginning at the right, and write the sum, if it is less than 10, under the column added.*

(3) *If the sum of any column is 10 or more, write the unit figure under the column added, and add the tens with the next column.*

```
 67
 86
 34
---
 17  sum of units
17   sum of tens
---
187  sum
```

This is done in full in the example shown above. In practice, however, we do not write the sum of each column by itself, but proceed as here shown. We think of the work as follows: "4, 10, 17," writing the 7 and adding the 1 to the tens' column; then think: "1, 4, 12, 18," and write the 8 under the tens' column and the 1 in the hundreds' place.

```
 $67
  86
  34
----
$187
```

The dollar sign before the top number in a column indicates that all the numbers in the column represent dollars.

EXERCISE 6

Add the following and check your work:

1.	2.	3.	4.	5.	6.
21	30	6	34	72	$32
32	47	57	9	89	21
46	22	61	68	67	49

7.	8.	9.	10.	11.	12.
34	62	75	62	47	$25
62	48	25	48	63	30
81	31	30	8	92	35
92	43	7	75	81	40

Copy the numbers in lines 13 to 22 inclusive and add; then add the columns numbered 23 to 30 inclusive:

	23.	24.	25.	26.	27.	28.	29.	30.
13.	10	11	14	34	15	20	31	12
14.	22	12	33	21	22	34	6	9
15.	13	21	20	12	31	26	15	34
16.	31	23	6	51	28	35	42	55
17.	42	31	14	62	61	77	82	69
18.	14	43	35	71	38	56	77	86
19.	31	37	7	33	46	59	68	77
20.	22	42	62	14	84	73	82	91
21.	30	61	70	68	59	38	47	65
22.	50	12	15	73	77	82	39	46

Proceed in the same way for lines numbered 31 to 45 inclusive, and for columns numbered 46 to 53 inclusive:

	46.	47.	48.	49.	50.	51.	52.	53.
31.	12	20	23	14	86	77	80	78
32.	6	10	15	17	83	85	90	94
33.	7	9	12	23	87	88	91	93
34.	13	15	14	36	42	23	31	46
35.	21	31	21	41	28	17	16	20
36.	42	24	33	23	14	23	41	26
37.	9	16	40	31	15	17	48	62
38.	8	33	62	14	26	33	27	15
39.	10	27	19	17	14	23	73	85
40.	23	18	28	62	17	15	52	38
41.	37	26	30	31	13	89	59	74
42.	52	58	42	48	58	77	84	80
43.	46	39	71	73	72	83	69	54
44.	51	62	16	92	86	77	9	7
45.	73	47	86	87	85	83	10	16

25. Addition of Larger Numbers. You have learned how to add numbers of more than two figures, and the directions given on page 16 for adding two-figure numbers apply to all cases.

2936	
8798	
14	sum of units
12	sum of tens
16	sum of hundreds
10	sum of thousands
11734	sum

Thus, to add 2936 and 8798 we might proceed as shown in the left-hand solution, but the process shown in the right-hand solution is better because it is shorter and allows the work to be performed more rapidly.

2936
8798
11734

EXERCISE 7

Add the following, checking your work and timing yourself:

1.	2.	3.	4.	5.
307	607	426	462	336
268	73	427	617	774
111	803	700	296	340
702	727	99	287	760

6.	7.	8.	9.	10.
4732	2732	596	4884	8492
6239	698	69	6939	3687
5268	1596	3492	4008	4009
3761	998	9873	7227	698

11.	12.	13.	14.	15.
64,283	41,294	28,009	43,218	52,988
2,796	58,706	77,986	37,298	13,678
12,426	14,261	20,811	42,361	60,211
20,431	2,143	30,021	20,000	72,010

Add the following:

16.	17.	18.	19.	20.
73,296	28,428	44,307	30,072	78,327
41,427	73,211	62,320	14,826	4,162
33,291	70,092	40,072	4,999	899
41,627	61,778	63,142	12,000	10,090
20,623	31,732	40,012	30,200	41,009

21. Recently we had 9196 steamers and 12,831 other American vessels engaged in trade along our coast. What was the total number?

22. In our country there were reported as built recently in one year 147 sailing vessels, 674 steam vessels, 62 canal boats, and 274 barges, not counting many smaller boats. What was the total of those reported?

23. In a recent year our country sold abroad $32,305,412 worth of manufactured cotton, and $2,287,936 worth of manufactured wool. What was the total value?

The following table shows certain statistics relating to our country in 1800, and the increase in 100 years. Add the increase to the column for 1800 and find the statistics for 1900.

	1800	Increase
24. Number of square miles	827,844	2,198,945
25. Number of bales of cotton grown	155,556	9,280,860
26. Receipts of post offices	$280,804	$102,073,775
27. Gold and silver money	$16,000,000	$736,856,806
28. Total circulation of money	$26,500,000	$2,028,650,998
29. Population	5,308,483	70,994,904

26. Addition of United States Money. In adding United States money simply proceed as with ordinary integers, keeping units of the same order in the same column as here shown.

$12.50
.75
23.
1.05
$37.30

We can easily keep the units of the same order in the same column by placing the decimal points under one another.

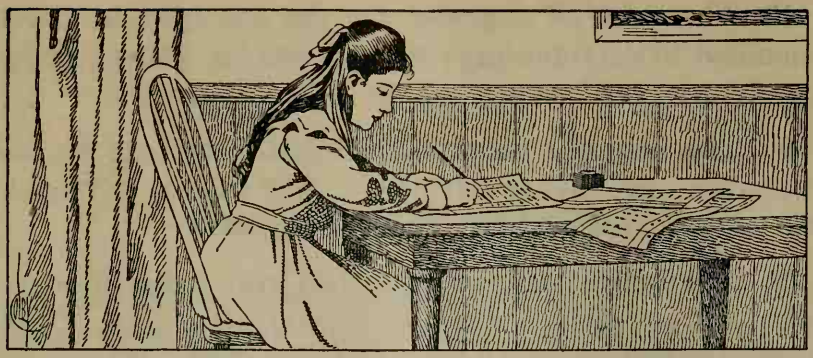
EXERCISE 8

Add the following:

1.	2.	3.	4.	5.
$2.34	$1.45	$7.40	$5.06	$6.92
1.35	.75	3.80	1.04	7.21
4.26	3.00	6.20	3.07	3.87
5.72	1.26	5.90	4.83	4.

6.	7.	8.	9.	10.
$2.93	$6.42	$5.42	$2.50	$23.50
3.81	7.08	.58	3.62	7.50
4.26	3.91	.39	0.48	18.75
5.35	.75	3.27	4.71	32.10
.75	.25	.58	3.	4.05
4.	6.	1.25	1.96	5.10

11.	12.	13.	14.	15.
$6.42	$4.08	$5.25	$2.93	$14.08
7.81	2.75	1.27	4.62	.92
3.10	3.02	3.06	8.21	26.29
2.07	6.72	4.81	4.26	53.07
.56	8.05	2.72	8.02	62.75
.82	9.	3.25	7.33	41.05
1.49	.33	4.05	.85	35.
7.23	6.44	6.00	3.82	14.75



A mother has told her daughter to add some grocer's bills as quickly as possible, but to be sure the results are correct. These are the bills (Exs. 16–19). Add them:

16.

Broom	$0.35
6 cakes soap	.48
3 cakes sapolio	.21
6 cakes common soap	.25
1 gal. oil and can	.38
1 doz. oranges	.60
½ lb. tea	.30
Lettuce	.10
Box silver polish	.25
Can peas	.20
Celery	.20
Box matches	.05
Box cornstarch	.12

17.

Cauliflower	$0.15
½ doz. oranges	.25
Lemons	.15
Nutmeg	.20
1 gal. oil	.18
Lettuce	.10
½ lb. tea	.35
½ doz. bananas	.13
3 lb. sugar	.21
1 package cereal	.13
6 cakes soap	.30
Butter	.34
Cloves	.05

18. The prices on another bill were as follows, the numbers indicating dollars and cents: 1.25, .27, .35, 2.43, 1.96, .82, 3.40, 1.75, 2.80, .65, .72, .38, 1.96.

19. The prices on another bill were as follows: 1.30, .75, .62, 4.60, 1.20, 3.20, .72, .05, .17, .07, 1.60, .50, .30, .13, .15.

20. The sales of a grocer for the six days in a week amounted to the following: \$63.45, \$52.70, \$86.50, \$75.32, \$89.73, \$123.48. What was the total amount?

21. The sales of a department store for the four weeks of December amounted to \$32,368.42, \$38,426.54, \$41,286.35, \$53,342.60. What was the total amount?

22. The approximate areas of the Great Lakes in square miles are as follows: Superior, 32,000; Michigan, 22,400; Huron, 23,000; Erie, 10,000; Ontario, 6700. What is the total area?

23. The area of Germany is 208,830 square miles; of France, 207,054 square miles; of Holland, 12,648 square miles; and of Belgium, 11,373 square miles. What is the area of the four countries together?

24. The sales of a dry goods merchant for the twelve months in a year were as follows: \$3772.60, \$4237.40, \$3816.75, \$4627.42, \$3278.75, \$3846.50, \$3121.27, \$3016.50, \$3827.52, \$4126.35, \$4335.50, \$4925.75. What was the total amount?

25. The number of pupils recently enrolled in the common schools of certain of our south central states was as follows: Kentucky, 501,482; Tennessee, 508,316; Alabama, 400,000; Mississippi, 482,208; Louisiana, 225,008; Texas, 705,305; Arkansas, 340,182; Oklahoma, 291,528. What was the total number?

26. The number of pupils recently enrolled in the common schools of certain of our north central states was as follows: Ohio, 827,414; Indiana, 538,881; Illinois, 983,921; Michigan, 521,463; Wisconsin, 465,490; Minnesota, 429,012; Iowa, 549,449; Missouri, 741,745; North Dakota, 124,000; South Dakota, 110,094; Nebraska, 279,532; Kansas, 381,595. What was the total number?

CHAPTER III

SUBTRACTION OF INTEGERS

27. Subtraction. The process of taking one number from another, or of finding what number must be added to one of two numbers to make the other, is called *subtraction.*

That is, if we take 9 things from 17 things we have 8 things left; or we may think of what must be added to 9 to make 17.

28. Terms. The number taken away is called the *subtrahend;* the number from which the subtrahend is taken is called the *minuend;* the result of the subtraction is called the *remainder* or *difference.*

29. Symbol. The symbol of subtraction is a short horizontal line, —. It is read *minus*, a word meaning *less.*

Thus $17 - 9 = 8$ is read "17 minus 9 equals 8."

30. Process of Subtraction. Suppose we wish to find the difference between 468 and 143. Of several methods there are two in common use:

(1) *The taking-away method.* Here we say: 3 from 8, **5**; 4 from 6, **2**; 1 from 4, **3**.

468	minuend
143	subtrahend
325	remainder

(2) *The making-change, or adding method.* Here we find what must be added to 143 to make 468, saying: $3 + 5 = 8$; $4 + 2 = 6$; $1 + 3 = 4$.

Each teacher should decide which method he will use, and follow that. The first is the more common; the latter is always used in making change, is the more rapid, and is growing in favor.

31. Check. To check the result, add the subtrahend and remainder; the sum should be the minuend.

EXERCISE 9

Subtract, checking your work and timing yourself:

1.	2.	3.	4.	5.
673	742	396	485	981
261	420	104	172	651

6.	7.	8.	9.	10.
908	659	485	398	667
702	128	152	287	421

11.	12.	13.	14.	15.
$7.25	$5.36	$9.28	$6.75	$8.82
1.02	2.15	1.05	1.25	2.31

16.	17.	18.	19.	20.
$15.75	$23.50	$37.55	$52.85	$75.88
2.20	2.40	15.20	21.70	13.52

21.	22.	23.	24.	25.
$82.96	$95.63	$86.75	$73.92	$48.52
71.42	15.02	14.25	13.92	24.10

26.	27.	28.	29.	30.
$175.40	$142.65	$235.75	$468.92	$536.48
21.30	21.02	101.20	107.02	121.25

31. 62,815 − 11,702.
32. 81,482 − 70,141.
33. 96,348 − 62,123.
34. 27,984 − 14,521.
35. 31,628 − 20,407.
36. 82,489 − 70,126.
37. 68,925 − 61,420.
38. 34,876 − 10,724.
39. 42,929 − 30,009.
40. 99,999 − 47,628.

41. A grocer had 1360 pounds of sugar and sold 1040 pounds. How many pounds had he left?

42. A farmer had 2445 bushels of corn and sold 2120 bushels. How many bushels had he left?

43. A man's income for a year was $1975 and his expenses were $1630. How much had he left?

44. A dealer had 2796 tons of coal and sold 1385 tons. How many tons had he left?

45. A dry goods merchant had 1440 spools of thread and sold 1200 spools. How many spools had he left?

46. A city school had 1296 pupils of whom 184 were in the first-year class. How many were in all the other classes together?

47. A real estate dealer had 1278 acres of land for sale, and sold 1163 acres. How many acres had he left unsold?

48. The distance from New York to Albany is 143 miles, and from New York to Chicago by way of Albany is 975 miles. How far is it from Albany to Chicago by this route?

49. The area of Indiana is 36,350 square miles, and that of Rhode Island 1250 square miles. What is the difference in area?

50. The area of Alabama is 52,250 square miles and that of California is 158,360 square miles. What is the difference in area?

51. The area of Nebraska is 77,510 square miles, and that of South Dakota is 77,650 square miles. What is the difference in area?

52. In a recent year Pennsylvania produced 31,816,496 bushels of oats, and Massachusetts 214,472 bushels. What was the difference in the amounts?

32. General Case of Subtraction. Suppose we wish to find the difference between 652 and 476. Here are the two methods stated in § 30, either of which may be used.

652 minuend
476 subtrahend
176 remainder

(1) Since we cannot take 6 from 2 we change one of the 5 tens to units, and this with the 2 units makes 12 units; then $12 - 6 = 6$. Since we have used one of the tens in the 5 tens we have 4 tens left, from which we cannot take 7 tens. We change one of the 6 hundreds to tens, and this with the 4 tens makes 14 tens; then 14 tens − 7 tens = 7 tens. We now have 5 hundreds left of the 6 hundreds, and 5 hundreds − 4 hundreds = 1 hundred. The remainder is therefore 176. It will make the work clearer to see the numbers separated as here shown.

$$\begin{array}{r} 500 + 140 + 12 \\ 400 + \ 70 + \ 6 \\ \hline 100 + \ 70 + \ 6 \end{array}$$

(2) The second method uses this principle: *If the same number is added to both minuend and subtrahend, the remainder is not changed.* Thus 8 − 2 gives the same result as 18 − 12 or 108 − 102.

We now consider what added to 6 makes 2. Since this is impossible, what added to 6 makes 12? Evidently 6, which we write below. But we increased the minuend from 2 to 12 by adding 10, so we add 10 to the subtrahend, making the 7 into an 8. Then, since no number added to 8 makes 5, what added to 8 makes 15? Evidently 7, which we write below. But we increased the minuend from 5 tens to 15 tens, by adding 100 (or 10 tens), so we add 100 to the subtrahend, making the 4 a 5. Then $5 + 1 = 6$, and we write the 1 below.

$$\begin{array}{r} 652 \\ 476 \\ \hline 176 \end{array}$$

Those who get used to this second method find it more rapid than the first, but either method is good.

33. Special Cases. In a case like 5000 − 2765 we may shorten the work by thinking of the 5000 as 4990 + 10, and of 2765 as 2760 + 5, beginning to subtract at the left.

$$\begin{array}{r} 4 \ 9 \ 9 \ 10 \\ 2 \ 7 \ 6. \ 5 \\ \hline 2 \ 2 \ 3 \ 5 \end{array}$$

This amounts to taking the 2 from 4 and every other figure from 9 except the units, which we take from 10. The remainder is 2235.

In subtracting United States money place the decimal points under each other and proceed as with integral numbers.

EXERCISE 10

Subtract, checking your work and timing yourself:

1. 274 — 62.
2. 378 — 69.
3. 465 — 57.
4. 582 — 44.
5. 673 — 65.
6. 766 — 49.
7. 857 — 28.
8. 939 — 47.
9. 354 — 62.
10. 268 — 85.
11. 325 — 63.
12. 436 — 94.
13. 742 — 70.
14. 819 — 37.
15. 352 — 63.
16. 603 — 25.
17. 760 — 75.
18. 504 — 68.
19. 320 — 33.
20. 404 — 27.
21. 570 — 92.
22. 700 — 30.
23. 600 — 43.
24. 300 — 56.
25. 400 — 90.
26. 500 — 99.
27. 638 — 126.
28. 427 — 127.
29. 636 — 318.
30. 342 — 225.
31. 596 — 379.
32. 758 — 449.
33. 342 — 207.
34. 467 — 208.
35. 621 — 416.
36. 733 — 329.
37. 570 — 444.
38. 392 — 288.
39. 340 — 225.
40. 628 — 419.
41. 492 — 284.
42. 462 — 373.
43. 707 — 528.
44. 981 — 694.
45. 606 — 428.
46. 702 — 513.
47. 603 — 427.
48. 402 — 236.
49. 520 — 348.
50. 630 — 469.
51. 800 — 532.
52. 900 — 687.
53. 2782 — 120.
54. 3648 — 439.
55. 5276 — 168.
56. 8496 — 229.
57. 2349 — 256.
58. 4872 — 380.
59. 6463 — 291.
60. 4486 — 395.
61. 7286 — 193.
62. 2872 — 385.
63. 5342 — 264.
64. 3353 — 296.
65. 4374 — 287.
66. 6681 — 599.
67. 6720 — 535.
68. 7872 — 5785.
69. 3526 — 2239.
70. 7280 — 2385.
71. 6807 — 4938.
72. 38,265 — 23,406
73. 34,635 — 25,296.
74. 48,406 — 26,573.
75. 62,068 — 48,679.
76. 85,723 — 67,836.
77. 92,631 — 75,842.
78. 80,050 — 69,967.

79.	80.	81.	82.	83.
$24.71	$32.68	$41.87	$72.91	$65.30
12.80	16.48	32.78	42.63	48.70

84.	85.	86.	87.	88.
$81.72	$49.26	$82.48	$91.75	$83.45
26.54	37.62	48.56	41.80	35.57

89.	90.	91.	92.	93.
$125.30	$132.40	$148.50	$175.75	$240.70
63.50	48.75	62.95	37.80	65.85

94.	95.	96.	97.	98.
$230.42	$420.63	$575.00	$280.00	$100.00
125.75	140.75	206.05	142.35	75.65

99.	100.	101.	102.	103.
$1000.00	$2000.00	$4000.00	$5000.00	$7000.00
75.50	275.50	148.50	325.25	625.75

104.	105.	106.	107.	108.
$3000.00	$6000.00	$2000.00	$4000.00	$7000.00
125.50	1240.50	1060.75	2075.50	1500.75

109. What number added to 1756 makes 4875 ?

110. What number subtracted from 5283 leaves 1924 ?

111. How much greater is 5481 than 2962 ?

112. How much less is 1782 than 2941 ?

113. If a man has $245.50, how much must he add to it to have $1000 ?

114. If a man has an income of $1500 a year, and his expenses are $1275.75, how much does he save ?

34. Cash Accounts. In keeping a cash account it is customary to write the receipts on the left side (called the *debit* side), and the expenditures on the right side (called the *credit* side), thus:

	19—					19—						
a	March	4	Cash on hand	$125	50	March	5	Groceries	$17	22	*f*
b		6	P. F. Roberts	30			6	Meat bill	6	90	*g*
c		7	R. S. Ball	72	25		7	Balance	203	63	*h*
d				227	75				227	75	*i*
e	March	7	Balance	203	63						

Here *a* shows the balance on hand when this page of the account is begun; *b* and *c* are receipts from Messrs. Roberts and Ball; *d* is the sum of these items on the day that the account is added to see how it stands. On the right side *f* and *g* are bills paid. To find how much is now on hand we subtract the sum of $17.22 and $6.90, or $24.12, from $227.75, and we find that the remainder (in accounts called the *balance*) is $203.63, which is written on the left, at *e*, and on the right, at *h*. To check the work, *f*, *g*, and *h* are added, and the sums at *d* and *i* must agree.

EXERCISE 11

Make out cash accounts and balance them as above:

1. Receipts: April 3, cash on hand, $25.30; April 4, J. B. Fuller, $30; April 7, R. H. Bates, $4.30. Expenses: April 4, groceries, $9.32; April 6, meat, $7.28. Balanced on April 8.

2. Receipts: Feb. 6, cash on hand, $135.60; Feb. 6, R. S. Roberts, $75.50; Feb. 7, M. L. Richards, $87.90; Feb. 8, R. P. Jasper, $42.25. Expenses: Feb. 6, rent, $75; Feb. 7, salaries, $60; Feb. 8, coal, $15.30; Feb. 8, account book, $2.25; Feb. 8, telephone rent, $6.50; Feb. 8, gas bill, $3.75. Balanced on Feb. 9.

Insert dates and items as in Exs. 1 and 2, and balance the following accounts:

3. Receipts, \$125, \$32.60, \$28.90, \$50; expenses, \$35, \$15.30, \$28.40.

4. Receipts, \$16.30, \$15.50, \$10.10, \$3.50; expenses, \$4.80, \$3.45, \$20, \$4.75.

5. Receipts, \$10.30, \$5.25, \$6.32, \$7.20; expenses, \$5.30, \$2.40, \$3.80, \$1.50.

6. Receipts, \$34.30, \$15.30, \$10.75, \$6.32; expenses, \$32.50, \$8.20, \$4.30, \$3.

7. Receipts, \$42.60, \$18.50, \$13.60, \$2.80, \$4; expenses, \$15.40, \$15, \$32.75, \$4.60.

8. Receipts, \$62.30, \$15.40, \$14.75, \$21.32, \$5; expenses, \$35.75, \$10.42, \$29.60, \$28.40.

9. Receipts, \$75.50, \$62.30, \$4.72, \$16, \$25.30; expenses, \$3.25, \$80, \$14.20, \$15, \$3.25.

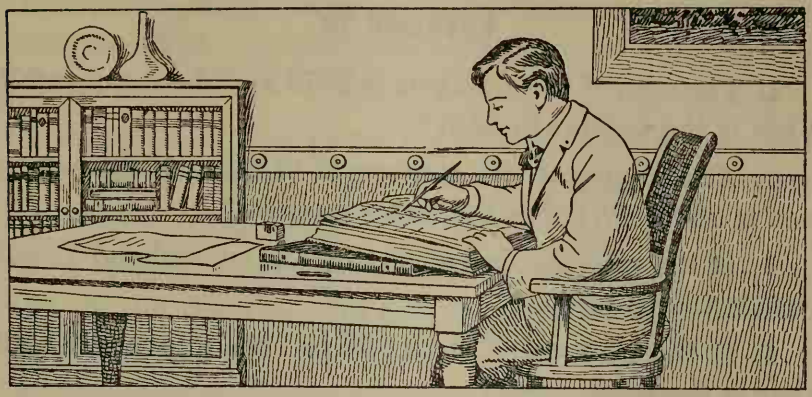
10. Receipts, \$1250.50, \$620, \$300, \$175, \$16.25, \$120.50, \$32.75, \$68.50; expenses, \$600, \$360, \$166.67, \$33.33, \$240.

11. Receipts, \$625.75, \$130.50, \$200, \$75, \$50, \$15.20, \$14.60, \$33.80; expenses, \$430, \$250, \$100, \$32.75, \$28.50, \$30, \$2.12.

12. Receipts, \$128.50, \$32.75, \$68.75, \$43.50, \$28.90; expenses, \$62.30, \$28.50, \$75.50, \$38.40, \$25.25, \$20.30, \$2, \$18.05.

13. Receipts, \$875.80, \$260.50, \$350, \$400, \$80, \$25.50, \$380, \$125; expenses, \$250, \$375, \$480.70, \$200, \$75, \$30.25, \$150.

14. Receipts, \$1025.30, \$535.40, \$287.50, \$126.75, \$324.80, \$125.45, \$135.42, \$12.87; expenses, \$225, \$430.75, \$223.50, \$121.60, \$123.75.



15. This boy is keeping a cash account for himself. He finds that he received $1.00 from his father on Oct. 15, 20 cents for work on Oct. 16, 35 cents for picking apples on Oct. 17, 15 cents for work on Oct. 18, 25 cents for work on Oct. 19, and 50 cents for some chestnuts that he gathered on Oct. 20. He paid 35 cents for a book on Oct. 16, 5 cents for some marbles on Oct. 17, 20 cents for paper and ink on Oct. 18, 40 cents for a knife on Oct. 20, and on Oct. 21 he put the balance that he had on hand in the bank, where he already had $17.50. Make out the cash account and find how much he then had in the bank.

16. For his father he made out a statement of the farm property, which showed the following items: value of the land and buildings, $7250; standing wood and timber, $1600; wagons and machinery, $750; cows and horses, $1975; oats, corn, and wheat on hand, $490; hay and straw on hand, $230; poultry, $35; sewing machine and household furniture, $325; due from creamery for milk furnished, $72.75; due from Mr. Young for hay, $28.50. The father still owed $2250 on the farm, and there was a grocery bill of $12.80 due. Make a statement like a cash account, showing the value of the property after deducting what was owed on the farm and at the grocery.

EXERCISE 12

1. The sum of two numbers is 2872 and the lesser is 929. What is the greater number?

2. The greater of two numbers is 17,214 and their difference is 2685. What is the lesser number?

3. If the minuend is 15,293 and the subtrahend 12,525, what is the remainder?

4. If the subtrahend is 32,658 and the minuend 61,229, what is the remainder?

5. If the minuend is 21,712 and the remainder 2564, what is the subtrahend?

6. If the subtrahend is 16,567 and the remainder 7543, what is the minuend?

7. A merchant sold some goods for $2962.50, and gained $548.75. What did they cost him?

8. What must be added to the sum of $175.50 and $62.75 to make $300?

9. What must be added to the sum of $62.30, $41.75, and $25, to make $500?

10. How much greater is the sum of $75.30 and $21.75, than the difference between these amounts?

11. What is the difference between $725.25 and the sum of $15.50, $17.92, $25.36, $48.75, $19.36, and $42.75?

12. What must be subtracted from $1000 to make the remainder equal to the sum of $175 and $42.50?

13. A man borrowed $1250 and paid back $125 at one time and $960 at another. How much did he still owe?

14. A man having $375.50 in the bank deposited at different times $175, $320, and $15.40. He then drew out $645. How much had he left in the bank?

15. Find the difference between the sum of \$62.30, \$31.70, \$42.65, and \$85, and the sum of \$21.75, \$28.50, \$15.75, and \$125.

16. A man bought a house for \$7250, and after expending \$321.50 for repairs he sold it for \$9125. How much did he gain?

17. A dealer bought three lots of goods for \$175.50, \$1280.60, and \$932.80. He sold them all for \$2875. How much did he gain?

18. In a certain year the number of births in Great Britain is given as 1,057,659 and in France as 827,297. What is the difference?

19. In one year there was mined in this country 4,178,592 ounces of gold, and in the following year 4,565,333 ounces. What was the increase?

20. In one year there was mined in this country 58,938,355 ounces of silver, and in the following year 56,517,900 ounces. What was the decrease?

21. In one year there were 8,027,192 depositors in the savings banks of this country, and in the following year 8,588,811 depositors. What was the increase?

22. In one year the United States produced 2,927,416,000 bushels of corn, and Argentina 195,000,000 bushels. By how much did our product exceed that of Argentina?

23. In one year the income from all the post offices of this country was \$167,932,782, and in the following year \$183,585,005. What was the increase?

24. In a certain year the public debt of France was quoted as \$5,655,134,825 and that of Russia as \$4,038,199,722. How much greater was the debt of France than that of Russia?

25. In one year this country produced 735,260,970 bushels of wheat and 964,904,522 bushels of oats. How many more bushels of oats than of wheat did we raise?

26. In one year the United States sold to other countries $820,863,405 worth of farm products, and the next year $969,457,906 worth. How much was the increase?

27. In a certain year the revenue of the government of Great Britain was $700,666,869 and the expenditure was $683,801,430. How much greater was the revenue than the expenditure?

28. In a certain year Wisconsin manufactured goods to the value of $411,139,681 and New Jersey to the value of $774,369,025. Find the excess of the value of the production of New Jersey over that of Wisconsin.

29. In one year there was issued in the post offices of New York state $42,283,196.46 worth of money orders, and in Pennsylvania $41,090,633.14 worth. How much greater was the New York issue?

30. In the year when the United States had 62,665 post offices Germany had 39,052, and Great Britain and Ireland 23,283. By how many did the United States exceed Germany and Great Britain and Ireland together?

31. In a certain year the cotton crop of Texas was 4,073,000 bales, that of Georgia 1,728,000, and that of Mississippi 1,548,000. How much greater was the cotton crop of Texas than of Georgia and Mississippi combined?

32. In one year the anthracite coal mined in the United States was 78,731,523 tons, and the bituminous coal mined was 308,344,613 tons. In the following year the anthracite coal amounted to 72,209,566 tons, and the bituminous to 341,629,113 tons. What was the decrease in the anthracite and the increase in the bituminous?

CHAPTER IV

MULTIPLICATION OF INTEGERS

35. Multiplication. The process of taking a number as many times as there are units in another number is called *multiplication.*

This means multiplication by an integer. Multiplication by a fraction is considered later.

36. Terms. The number that is multiplied is called the *multiplicand;* the number that shows how many times the multiplicand is taken is called the *multiplier;* the result of the multiplication is called the *product.*

4 multiplicand
3 multiplier
12 product

37. Multiple. The product of two or more integers is called a *multiple* of each.

For example, 15 is a multiple of 3 and 5, and 40 is a multiple of 2, 4, 5 and also of 8, 10, 20.

38. Factors. Integral numbers whose product is a given number are called *factors* of that number; the multiplicand and the multiplier, for example, are factors of the product.

39. Symbol. The symbol of multiplication is an oblique cross, $\times$. It is read *times* or *multiplied by.*

Thus $3 \times 4 = 12$ is read "3 times 4 equals 12."

The multiplier may be written either before or after the multiplicand, but the tendency in this country is to write the multiplier first, as we naturally read it. For example, $3 \times \$4$ is read "3 times \$4," and $\$4 \times 3$ is read "\$4 multiplied by 3."

40. The Multiplication Table. The following table, already learned, must be reviewed so that you can tell instantly any product when the teacher tells you the factors:

$2 \times 1 = 2$	$3 \times 1 = 3$	$4 \times 1 = 4$	$5 \times 1 = 5$
$2 \times 2 = 4$	$3 \times 2 = 6$	$4 \times 2 = 8$	$5 \times 2 = 10$
$2 \times 3 = 6$	$3 \times 3 = 9$	$4 \times 3 = 12$	$5 \times 3 = 15$
$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$	$5 \times 4 = 20$
$2 \times 5 = 10$	$3 \times 5 = 15$	$4 \times 5 = 20$	$5 \times 5 = 25$
$2 \times 6 = 12$	$3 \times 6 = 18$	$4 \times 6 = 24$	$5 \times 6 = 30$
$2 \times 7 = 14$	$3 \times 7 = 21$	$4 \times 7 = 28$	$5 \times 7 = 35$
$2 \times 8 = 16$	$3 \times 8 = 24$	$4 \times 8 = 32$	$5 \times 8 = 40$
$2 \times 9 = 18$	$3 \times 9 = 27$	$4 \times 9 = 36$	$5 \times 9 = 45$
$2 \times 10 = 20$	$3 \times 10 = 30$	$4 \times 10 = 40$	$5 \times 10 = 50$
$6 \times 1 = 6$	$7 \times 1 = 7$	$8 \times 1 = 8$	$9 \times 1 = 9$
$6 \times 2 = 12$	$7 \times 2 = 14$	$8 \times 2 = 16$	$9 \times 2 = 18$
$6 \times 3 = 18$	$7 \times 3 = 21$	$8 \times 3 = 24$	$9 \times 3 = 27$
$6 \times 4 = 24$	$7 \times 4 = 28$	$8 \times 4 = 32$	$9 \times 4 = 36$
$6 \times 5 = 30$	$7 \times 5 = 35$	$8 \times 5 = 40$	$9 \times 5 = 45$
$6 \times 6 = 36$	$7 \times 6 = 42$	$8 \times 6 = 48$	$9 \times 6 = 54$
$6 \times 7 = 42$	$7 \times 7 = 49$	$8 \times 7 = 56$	$9 \times 7 = 63$
$6 \times 8 = 48$	$7 \times 8 = 56$	$8 \times 8 = 64$	$9 \times 8 = 72$
$6 \times 9 = 54$	$7 \times 9 = 63$	$8 \times 9 = 72$	$9 \times 9 = 81$
$6 \times 10 = 60$	$7 \times 10 = 70$	$8 \times 10 = 80$	$9 \times 10 = 90$

It is also useful to know the following:

$11 \times 1 = 11$	$11 \times 4 = 44$	$11 \times 7 = 77$	$11 \times 10 = 110$
$11 \times 2 = 22$	$11 \times 5 = 55$	$11 \times 8 = 88$	$11 \times 11 = 121$
$11 \times 3 = 33$	$11 \times 6 = 66$	$11 \times 9 = 99$	$11 \times 12 = 132$
$12 \times 1 = 12$	$12 \times 4 = 48$	$12 \times 7 = 84$	$12 \times 10 = 120$
$12 \times 2 = 24$	$12 \times 5 = 60$	$12 \times 8 = 96$	$12 \times 11 = 132$
$12 \times 3 = 36$	$12 \times 6 = 72$	$12 \times 9 = 108$	$12 \times 12 = 144$

41. Multiplying by a One-Figure Multiplier. Required the product of 587 by 6.

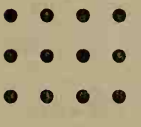
We may multiply each part of 587 by 6 and add the results, as shown in the upper solution. This method is long, so we use it only for explanation, and write the work as in the lower solution. Beginning at the right, we have $6 \times 7 = 42$; we write the 2 in the units' place and reserve the 4 tens to add to the product of the tens. Then 6×8 tens $= 48$ tens, which, with the 4 tens reserved, makes 52 tens, or 5 hundreds and 2 tens. We therefore write the 2 tens in the tens' place, and reserve the 5 hundreds to add to the product of the hundreds. Then 6×5 hundreds $= 30$ hundreds, which, with the 5 hundreds reserved, makes 35 hundreds, or 3 thousands and 5 hundreds, which we write in the thousands' and hundreds' places. The product is therefore 3522.

587 multiplicand
6 multiplier

42 6×7
48 6×8 (tens)
30 6×5 (hundreds)
3522 6×587 product

587
6
3522

42. Order of the Factors. The product is the same whatever the order of the factors.



That is, $3 \times 4 = 4 \times 3$. If we read these dots by rows, we have 3×4 dots; if we read by columns, we have 4×3 dots. But the dots do not change, so $3 \times 4 = 4 \times 3$. This is true for any number of rows and columns.

For this reason we may always use the smaller factor as the multiplier, thus making the work easier.

Thus, instead of multiplying 3 by 478, we would multiply 478 by 3. In the same way, if we wished to find the product of 478 × \$3, we would take the product of 3 × \$478, the answer being the same.

43. Nature of the Factors. Since the multiplier shows how many times the multiplicand is taken, we have the following:

The multiplier is always an abstract number.

The multiplicand may be either abstract or concrete, and it and the product will always be like numbers.

In $3 \times \$4 = \$12$, 3 is abstract and \$4 and \$12 are like numbers.

EXERCISE 13

Find the product of:

1. 2×207.	**21.** 2×6087.	**41.** $2 \times 41{,}203$.
2. 3×409.	**22.** 3×6074.	**42.** $3 \times 26{,}207$.
3. 4×306.	**23.** 4×8096.	**43.** $4 \times 35{,}705$.
4. 5×704.	**24.** 5×7048.	**44.** $5 \times 42{,}609$.
5. 6×508.	**25.** 6×3409.	**45.** $6 \times 30{,}926$.
6. 7×802.	**26.** 7×4706.	**46.** $7 \times 40{,}289$.
7. 8×605.	**27.** 8×6808.	**47.** $8 \times 70{,}443$.
8. 9×909.	**28.** 9×4307.	**48.** $9 \times 80{,}625$.
9. 2×437.	**29.** 2×2763.	**49.** $2 \times 77{,}234$.
10. 3×526.	**30.** 3×4496.	**50.** $3 \times 82{,}629$.
11. 4×823.	**31.** 4×8277.	**51.** $4 \times 47{,}726$.
12. 5×715.	**32.** 5×7879.	**52.** $5 \times 83{,}921$.
13. 6×412.	**33.** 6×9236.	**53.** $6 \times 72{,}277$.
14. 7×813.	**34.** 7×8943.	**54.** $7 \times 63{,}429$.
15. 8×612.	**35.** 8×9448.	**55.** $8 \times 48{,}296$.
16. 9×730.	**36.** 9×7367.	**56.** $9 \times 36{,}436$.
17. 2×749.	**37.** 6×9982.	**57.** $6 \times 78{,}879$.
18. 3×684.	**38.** 7×7347.	**58.** $7 \times 89{,}986$.
19. 8×777.	**39.** 8×8888.	**59.** $8 \times 56{,}789$.
20. 7×648.	**40.** 9×9999.	**60.** $9 \times 99{,}999$.

61. What will 48 sheep cost at \$7 each?

62. What will 480 dozen pairs of hose cost at \$5 a dozen?

63. A farmer shipped 9 cases, each containing 360 eggs. How many eggs did he ship?

64. A farmer had 7 acres of onions, and the yield was 562 bushels per acre. How many bushels did he get in all?

44. Multiplying United States Money. The process here is the same as with other numbers. Simply keep the decimal point after the dollars.

$$\begin{array}{r} \$23.05 \\ \times\quad 7 \\ \hline \$161.35 \end{array}$$

Thus, to multiply $23.05 by 7, proceed as here shown. The explanation is the same as in § 41.

EXERCISE 14

Find the product of:

1. 3 × $1.25.	**22.** 3 × $15.25.	**43.** 3 × $121.50.
2. 3 × $2.70.	**23.** 3 × $21.70.	**44.** 3 × $271.20.
3. 4 × $2.65.	**24.** 4 × $12.62.	**45.** 4 × $125.25.
4. 4 × $3.25.	**25.** 4 × $32.15.	**46.** 4 × $306.70.
5. 5 × $2.63.	**26.** 5 × $17.20.	**47.** 5 × $321.51.
6. 5 × $6.31.	**27.** 5 × $23.11.	**48.** 5 × $248.75.
7. 6 × $2.48.	**28.** 6 × $18.90.	**49.** 6 × $328.90.
8. 6 × $3.72.	**29.** 6 × $37.50.	**50.** 6 × $542.75.
9. 7 × $4.28.	**30.** 7 × $41.16.	**51.** 7 × $128.75.
10. 7 × $5.75.	**31.** 7 × $37.75.	**52.** 7 × $305.20.
11. 8 × $3.62.	**32.** 8 × $21.63.	**53.** 8 × $211.75.
12. 8 × $6.41.	**33.** 8 × $82.76.	**54.** 8 × $425.25.
13. 9 × $3.25.	**34.** 9 × $35.50.	**55.** 9 × $372.72.
14. 9 × $7.28.	**35.** 9 × $47.63.	**56.** 9 × $987.68.
15. 452 × $7.	**36.** 2142 × $6.	**57.** 42,341 × $2.
16. 326 × $6.	**37.** 3142 × $5.	**58.** 21,421 × $3.
17. 281 × $5.	**38.** 4126 × $3.	**59.** 26,401 × $4.
18. 429 × $8.	**39.** 2107 × $2.	**60.** 31,620 × $5.
19. 532 × $3.	**40.** 4340 × $7.	**61.** 77,217 × $7.
20. 799 × $9.	**41.** 2989 × $9.	**62.** 60,509 × $9.
21. 867 × $4.	**42.** 5842 × $8.	**63.** 84,748 × $6.

64. At \$3 each, what will 37 kegs of nails cost?

65. At \$3.25 each, what will 7 door casings cost?

66. How much will 2 doors for a house cost at \$6.75 each?

67. How much will 7 windows for a house cost at \$3.70 each?

68. At \$4.50 per thousand, what will 7 thousand shingles cost?

69. At \$17.75 per thousand, what will 8 thousand feet of lumber cost?

70. At \$42.75 per thousand, what will 4 thousand feet of pine siding cost?

71. If a man builds 4 cottages costing \$1728.75 each, what is the total cost?

72. At \$35.50 per thousand, what will 2 thousand feet of matched pine flooring cost?

73. At \$2.28 per thousand square feet, what will 3 thousand square feet of building paper cost?

74. At \$4.48 each for doors, and \$5.62 each for windows, what will 6 doors and 9 windows cost?

75. If a dealer sells 8 building lots at an average price of \$287.50 each, how much does he receive for all?

76. If a real estate dealer pays taxes of \$37.75 on each of 9 lots, what is the total tax he pays?

77. How much will it cost to give some rooms in a house 3 coats of paint, if the cost of each averages \$37.50?

78. If a builder has 7 men employed at \$2.75 a day and 3 at \$3.50 a day, how much does he pay them all a day?

79. If a contractor employs 8 men at \$3.75 a day, 6 men at \$2.90 a day, and 7 men at \$2.65 a day, how much does he pay them all a day?

45. Powers. The result of taking a number any number of times as a factor is called a *power* of the number.

For example, $3 \times 3 = 9$, and 9 is called the *second power*, or *square*, of 3. So $3 \times 3 \times 3$, or 27, is the *third power*, or *cube*, of 3. The successive powers of 10 are 10, 100, 1000, and so on.

46. Multiplying by Powers of 10. To multiply an integer by 10, annex a 0; by 100, two 0's; by 1000, three 0's.

To multiply United States money by 10, move the decimal point 1 place to the right; by 100, 2 places; by 1000, 3 places.

27	\$25.16	360
40	300	400
1080	\$7548	144000

That is, to multiply 27 by 40, multiply by 4 and annex a 0.

Also to multiply \$25.16 by 300, multiply by 3 and move the decimal point 2 places to the right.

And, in general, consider the case of 400×360. Here we see the multiplicand is 36×10, that is, 360; the multiplier is 4×100, that is, 400; therefore the product is 144×1000, that is, 144,000.

Therefore, *when the multiplicand and multiplier are integers ending in zeros, or either ends in zeros, multiply without regard to the zeros, but annex to the product as many zeros as there are at the end of the multiplicand and of the multiplier.*

EXERCISE 15

Find the product of:

1. 30×59.	**8.** $70 \times \$430$.	**15.** 300×427.
2. 40×477.	**9.** $80 \times \$3.55$.	**16.** 500×640.
3. 60×238.	**10.** $90 \times \$4.88$.	**17.** 700×4200.
4. 50×470.	**11.** $70 \times \$2.66$.	**18.** 900×3500.
5. 60×930.	**12.** $80 \times \$4.32$.	**19.** $800 \times \$3.75$.
6. 20×765.	**13.** $70 \times \$7.80$.	**20.** $500 \times \$4.80$.
7. 50×642.	**14.** $90 \times \$5.60$.	**21.** $900 \times 99{,}000$.

47. Multiplying by Larger Multipliers. Required to multiply 4674 by 649.

The multiplier is 600 + 40 + 9. The product is obtained by multiplying by 9, then by 40, and then by 600, and adding the products.

	(1)	(2)
	4674	4674
	649	649
9 times the multiplicand =	42066	42066
40 times the multiplicand =	186960	18696
600 times the multiplicand =	2804400	28044
649 times the multiplicand =	3033426	3033426

Zeros at the right of the partial products, since they do not affect the result of the addition, should be omitted as in (2).

Care must be taken, however, to put the right-hand figure of each partial product directly under the figure of the multiplier used to obtain it.

Find the product of 2007 times $45.87.

The products corresponding to the zeros in the multiplier will be zero, and therefore they need not be written. With this exception the multiplication is performed as in the preceding example. The decimal point is placed before the cents as in the multiplicand.

$45.87
2007
321.09
9174
$92061.09

EXERCISE 16

Find the product of:

1. 21 × 2648.
2. 31 × 4872.
3. 42 × 2634.
4. 62 × 3108.
5. 44 × 2177.
6. 54 × 3623.
7. 47 × 3926.
8. 56 × 7793.
9. 67 × 2987.
10. 74 × 3692.
11. 78 × 8269.
12. 89 × 4871.
13. 93 × 3642.
14. 96 × 2996.
15. 73 × 8989.
16. 48 × 7642.
17. 54 × 8387.
18. 73 × 9874.
19. 36 × 7448.
20. 95 × 6877.
21. 68 × 7482.
22. 73 × 8929.
23. 85 × 4832.
24. 63 × 6309.

25. 84×7470.
26. 96×3998.
27. 74×8647.
28. 98×8766.
29. 99×9999.
30. 211×7361.
31. 311×4872.
32. 401×3664.
33. 503×2763.
34. 630×4876.
35. 580×6924.
36. 770×3871.
37. 860×3926.
38. 722×3322.
39. 421×7232.
40. 512×3211.
41. 643×7806.
42. 428×3605.
43. 826×8207.
44. 407×8643.
45. 502×8926.
46. 708×3971.
47. 637×9286.
48. 843×8792.
49. 695×5764.
50. 783×9871.
51. 847×6476.
52. 972×3502.
53. 866×4009.
54. 707×8763.
55. 5002×7983.
56. 6008×3976.
57. 4003×6648.
58. 5009×9987.
59. $300 \times \$75.35$.
60. $400 \times \$82.25$.
61. $500 \times \$61.32$.
62. $600 \times \$43.35$.
63. $700 \times \$27.73$.
64. $800 \times \$51.36$.
65. $900 \times \$83.42$.
66. $800 \times \$4200$.
67. $700 \times \$7500$.
68. $600 \times \$3200$.
69. $500 \times \$9600$.
70. $283 \times \$48.25$.
71. $342 \times \$29.75$.
72. $276 \times \$49.42$.
73. $396 \times \$64.40$.
74. $428 \times \$75.19$.
75. $532 \times \$81.62$.
76. $437 \times \$29.86$.
77. $522 \times \$62.23$.
78. $729 \times \$72.30$.
79. $8004 \times \$91.76$.
80. $9006 \times \$38.92$.
81. $7052 \times \$47.80$.

82. What will 175 chairs cost at \$2.35 each?

83. What will 630 barrels of salt cost at \$3.08 a barrel?

84. What will it cost to cart 168 loads of produce at \$3.15 a load?

85. What will 475 bushels of clover seed cost at \$3.12 a bushel?

86. What will a wholesale dealer have to pay for 240 yards of velvet at \$1.85 a yard, 480 yards of silk at \$1.38, 520 yards of lace at \$1.62, and 250 yards of broadcloth at \$1.75?

EXERCISE 17

Review Problems

1. How much will 30 cows cost at $35 a head?

2. A mile contains 5280 feet. How many feet in 8 miles?

3. How much will 440 bales of hops weigh at 200 pounds a bale?

4. If a square lot is 66 feet on a side, what is the distance around it?

5. If sound travels 1125 feet in 1 second, how far will it travel in 20 seconds?

6. A farmer bought 7 bushels of grass seed at $3.40 a bushel. How much did it all cost?

7. A city school bought 20 dictionaries at $9.75 each. How much did they all cost?

8. If a salesman receives $25 a week and works 48 weeks a year, what is his income for the year?

9. At $85 a month, how much will the rent of a city apartment amount to in a year?

10. A lady rents a piano for $4.25 a month. How much is the rent for 6 months?

11. At $250 a month, what will a man's salary amount to in a year?

12. If a man grinds 35 bushels of corn of 56 pounds each, how many pounds does he grind?

13. A farmer adds to his property a field of 65 acres, paying $75 an acre for it. How much does the field cost?

14. A dealer sells 15 automobiles at $925 each. How much does he receive for them all?

15. If 38 cows average 408 pounds of milk apiece a month, how many pounds do the cows produce in a month?

16. There being 12 inches in 1 foot, how many inches are there in 175 feet?

17. There being 5280 feet in 1 mile, how many feet are there in 24 miles?

18. There being 144 square inches in 1 square foot, how many square inches are there in 93 square feet?

19. There being 128 cubic feet in a cord, how many cubic feet are there in 152 cords of wood?

20. If a farm averages 28 bushels of buckwheat to the acre, how many bushels will 175 acres produce?

21. If a farm averages 18 bushels of winter rye to the acre, how many bushels will 136 acres produce?

22. A manufacturer sells 175 sets of harness at $24 a set. How much does he receive for them all?

23. There being 640 acres in 1 square mile, how many acres are there in 135 square miles?

24. If a farm averages 2256 pounds of hay to the acre, how much hay will 275 acres produce?

25. A cubic foot of hard coal weighing 84 pounds, how much will 246 cubic feet of hard coal weigh?

26. A cubic foot of ice weighing 58 pounds, how much will 1070 cubic feet of ice weigh?

27. Allowing 5 ounces to a cubic inch, how much will a cubic foot (1728 cubic inches) of copper weigh?

28. If a man averages 2265 steps in walking a mile, how many steps will he take in walking 23 miles?

29. A barrel of flour weighing 196 pounds, how much will 144 barrels weigh?

30. Find the cost of 42 head of cattle at $37.50 each.

31. If a farmer buys 75 acres of land at $62.25 an acre, what is the total cost?

32. How much will 65 horses cost at $82.75 each?

33. At $5.75 a ton, how much will 356 tons of coal cost?

34. A book dealer sells 206 copies of a certain book at $1.25 each. How much does he receive for them all?

35. At $2.98 a ton at the mine, how much will 478 tons of iron cost?

36. When sound travels 1125 feet in a second, how far will it travel in 2 minutes?

37. If a cubic foot of cast iron weighs 447 pounds, how much will 12 posts weigh, each containing 3 cubic feet?

38. If a cubic foot of wrought iron weighs 486 pounds, how much will 17 beams of 2 cubic feet each weigh?

39. If a cubic foot of ice weighs 58 pounds, what will 32 cakes weigh, each containing 3 cubic feet?

40. The earth moves about the sun 101,090 feet in a second. How many feet does it move in a minute?

41. If a steam engine uses 2 tons of coal a day, how many tons will it consume in January if it runs every day?

42. If a man earns $3.50 a day, and works 6 days in the week for 48 weeks, how much does he earn in a year?

43. A merchant bought 7 pieces of cloth of 33 yards each and paid $1.60 a yard for it. What was the total cost?

44. A grocer sold 4 boxes of oranges, each box containing 196 oranges, at 4 cents apiece. How much did he receive for all?

45. Which will cost the more, 48 cows at $37.50 each, or 22 horses at $88 each? How much more?

46. If electricity travels on a wire at the rate of 288,000 miles per second, how long would a wire have to be in order that an electric current would travel from one end to the other in one minute?

CHAPTER V

DIVISION OF INTEGERS

48. Nature of Division. In multiplication two factors are given to find the product. In division the product and one of two factors are given to find the other factor.

That is, in the case of $2 \times \$4 = \$8$, we have given the factors 2 and \$4, from which to find the product \$8.

In division we have given the product \$8 and the factor 2, to find the other factor \$4; or the factor \$4, to find the other factor 2.

49. Division. If the product and one of two factors are given, the process of finding the other factor is called *division.*

50. Symbol. The symbol for division is $\div$. It is read *divided by.*

For example, $15 \div 5 = 3$. This is also indicated by another symbol, thus: $\frac{15}{5} = 3$. That is, $15 \div 5$ and $\frac{15}{5}$ have the same meaning.

51. Dividend. The given product, that is, the number to be divided, is called the *dividend.*

52. Divisor. The given factor, that is, the number by which we divide, is called the *divisor.*

53. Quotient. The required factor, that is, the number found by division, is called the *quotient.*

54. Remainder. Often a divisor does not exactly divide the dividend. Then the part of the dividend that is left is called the *remainder.*

Thus $14 \div 3 = 4$ with remainder 2. The quotient is usually written $4\frac{2}{3}$, meaning 4 with 2 still to be divided into 3 equal parts.

55. Two Kinds of Division. Since $2 \times \$4 = \$8$, therefore $\$8 \div \$4 = 2$, and $\$8 \div 2 = \$4$. That is,

If the dividend and the divisor are concrete, they must be like numbers, and the quotient is abstract.

For example, $\$8 \div \$4 = 2$. This is sometimes called *measuring* because the \$8 is measured by the \$4.

If the dividend is concrete and the divisor abstract, the quotient is like the dividend.

For example, $\$8 \div 2 = \$4$. This is sometimes called *partition*, because the \$8 is separated into 2 equal parts.

56. Check. If there is no remainder, the dividend is the product of the quotient and the divisor. To check the work in division, therefore, we multiply the quotient by the divisor; this product added to the remainder, if any, should equal the dividend.

For example, $45 \div 5 = 9$; then $5 \times 9 = 45$. Again $48 \div 5 = 9$ with remainder 3; then $5 \times 9 = 45$, and $45 + 3 = 48$.

57. Short Division. When the divisor is so small that the work may be done mentally the process is called *short division.*

(1) Required to divide 342 by 3.

$3)\overline{300 + 30 + 12}$
$100 + 10 + 4 = 114$

If 342 is separated as shown, we can divide each part by 3. This is what we do mentally below. We write the numbers as shown; then 3 is contained in 3 once (that is, 100 times in 300); 3 is contained in 4 once with remainder 1 (that is, 10 times in 40 with remainder 10); this 1 is 10 units, and with the 2 makes 12. Then 3 is contained in 12 four times. Therefore the quotient is 114. We think: "3 in 3, 1; in 4, 1; in 12, 4."

$3)\overline{342}$
114

(2) Required to divide 42,364 by 7.

$7)\overline{42364}$
6052

Here 7 is not contained in 4, but 7 is contained in 42 six times. Then 7 is not contained in 3, so we write a 0 beneath the 3. Then 7 is contained in 36 five times with remainder 1, and in 14 twice. Therefore the quotient is 6052.

(3) Required to divide 35,282 by 9.

$$9\overline{)35282}\quad 3920$$

Here 9 is contained in 35 three times with remainder 8; in 82 nine times with remainder 1; in 18 twice; in 2 no times with remainder 2. Therefore the quotient is 3920 with remainder 2, and this is usually written $3920\frac{2}{9}$.

(4) Required to divide \$27.53 by 4.

$$4\overline{)\$27.53}\quad \$6.88$$

Dividing as before, we have $\$27 \div 4 = \$6$, with \$3 remaining; $\$3.50 \div 4 = \$0.8$ with \$0.30 remaining; $\$0.33 \div 4 = \$0.08$, with \$0.01 remaining. Therefore the quotient is $\$6.88\frac{1}{4}$.

EXERCISE 18

Find the quotient of:

1. $232 \div 2$.
2. $345 \div 3$.
3. $416 \div 4$.
4. $325 \div 5$.
5. $738 \div 6$.
6. $924 \div 7$
7. $592 \div 8$.
8. $837 \div 9$.
9. $3242 \div 2$.
10. $4122 \div 3$.
11. $6004 \div 4$.
12. $3015 \div 5$.
13. $4032 \div 6$.
14. $3073 \div 7$.
15. $7368 \div 8$.
16. $8019 \div 9$.
17. $5555 \div 2$.
18. $6014 \div 3$.
19. $4213 \div 4$.
20. $2893 \div 5$.
21. $6271 \div 6$.
22. $7356 \div 7$.
23. $9123 \div 8$.
24. $8077 \div 9$.
25. $\$47.24 \div 2$.
26. $\$33.15 \div 3$.
27. $\$28.72 \div 4$.
28. $\$32.75 \div 5$.
29. $\$21.30 \div 6$.
30. $\$29.47 \div 7$.
31. $\$65.68 \div 8$.
32. $\$83.61 \div 9$.
33. $81,004 \div 4$.
34. $29,005 \div 5$.
35. $31,266 \div 6$.
36. $40,040 \div 7$.
37. $70,328 \div 8$.
38. $32,013 \div 9$.
39. $31,201 \div 2$.
40. $34,078 \div 3$.
41. $26,231 \div 4$.
42. $70,023 \div 5$.
43. $80,335 \div 6$.
44. $29,321 \div 7$.
45. $42,345 \div 8$.
46. $76,209 \div 9$.
47. $\$35.21 \div 2$.
48. $\$72.43 \div 3$.
49. $\$61.23 \div 4$.
50. $\$72.36 \div 5$.
51. $\$82.01 \div 6$.
52. $\$42.03 \div 7$.
53. $\$65.00 \div 8$.
54. $\$93.65 \div 9$.

EXERCISE 19

1. If 3 head of cattle cost $117, how much will 1 head cost?

2. It is 108 feet around a square. What is the length of each side?

3. There being 7 days in a week, how many weeks are there in 364 days?

4. There being 3 feet in a yard, how many yards are there in 432 feet?

5. There being 4 quarts in a gallon, how many gallons are there in 236 quarts?

6. It is 132 feet around a triangle whose sides are all equal. What is the length of each side?

7. At $9 a dozen, how many dozen cups and saucers can be bought for $1116?

8. At $4 a dozen, how many dozen handkerchiefs can be bought for $1152?

9. A dealer pays $1224 for 9 wagons. What is the average price?

10. A wholesale grocer pays $1728 for a shipment of chocolate at $3 per dozen cans. How many dozen cans does he buy?

11. A wholesale grocer pays $4320 for some cocoa at $6 per dozen cans. How many dozen cans does he buy?

12. A hotel pays $13.20 for 8 dozen packages of breakfast food. How much does it pay per dozen?

13. If a grocer pays $12.15 for 9 dozen packages of breakfast cereal, how much does he pay per dozen?

14. If he pays $19.84 for 8 dozen tins of biscuit, how much does he pay per dozen?

58. Dividing by 10 and its Powers. To divide by 10 is merely to find how many tens there are in the number, and this we can tell at a glance.

Thus in 1230 there are 123 tens and no units. In 2373 there are 237 tens and 3 units remainder; that is, $2373 \div 10 = 237\frac{3}{10}$.

Therefore, *to divide an integer by 10 cut off the right-hand figure of the dividend. The result is the quotient, and the right-hand figure expresses the remainder.*

To divide an integer by any power of 10 cut off as many figures from the right of the dividend as there are zeros at the right of the divisor. The result is the quotient, and the right-hand figures express the remainder.

Thus $12{,}400 \div 100 = 124$; $43{,}203 \div 100 = 432\frac{3}{100}$; $624{,}000 \div 1000 = 624$; $736{,}117 \div 1000 = 736\frac{117}{1000}$.

59. Dividing by Multiples of Powers of 10. In this case we divide by the power of 10, as in § 58, and then by the number indicating the multiple.

$$\begin{array}{r} 2\cancel{0}\cancel{0}\,)\,428\cancel{0}\cancel{0} \\ \hline 214 \end{array}$$

$$\begin{array}{r} 3\cancel{0}\cancel{0}\,)\,274\cancel{0}\cancel{7} \\ \hline 91\frac{107}{300} \end{array}$$

For example, to divide 42,800 by 200 we cut off two zeros from divisor and dividend and then divide by 2.

So to divide 27,407 by 300 we cut off two zeros from the divisor and two figures from the right of the dividend. We then divide by 3, and the quotient is 91, with remainder 1 in the hundreds' place, and to this we join the 07 cut off, obtaining the remainder 107.

EXERCISE 20

Find the quotient and the remainder (if any):

1. $340 \div 10$.
2. $720 \div 20$.
3. $540 \div 30$.
4. $640 \div 40$.
5. $750 \div 50$.
6. $840 \div 60$.
7. $840 \div 70$.
8. $960 \div 80$.
9. $360 \div 90$.
10. $720 \div 90$.
11. $273 \div 10$.
12. $423 \div 20$.

Find the quotient and the remainder (if any):

13. $325 \div 30$.	**21.** $20{,}300 \div 100$.	**29.** $98{,}207 \div 900$.
14. $275 \div 40$.	**22.** $27{,}400 \div 200$.	**30.** $126{,}000 \div 3000$.
15. $236 \div 50$.	**23.** $64{,}200 \div 300$.	**31.** $328{,}000 \div 4000$.
16. $327 \div 70$.	**24.** $72{,}400 \div 400$.	**32.** $465{,}000 \div 5000$.
17. $542 \div 70$.	**25.** $31{,}501 \div 500$.	**33.** $564{,}700 \div 6000$.
18. $391 \div 80$.	**26.** $27{,}403 \div 600$.	**34.** $429{,}820 \div 7000$.
19. $278 \div 90$.	**27.** $59{,}721 \div 700$.	**35.** $742{,}623 \div 8000$.
20. $777 \div 90$.	**28.** $42{,}348 \div 800$.	**36.** $857{,}542 \div 9000$.

37. At \$90 each, how many horses can I buy for \$2160?

38. There being 60 minutes in an hour, how many hours are there in 1020 minutes?

39. There being 60 seconds in a minute, how many minutes are there in 1620 seconds?

40. At \$30 each, how many harnesses must a manufacturer sell to receive \$1110?

41. A farmer pays \$6600 for some land at \$40 an acre. How many acres does he buy?

42. A dealer sells some village lots for \$10,200, at \$600 a lot. How many lots does he sell?

43. An investor has \$87,750 with which to buy some bonds at \$500 each. What is the greatest number he can buy, and how much will he have left?

44. An automobile dealer has \$35,100 to invest in automobiles at \$2000 each. What is the greatest number he can buy for this sum, and how much will he have left?

45. A ton is 2000 pounds. If a coal pocket contains 583,260 pounds of coal, how many tons does it contain, and how many pounds are left over?

EXERCISE 21

Problems of the Farm

1. A farmer sowed 60 acres of land, using 780 quarts of timothy seed. How much seed did he use per acre?

2. He also sowed 70 acres, using 1050 quarts of timothy and clover mixed. How much seed did he use per acre?

3. Another farmer sowed 90 acres, using 2520 quarts of timothy and redtop mixed equally. How many quarts of each kind did he use per acre?

4. A Maryland farmer experimented with Italian rye grass, using 3250 pounds to sow 50 acres. How many pounds did he use per acre?

5. A farmer found that 30 sheep ate 11,550 pounds of dry fodder in a year. What was the average amount per sheep?

6. He found that 20 horses consumed 162,400 pounds of hay and grain in a year. What was the average amount per horse?

7. A dealer found that 60 bushels of white clover seed weighed 3780 pounds. How much did it weigh per bushel?

8. He told a customer 1080 pounds would seed 90 acres. How much is that per acre? If he mixed other seed with the clover so as to use this amount to cover 4 acres, how much clover seed would he use per acre?

9. A farmer found that it took 980 pounds of red clover for 70 acres, and 1200 pounds of crimson clover for 80 acres. What was the amount of each used per acre?

10. How many pounds of Paris green will be needed to spray 300 trees, allowing 5 gallons to a tree, if every 150 gallons contains 1 pound of Paris green?

60. Long Division. When the divisor has two or more figures, except in such cases as are given in § 58 and § 59, it is customary to use a process known as *long division.*

We shall first consider the simplest case, that in which the units' figure is 1.

Required to divide 7182 by 21.

If all the work should be written, it would appear as in the first of these forms, but for practical purposes the second is used.

(1)

```
  342
21)7182
  6300 = 300 times the divisor
  882
  840 =  40 times the divisor
  42
   42 =   2 times the divisor
```

(2)

```
  342
21)7182
  63
  88
  84
  42
  42
```

First we will consider (1), the more complete form. We see that 21 is evidently not contained in 7, so there are no thousands in the quotient. The divisor is contained 3 times in 71, and since the 71 is hundreds, the 3 is written above, over the hundreds' place. Then $300 \times 21 = 6300$, and there remains 882 still to be divided; 21 is contained in 88 four times, and since the 88 is tens, the 4 of the quotient is written over the tens' place. Then $40 \times 21 = 840$, and there remains 42 to be divided; 21 is contained in 42 twice, and 2 is written over the units' place. The quotient is therefore 342. It will be seen that 6300, 840, and 42 make up 342×21, as should be the case.

Consider now the second form, which is always used in practice. Since at any time in this example we need only two figures of the dividend or two figures of such a partial dividend as 882, we may omit the zeros in 6300 and 840, and use only the 88 in the remainder 882, thus making the work shorter.

We may check our result by multiplying 342 by 21. Then $21 \times 342 = 7182$, the dividend.

The pupil should check his work in enough examples to become thoroughly familiar with the process.

Required to divide 6447 by 21.

$$\begin{array}{r} 307 \\ 21\overline{)6447} \\ \underline{63} \\ 147 \\ \underline{147} \end{array}$$

Here we have $64 \div 21 = 3$, and since the 64 is hundreds, the 3 is written over the hundreds' place. Then $3 \times 21 = 63$, and when 63 is subtracted, there is a remainder 1. Bringing down one figure as before, we have 14 tens to be divided by 21. Since 21 is not contained in 14, we write 0 in the tens' place to show that there are no tens, and bring down the next figure 7. Then $147 \div 21 = 7$, and $7 \times 21 = 147$ exactly, so there is no remainder.

EXERCISE 22

Find the quotient of:

1. $352 \div 11$.
2. $451 \div 11$.
3. $682 \div 11$.
4. $968 \div 11$.
5. $858 \div 11$.
6. $957 \div 11$.
7. $737 \div 11$.
8. $913 \div 11$.
9. $252 \div 21$.
10. $294 \div 21$.
11. $441 \div 21$.
12. $672 \div 21$.
13. $777 \div 21$.
14. $819 \div 21$.
15. $987 \div 21$.
16. $868 \div 31$.
17. $779 \div 41$.
18. $765 \div 51$.
19. $1037 \div 61$.
20. $1312 \div 41$.
21. $1562 \div 71$.
22. $2673 \div 81$.
23. $3367 \div 91$.
24. $2331 \div 21$.
25. $2583 \div 21$.
26. $3472 \div 31$.
27. $2121 \div 21$.
28. $6789 \div 31$.
29. $8547 \div 21$.
30. $3317 \div 31$.
31. $10{,}731 \div 21$.
32. $14{,}322 \div 31$.
33. $16{,}461 \div 31$.
34. $25{,}297 \div 41$.
35. $18{,}879 \div 31$.
36. $28{,}741 \div 41$.
37. $40{,}953 \div 51$.
38. $49{,}349 \div 61$.
39. $18{,}696 \div 41$.
40. $52{,}582 \div 61$.
41. $62{,}267 \div 71$.
42. $63{,}048 \div 71$.
43. $71{,}280 \div 81$.
44. $90{,}090 \div 91$.
45. $90{,}909 \div 91$.
46. $50{,}337 \div 51$.
47. $50{,}898 \div 51$.
48. $62{,}937 \div 81$.
49. $71{,}928 \div 81$.
50. $60{,}939 \div 61$.
51. $34{,}587 \div 61$.
52. $73{,}629 \div 81$.
53. $65{,}448 \div 81$.
54. $73{,}619 \div 91$.

61. Long Division Continued. We have considered the simple case in which the units' figure is 1, and we shall now consider the more general case of a two-figure divisor.

Required to divide 19,224 by 54.

The divisor is not contained in 1 or in 19, so there are no ten-thousands and no thousands in the quotient. But since $3 \times 54 = 162$, the divisor is contained 3 times in 192 with remainder 30. Since this is hundreds, the quotient figure 3 is written over the hundreds' place. Bringing down the next figure of the dividend, 2, we have 302 tens to be divided. Since $5 \times 54 = 270$, the divisor is contained 5 times in 302, with remainder 32. Since this is tens, the quotient figure 5 is written over the tens' place. Bringing down the next figure of the dividend, 4, we have 324 still to be divided. Since 324 is exactly 6×54, the units' figure is 6 and there is no remainder. Hence the quotient is 356.

```
      356
54 ) 19224
     162
     ---
      302
      270
      ---
       324
       324
       ---
```

62. Product Tables. It is sometimes helpful for beginners to prepare a table of products of the divisor, as follows:

$2 \times 54 = 108$	$4 \times 54 = 216$	$6 \times 54 = 324$	$8 \times 54 = 432$
$3 \times 54 = 162$	$5 \times 54 = 270$	$7 \times 54 = 378$	$9 \times 54 = 486$

With such a table made out in advance, we can tell at once, without making several trials, what the next figure of the quotient will be.

EXERCISE 23

Find the quotient of:

1. $3796 \div 52$.	8. $5929 \div 77$.	15. $98{,}901 \div 99$.
2. $8464 \div 92$.	9. $2451 \div 19$.	16. $41{,}831 \div 59$.
3. $6510 \div 62$.	10. $8514 \div 66$.	17. $55{,}821 \div 69$.
4. $4599 \div 73$.	11. $22{,}311 \div 67$.	18. $54{,}439 \div 77$.
5. $8772 \div 43$.	12. $31{,}950 \div 75$.	19. $118{,}216 \div 56$.
6. $2275 \div 35$.	13. $63{,}204 \div 69$.	20. $301{,}184 \div 64$.
7. $2736 \div 57$.	14. $74{,}994 \div 87$.	21. $891{,}891 \div 99$.

22. At 48 pounds to the bushel, how many bushels in 36,000 pounds of buckwheat?

23. At 32 pounds to the bushel, how many bushels in 21,184 pounds of oats?

24. At 56 pounds to the bushel, how many bushels in 13,160 pounds of shelled corn?

25. At 55 pounds to the bushel, how many bushels in 42,625 pounds of sweet potatoes?

26. At 26 pounds to the bushel, how many bushels in 34,476 pounds of dried apples?

27. At 44 pounds to the bushel, how many bushels in 66,220 pounds of blue-grass seed?

28. At 48 pounds to the bushel, how many bushels in 96,096 pounds of corn meal?

29. At 45 pounds to the bushel, how many bushels in 93,375 pounds of timothy seed?

30. At 57 pounds to the bushel, how many bushels in 74,784 pounds of onions?

31. At 34 pounds to the bushel, how many bushels in 93,636 pounds of malt?

32. At 46 pounds to the bushel, how many bushels in 90,850 pounds of castor beans?

33. At 55 pounds to the bushel, how many bushels in 92,565 pounds of turnips?

34. At 62 pounds to the bushel (the weight allowed in some parts of the country), how many bushels in 181,598 pounds of fine salt?

35. A dealer pays $2356 for 62 cows. He also buys 31 horses, and his bill for the cows and horses together is $5301. How much apiece does he pay for the cows and for the horses?

36. If a steamship sails 6336 miles in 18 days, what is the rate per day?

37. How many 25-pound packages can be made from 18,000 pounds of paper?

38. How many 49-pound sacks can be filled from 150,675 pounds of flour?

39. The product of two numbers is 98,441, and one of them is 49. What is the other?

40. A man paid $28,800 for a tract of land at $45 an acre. How many acres did he buy?

41. A man paid $19,125 for a farm, the price being $85 an acre. How many acres did he buy?

42. A certain book averages 29 lines to a page and contains 10,034 lines in all. How many pages are there?

43. If a desk company sells $20,184 worth of desks in a year at $29 each, how many does it sell?

44. If a factory sells $18,648 worth of harnesses in a season at $37 a set, how many sets does it sell?

45. A canning factory ships 30,000 cans of fruit in boxes containing 4 dozen cans each. How many boxes does it require?

46. There being 27 cubic feet in a cubic yard, how many loads (cubic yards) of earth must be removed to excavate 108,675 cubic feet?

47. One of the routes around the world is 18,424 miles long. If a man makes the trip in 56 days, what is the average distance per day?

48. If the circumference of a wagon wheel is 15 feet, and there are 5280 feet in a mile, how many times will the wheel turn in going 9 miles? How many times in going 25 miles?

63. Long Division Continued. We may now consider divisors of three or more figures, and cases of division in which there are remainders.

Required to divide 341,597 by 248.

The divisor is not contained in 3 or in 34, so there are no hundred-thousands or ten-thousands in the quotient. But it is contained in 341 once with remainder 93. Since this is thousands, the quotient figure, 1, is written over the thousands' place. Bringing down the next figure of the dividend, 5, we have 935 hundreds to be divided. It might seem as if the next figure would be 4, and here a product table (§ 62) would be helpful; but $4 \times 248 = 992$, and this is larger than 935. Trying 3 for the next figure, we have $3 \times 248 = 744$, and there is a remainder 191. Since we have divided hundreds by 248, the 3 is hundreds and is written over that place. Bringing down the next figure, we find that $1919 \div 248 = 7$, with 183 remaining. Since the 7 is tens, we write it over the tens' place. Then, bringing down the last figure, we have $1837 \div 248 = 7$, with 101 remaining. Therefore the quotient is 1377, and the remainder is 101. The complete quotient may be written $1377\frac{101}{248}$.

```
  1377
248)341597
   248
   935
   744
  1919
  1736
  1837
  1736
   101
```

We may check the work by adding 101 to the product of 248×1377, the result being 341,597, the dividend.

64. General Directions. It may assist us if we now state the general directions we have found for long division:

Write the divisor to the left of the dividend.

Take for the first partial dividend the least number of left-hand figures that contains the divisor, and write the quotient over the right-hand figure of this partial dividend.

Multiply the divisor by this quotient, and place the product under the partial dividend used.

Subtract this product, and to the remainder annex the next figure of the dividend for a new partial dividend.

Divide as before, and continue this process until all the figures of the dividend have been used.

EXERCISE 24

Find the quotient of:

1. 25,284 ÷ 301.
2. 51,742 ÷ 631.
3. 47,348 ÷ 623.
4. 153,186 ÷ 211.
5. 475,950 ÷ 501.
6. 972,360 ÷ 111.
7. 161,463 ÷ 321.
8. 324,360 ÷ 306.
9. 140,850 ÷ 225.
10. 183,750 ÷ 175.
11. 148,555 ÷ 407.
12. 234,901 ÷ 503.
13. 208,575 ÷ 309.
14. 870,672 ÷ 204.
15. 367,848 ÷ 786.
16. 102,465 ÷ 495.
17. 472,932 ÷ 604.
18. 349,206 ÷ 407.
19. 117,440 ÷ 320.
20. 419,640 ÷ 538.
21. 287,712 ÷ 216.
22. 615,000 ÷ 328.
23. 685,542 ÷ 286.
24. 997,108 ÷ 478.
25. 965,496 ÷ 9852.
26. 692,356 ÷ 8764.
27. 7684 ÷ 69.
28. 8522 ÷ 76.
29. 8562 ÷ 23.
30. 20,452 ÷ 56.
31. 18,113 ÷ 64.
32. 14,797 ÷ 57.
33. 19,140 ÷ 46.
34. 26,523 ÷ 47.
35. 27,559 ÷ 59.
36. 47,078 ÷ 69.
37. 50,602 ÷ 82.
38. 101,160 ÷ 232.
39. 110,370 ÷ 124.
40. 252,150 ÷ 412.
41. 120,830 ÷ 653.
42. 260,380 ÷ 422.
43. 192,892 ÷ 232.
44. 211,428 ÷ 5873.
45. 287,712 ÷ 5328.
46. 313,205 ÷ 8465.
47. 411,075 ÷ 5481.
48. 616,693 ÷ 8009.
49. 965,496 ÷ 9852.
50. 1,599,560 ÷ 2836.
51. 2,402,728 ÷ 6257.
52. 5,191,776 ÷ 5947.

53. If a man receives a salary of $1728 a year, how much does he receive a month?

54. If a man receives a salary of $1092 for the 52 weeks of a year, how much does he receive a week?

55. If a man pays $8569 for 209 acres of land, how much does it cost per acre?

56. If a book jobber pays $3402 for 42 sets of an encyclopedia, how much does he pay per set?

57. If a stock dealer buys some cattle for $71,961, paying $51 a head, how many head does he buy?

58. If a jobber sells during the year $82,350 worth of shoes at $61 a case, how many cases does he sell?

59. If a dealer sells $142,560 worth of linseed oil during a year at $22 a barrel, how many barrels does he sell?

60. If a factory sells 3684 ready-made suits of a certain grade for $92,100, what is the price per suit?

61. If the product of two numbers is 2,137,512, and one of them is 372, what is the other number?

62. If a state pays $2,325,798 for constructing 471 miles of model road, what is the average cost per mile?

63. If it costs $5,236,602 to construct 287 miles of railroad, what is the average cost per mile?

64. What is the smallest number that must be subtracted from 615,476 so that the result divided by 1875 gives a quotient with no remainder?

65. If a company employs 8009 men on the same schedule of wages, and the pay roll for these men is $6,247,020 a year, what is the annual wages of each?

66. If a company employs 250 men on the same schedule of wages, and the pay roll for these men is $270,000 a year, what is the monthly wages of each?

65. Long Division of United States Money. Division of United States money does not involve any new principle. We divide as with other numbers, placing the decimal point in the quotient above that in the dividend.

Required to divide $21,375.12 by 372.

$$
\begin{array}{r}
\$57.46 \\
372\overline{)\$21375.12} \\
1860 \\
\hline
2775 \\
2604 \\
\hline
1711 \\
1488 \\
\hline
2232 \\
2232 \\
\hline
\end{array}
$$

Since $2, $21, and $213 cannot be divided by 372 with an integer for a quotient, we divide $2137 by 372, and the quotient is $5, with remainder $277. Since the $2137 is in the tens' place of dollars, we write the $5 over that place of the dividend.

The rest of the division is as in cases already studied. The decimal point in the quotient is placed over that in the dividend, and the quotient is $57.46.

EXERCISE 25

Find the quotient of:

1. $972.36 ÷ 111.
2. $374.12 ÷ 199.
3. $1467.50 ÷ 50.
4. $6882.92 ÷ 11.
5. $12,018.30 ÷ 21.
6. $96,899.23 ÷ 23.
7. $36,147.06 ÷ 53.
8. $33,328.80 ÷ 54.
9. $33,747.20 ÷ 64.
10. $22,310.64 ÷ 72.
11. $30,796.80 ÷ 64.
12. $49,060.08 ÷ 72.
13. $34,689.75 ÷ 69.
14. $39,832.56 ÷ 81.
15. $15,391.25 ÷ 125.
16. $14,225.40 ÷ 420.
17. $734,986.12 ÷ 836.
18. $501,738.80 ÷ 538.
19. $102,465.00 ÷ 2070.
20. $692,356.00 ÷ 8764.
21. $616,693.00 ÷ 8009.
22. $388,591.00 ÷ 9037.
23. $421,203.20 ÷ 5983.
24. $5,191,731.00 ÷ 5947.
25. $5,236,602.00 ÷ 9123.
26. $3,851,640.00 ÷ 9876.

27. A man distributed the sum of $17,975.10 equally among fifteen relatives. How much did each receive?

28. A man paid $52.08 for 24 dozen cans of peas. How much did they cost him a dozen?

29. A man paid $150.72 for 96 dozen cans of tomatoes. How much did they cost him a dozen?

30. A grocer bought 94 dozen quart jars of preserved figs for $888.30. How much did they cost him a dozen?

31. A dealer paid $517.50 for 250 dozen packages of shredded cocoanut. How much did he pay a dozen?

32. If $2083.20 is divided equally among 672 men, how much does each receive?

33. If $2652.30 is divided equally among 842 men, how much does each receive?

34. A merchant pays $553.80 for 195 yards of silk. How much does he pay a yard?

35. A merchant pays $865.08 for 324 yards of velvet. How much does he pay a yard?

36. A wholesale grocer sells 175 dozen quart bottles of vinegar for $862.75. How much does he receive a dozen?

37. If $1991 a day is paid to 724 men who have the same wages in a certain shop, how much does each receive?

38. If 624 men, working for the same wages in a certain shop, receive $1996.80 a day, how much does each receive?

39. If 485 men, working for the same wages in a certain shop, receive $1503.50 a day, how much does each receive?

40. A wholesale dealer purchased 144 dozen pineapple cheeses for $1451.52. How much did he pay a dozen?

41. A shop has 664 men on its pay roll, receiving the same wages. For 6 days this pay roll amounts to $9960. How much does each man receive a day?

EXERCISE 26

Review Problems

1. This girl bought for her mother 7 yards of silk at $1.35 a yard. How much did it cost?

2. Her mother sent her to buy 5 yards of serge at $1.75 a yard. How much did it cost? How much change should she receive from a $10 bill?

3. If the girl's purchases at another time consist of toweling at 80 cents, muslin at $1, needles at 45 cents, thread at 16 cents, beads at 25 cents, and braid at 24 cents, how much change should she receive from $5?

4. She saw a lady buy 13 yards of broadcloth and pay $45.50 for it. How much did it cost a yard?

5. If the merchant charges $34.65 for 21 yards of velvet, how much does he charge a yard?

6. If the merchant sells tweeds at $3.25 a yard, how much will he receive for 18 yards? How much for 37 yards?

7. If a lady buys 27 yards of silk at $1.45, 3 yards of velvet at $1.75, and 22 yards of serge at $1.60, how much does she pay in all?

8. A man pays $2.52 for 12 pounds of meat. How much does it cost a pound?

9. How many inches are there in 1 foot? How many feet are there in 432 inches?

10. If the average length of a man's step is 27 inches, how many steps will he take in going 837 inches?

11. If the average length of a man's step is 27 inches, how many inches will he go in taking 275 steps?

12. If a man earns $2.65 in each of the 27 working days of a certain month, how much does he earn in all?

13. A dealer sold 75 barrels of sweet potatoes at $2.25 a barrel. How much did he receive?

14. A dealer sold 64 barrels of sweet potatoes at $1.80 a barrel, and 36 barrels at $2.40 a barrel. How much did he receive for them all?

15. If 3 farmers, having equal amounts of oats to thresh, pay $38.10 for the use of a threshing machine, how much should each pay?

16. If the altitude of Mt. Everest is 29,002 feet, and that of Mt. McKinley 20,464 feet, how much higher is Mt. Everest than Mt. McKinley?

17. If the total railway mileage in the United States was 193,346 miles at one time, and 229,951 miles a few years later, how much was the increase in mileage?

18. If a school buys 3 dozen blackboard pointers at $1.15 a dozen, a dictionary holder at $6.45, a dictionary at $6.75, and a teacher's desk at $13.50, what is the total cost?

19. If the greatest known depth of the Atlantic Ocean is 27,366 feet, and Mt. Washington is 6279 feet high, how high does Mt. Washington stand above the bottom of the Atlantic at its greatest known depth?

20. A grocer sold 7 barrels of flour for $38.43. How much did he receive per barrel?

21. In making the voyage from Southampton to New York, an ocean steamer consumes 2375 tons of coal in 5 days. What is the average amount consumed per day?

22. A special train made the distance from Chicago to New York in 16 hours. The distance by the route taken was 960 miles. What was the average speed per hour?

23. Two trains leave Chicago at the same time. One travels east at the rate of 48 miles an hour and the other travels west at the rate of 39 miles an hour. How far apart are they in 4 hours?

24. Two trains start at the same time, one from New York and the other from Chicago. They travel towards each other on the same line, the former at the rate of 53 miles an hour and the latter at the rate of 48 miles an hour. The distance between New York and Chicago being 980 miles by this route, how far apart are the trains in 4 hours?

25. A farmer sowed 68 acres to clover, using 13 pounds of seed to the acre. How many pounds did he use in all? If the crop averaged 4 tons to the acre, how many tons did the farmer cut in all?

26. A coal dealer buys coal at $2.87 a ton. He pays $0.67 a ton for railway transportation and $0.78 a ton for delivery, and sells the coal for $5.25 a ton. How much profit does he make per ton?

27. A watering trough holds 118 gallons. It is cracked so that water leaks out at the rate of 24 gallons an hour, but water flows in through a feed pipe at the rate of 83 gallons an hour. If the trough is empty when the water is turned on, how long will it take to fill the trough?

28. A workman receiving \$0.31 an hour works 48 hours one week and 43 hours the next week. How much does he receive for the two weeks' work?

29. A village lot was sold for \$745, which was \$130 less than it cost. For how much should it have been sold in order that the owner might have gained \$130?

30. A train that while running makes 45 miles an hour requires 2 hours and 6 minutes to go from New York to Philadelphia. It makes two stops of 3 minutes each. How far is it from New York to Philadelphia?

31. A clerk in a store receives \$60 a month, and works 12 hours a day. If he worked only 8 hours a day, how much would he receive if he were paid according to the number of hours?

32. A grocer has to pay the following bills on the first of the month: \$175.25, \$32.47, \$58.65, \$82.75, \$29.60, and \$59.80. What is the total amount? How much does the total amount lack of being \$500?

33. A man asked \$5825 for his farm, but was forced to sell it for \$3925. He thereby lost as much on what it cost him as he would have gained if he had sold it at his asking price. What did it cost him?

34. A man buys a house for \$3750. He pays \$1950 down and agrees to pay the balance in equal monthly payments of \$50 each. How many months will it take him to pay for the house? There being 12 months in a year, how many years will it take?

35. At an ordinary rate, 124 words a minute, how long will it take a man to deliver a speech of 12 pages, if each page contains 31 lines and the lines average 11 words each? How long would it take a slow speaker to deliver the same speech, if he averaged 93 words a minute?

36. How many times does a wheel 12 feet in circumference turn in going a mile (5280 feet)?

37. How many times does a wheel 11 feet in circumference turn in going half a mile?

38. A dealer purchased in one week 347 dozen eggs and paid $72.87 for them. How much did he pay a dozen?

39. How many squads of 15 men each can be formed from a company of 112 men, and how many men will be left over?

40. A delivery wagon of a city store traveled 3756 miles in 313 days last year. What was the average number of miles per day?

41. A man left an estate valued at $25,641.90 to be divided equally among six persons. How much was the share of each?

42. A railroad pays $14,227.20 to 192 men for working 26 days in a certain month. What is the average daily wages of each man?

43. A factory pays $127,078 to 140 workmen for working 313 days in a year. What is the average daily wages of each workman?

44. A factory pays $133,103.25 to 135 workmen for working 313 days in a year. What is the average daily wages of each workman?

45. A man left an estate of $30,125.50, and directed in his will that, after the expenses of settling his estate had been paid, $3000 should go to a college, and $500 to his church; that half of what remained should go to his wife, and the rest should be divided equally among his four children. The expense of settling the estate was $423.10. Required the share of each child.

46. A certain city pays 320 street cleaners \$4512 for 6 days' work. What is the daily wages of each man?

47. The area of Texas is 265,780 square miles, and that of the District of Columbia is 70 square miles. How many pieces the size of the latter could be cut from Texas, and how many square miles would then remain?

48. The area of Rhode Island is 1250 square miles. How many pieces the size of Rhode Island could be cut from Texas (see Ex. 47), and how many square miles would then remain?

49. At the time when the population of New York City was 4,318,035 the area was 327 square miles. What was the average population per square mile?

50. In 1820 Illinois had a population of only 55,162. When the population had reached 5,074,904 it was how many times that of 1820?

51. In 1820 Indiana had a population of 147,178. When the population had reached 2,502,026 it was how many times that of 1820?

52. If, when the population of the United States was 88,425,486, the number of children 12 years of age was 1,637,509, the total was how many times the latter?

53. The area of Iowa is 56,025 square miles, and that of Indiana is 36,350 square miles. Taking the area of the United States (including Alaska) as 3,616,484 square miles, how many times is the combined area of these two states exactly contained in it, and how many square miles remain over?

54. If the dividend is 56,136,000, the quotient 126,432, and the remainder 192, what is the divisor?

55. In 1800 the ordinary expenses of our government were \$7,411,370 a year. When these expenses had reached \$548,441,380, this was how many times the sum in 1800?

66. Principles of Division. It will be seen by the following examples that some interesting relations exist among the dividend, divisor, and quotient. If we consider the case of $48 \div 6 = 8$, we see that

(1) *Multiplying the dividend multiplies the quotient by the same number.*

That is, $96 \div 6 = 16$. The dividend, 48, has been multiplied by 2, and the quotient, 8, has also been multiplied by 2.

(2) *Dividing the dividend divides the quotient by the same number.*

That is, $24 \div 6 = 4$. The dividend, 48, has been divided by 2, and the quotient, 8, has also been divided by 2.

(3) *Multiplying the divisor divides the quotient by the same number.*

That is, $48 \div 12 = 4$. The divisor, 6, has been multiplied by 2, and the quotient, 8, has been divided by 2.

(4) *Dividing the divisor multiplies the quotient by the same number.*

That is, $48 \div 3 = 16$. The divisor, 6, has been divided by 2, and the quotient, 8, has been multiplied by 2.

(5) *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

That is, if instead of $48 \div 6$ we have $96 \div 12$ or $24 \div 3$, the quotient will still be 8.

The principle involved in (5) is a very convenient principle and is used much in arithmetic.

For example, instead of dividing 725 by 5 we may divide 1450 by 10 because it is much easier to multiply by 2 than to divide by 5. Likewise, if we wish to divide \$11.25 by \$3.25 we may multiply each number by 100 and then divide \$1125 by \$325, thus not being troubled with decimal points; again, if we wish to divide 27,235 by 25 we may multiply each number by 4 and divide 108,940 by 100, which can be done at a glance.

EXERCISE 27

Written Review Questions

1. Define units and give three illustrations.

2. Define numbers and give examples of two kinds.

3. What do you mean by integers, and what other names can you give to them?

4. What are abstract numbers? concrete numbers?

5. Distinguish between notation and numeration.

6. What are the two kinds of notation known to you? Why are they so called?

7. What do you mean by numerals? by digits?

8. What is meant by the place value of a figure?

9. What is the difference between orders and periods in writing numbers? Illustrate each.

10. How are periods indicated in the writing of numbers? Illustrate in two ways.

11. In writing United States money, how are the dollars separated from the cents? Illustrate.

12. What are the characters used in writing Arabic numbers, and those used in writing Roman numbers? Write four hundred forty-nine in each system.

13. What name is given to numbers that are to be added? What is the name of the result?

14. What is meant by like numbers? Illustrate by three pairs of like numbers.

15. How do you check the work in addition?

16. What names are given to the three numbers used in subtraction? Illustrate.

17. How do you check the work in subtraction?

18. What do you mean by multiplication? Show that it is a short process of addition.

19. What names are given to the three numbers used in multiplication? Illustrate.

20. What are factors? Illustrate.

21. May the multiplicand be abstract? May it be concrete? Illustrate both cases.

22. What is the effect of changing the order of the addends? also of the factors of a number? Illustrate by the cases of $4 + 7 + 9$, and $4 \times 7 \times 9$.

23. What is meant by a power of a number? by the square of a number? by the cube of a number? Illustrate.

24. What is the short way of multiplying an integer by 100? of multiplying United States money by 100?

25. What do you mean by division? In what way does division differ from multiplication?

26. Name the terms used in division and give an example showing the use of each.

27. If the dividend and divisor are concrete, what else may be said of their nature? What is the nature of the quotient? Illustrate.

28. May the dividend be concrete and the divisor abstract? Illustrate.

29. What is the short process of dividing an integer by 10? Illustrate by dividing 4750 by 10.

30. What is the effect on the quotient of multiplying the dividend or dividing the divisor? Illustrate each case.

31. What is the effect on the quotient of dividing the dividend or multiplying the divisor? Illustrate each case.

CHAPTER VI

FACTORS, MEASURES, MULTIPLES

67. Factors. Integral numbers whose product is a given number are called *factors* of that number.

In § 38 we met the case of two factors of a number, but we may also have several factors. Thus since $2 \times 3 \times 5 \times 7 = 210$, some of the factors of 210 are 2, 3, 5, and 7. Although every number has as factors itself and 1, we do not usually speak of these as factors.

68. Prime Numbers. A number that has no factors is called a *prime number.*

As stated in § 67, this means that the number itself and 1 are excluded. Thus 2, 3, 5, and 7 are prime numbers.

69. Prime Factors. A factor that is a prime number is called a *prime factor.*

Thus 2, 3, and 5 are the prime factors of 30. Although 6 is a factor of 30 it is not a prime factor. As stated in § 67, 1 is not considered a factor.

70. Composite Numbers. A number that is not a prime number is called a *composite number.*

Thus 4, 15, 100, and 250 are composite numbers.

71. Exact Divisor. A number that divides another without a remainder is called an *exact divisor* of that number.

The case is then said to be one of *exact division.* Thus 15 is *exactly divisible* by 5. Usually we speak of an exact divisor simply as a *divisor* of a number. We often speak of an exact divisor as a *measure* of a number because with it we may exactly measure the number, just as with a yardstick we may measure the length of a carpet.

72. Divisible Numbers. When we speak of one number as being *divisible* by another we mean exactly divisible, that is, divisible without a remainder.

73. Even Numbers. A number that is divisible by 2 is called an *even number.*

74. Odd Numbers. A number that is not an even number is called an *odd number.*

75. Divisibility by 2. *A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.*

Thus 36 must be divisible by 2 if 6 is; because 30, or any other number of tens, is divisible by 2, ten being 2×5.

Thus we may tell without dividing that 1,023,578 is divisible by 2.

76. Divisibility by 3. *A number is divisible by 3 if the sum of the digits is divisible by 3.*

We often use the word *digits* to mean the *value* of the symbols 0, 1, 2, and so on to 9, as well as the characters from 1 to 9.

Thus $54 = 5 \times 9 + 5 + 4$, and 5×9 contains the factor 3, so the number is divisible by 3 as $5 + 4$ is divisible by 3.

Likewise, 534 is divisible by 3 because $5 + 3 + 4$, or 12, is divisible by 3.

77. Divisibility by 5. *A number is divisible by 5 if it ends in 0 or 5.*

For if a number ends in 0 it is tens, and 10 is divisible by 5. If it ends in 5, it is tens plus 5, and both are divisible by 5.

There is no easy test for divisibility by 7.

78. Finding Prime Factors. Find the prime factors of 420.

By § 75, 2 is a factor of 420 and of 210; by § 76, 3 is a factor of 105; by § 77, 5 is a factor of 35, the other factor being 7. Hence the prime factors of 420 are 2, 2, 3, 5, 7.

$$\begin{array}{r} 2\overline{)420} \\ 2\overline{)210} \\ 3\overline{)105} \\ 5\overline{)35} \\ \hline 7 \end{array}$$

That is, *divide as often as possible by the smallest prime factor, and then by the next larger, and so on until a prime quotient is reached. The several divisors and the last quotient are the prime factors.*

79. Proving a Number Prime. Required to find if 239 is a prime number.

The last digit is not 0, 2, 4, 6, or 8, so 239 cannot contain the factor 2, and therefore cannot contain the factor 4, 6, 8, or any other multiple of 2. The sum of the digits does not contain the factor 3, and therefore 239 cannot contain the factor 3 or any multiple of 3. The last digit is not 0 or 5, and therefore 239 cannot contain the factor 5 or any multiple of 5. Therefore 2, 3, 4, 5, 6, 8, 9, 10, 12, 14, 15, 16, cannot be factors of 239. By trial we find that 7, 11, 13, 17, are not factors. We need not try any prime number higher than 17, as the quotient when 17 is tried is *less* than 17, and we have tried all prime factors below 17. Hence 239 is prime.

Try the various prime numbers, beginning with 2, to see if they are factors. If no prime factor is found before the quotient becomes less than the divisor, the number is prime.

EXERCISE 28

1. Select the even numbers: 432, 640, 287, 1024, 3036, 8292, 4427, 8329, 4878.

2. Select the numbers divisible by 3: 424, 636, 292, 444, 1002, 3027, 1104, 8822, 6777.

3. Select the numbers divisible by 5: 830, 272, 637, 4225, 3010, 7200, 6305, 9270, 4005.

4. Select the prime numbers: 53, 61, 97, 111, 121, 143, 187, 1005, 1001, 1203, 2112, 1331, 1337.

Find the prime factors of the following:

5. 120.	10. 175.	15. 1001.	20. 1728.
6. 210.	11. 225.	16. 1245.	21. 5280.
7. 150.	12. 303.	17. 1331.	22. 6006.
8. 220.	13. 750.	18. 3645.	23. 3737.
9. 170.	14. 666.	19. 3432.	24. 9768.

80. Common Measure. A number that exactly divides each of two or more numbers is called a *common measure* of those numbers.

Thus 7 is a common measure of 28 and 42, since it exactly divides each of them, as a foot length exactly divides a 10-foot length we are measuring. A common measure is also called a *common divisor.*

81. Greatest Common Measure. The greatest number that exactly divides each of two or more numbers is called the *greatest common measure* of those numbers.

Thus while 7 is a common measure of 28 and 42, 14 is the *greatest* common measure. It is also called the *greatest common divisor.* The letters G.C.M. stand for the words Greatest Common Measure, and G.C.D. for Greatest Common Divisor.

82. Numbers Prime to Each Other. If two integers have no common measure, always excepting 1, they are said to be *prime to each other.*

For example, 25 and 33 are prime to each other, although each has factors.

83. Finding the G.C.M. The greatest common measure of such numbers as we shall need in studying fractions is found easily by factoring.

For example, required the G.C.M. of 24, 30, and 36.

Finding the prime factors of each, we have the following:

2)24	2)30	2)36
2)12	3)15	2)18
2)6	5	3)9
3		3

The factor 2 occurs once in each of the numbers. The factor 3 occurs once in each of the numbers. No other factor occurs in *each one* of the numbers. Therefore the G.C.M. is 2×3, or 6.

This solution may be expressed as follows:

$$24 = 2 \times 2 \times 2 \times 3; \quad 30 = 2 \times 3 \times 5; \quad 36 = 2 \times 2 \times 3 \times 3.$$

Therefore the G.C.M. is 2×3, or 6.

Required the G.C.M. of 54, 81, and 108.

Now $54 = 2 \times 3 \times 3 \times 3;$

$81 = 3 \times 3 \times 3 \times 3;$

$108 = 2 \times 2 \times 3 \times 3 \times 3.$

Here 3 occurs three times as a factor in each number, and there is no other common factor. Hence the G.C.M. is $3 \times 3 \times 3$, or 27.

Hence, to find the G.C.M. of two or more numbers,

Separate the numbers into prime factors and find the product of the prime factors common to the numbers.

If one of the given numbers is a multiple of one of the others, it may be canceled, since it must contain all the factors of that other. Thus the G.C.M. of 48, 54, and 108 is the same as that of 48 and 54.

EXERCISE 29

Find the G.C.M. of the following:

1. 2, 8.
2. 4, 16.
3. 9, 33.
4. 12, 78.
5. 14, 49.
6. 15, 75.
7. 16, 56.
8. 17, 51.
9. 18, 81.
10. 19, 171.
11. 20, 175.
12. 21, 105.
13. 22, 121.
14. 23, 207.
15. 24, 246.
16. 6, 8, 42.
17. 9, 12, 33.
18. 8, 28, 84.
19. 9, 33, 81.
20. 10, 35, 85.
21. 12, 48, 68.
22. 14, 21, 77.
23. 16, 72, 96.
24. 18, 72, 81.
25. 20, 80, 95.
26. 24, 60, 96.
27. 30, 90, 96.
28. 32, 48, 96.
29. 36, 54, 90.
30. 39, 52, 91.
31. 63, 84, 126.
32. 33, 77, 121.
33. 30, 60, 250.
34. 32, 48, 128.
35. 51, 63, 132.
36. 36, 84, 132.
37. 60, 144, 576.
38. 38, 190, 760.
39. 32, 160, 808.
40. 33, 132, 143.
41. 50, 125, 250.
42. 64, 128, 256.
43. 64, 160, 640.
44. 63, 126, 333.
45. 75, 125, 375.

84. Finding the G.C.M. of Large Numbers. It is not often necessary to find the G.C.M. of large numbers that cannot be separated readily into factors. An example will illustrate a process often called the long-division method.

This method depends upon the principle that a common factor of two numbers is also a factor of their sum and of their difference.

Required the G.C.M. of 323 and 357.

```
323)357(1
    323
    ---
     34)323(9
        306
        ---
         17)34(2
            34
            --
```

Here we cannot easily factor the two numbers. We divide the larger number by the smaller to see if the smaller is the G.C.M. It is not because there is a remainder 34. We then divide the smaller number by this remainder and find there is a remainder 17. Hence 34 is not the G.C.M. Then we divide the last divisor 34 by the last remainder 17 and find that 17 is an exact divisor of 34. Then 17 is the G.C.M. required.

EXERCISE 30

Find the G.C.M. of the following:

1. 195 and 260.
2. 294 and 350.
3. 333 and 592.
4. 410 and 697.
5. 720 and 1728.
6. 1875 and 2500.
7. 1220 and 2013.
8. 1010 and 2121.
9. 2813 and 3973.
10. 2002 and 3010.
11. 7099 and 9389.
12. 2220 and 3293.
13. 2650 and 8003.
14. 1140 and 1767.
15. 3621 and 7100.
16. 3650 and 3796.

17. What is the largest number that will exactly divide 2378 and 3915?

18. What is the largest number that will exactly divide 551, 667, and 841?

85. Multiple. The product of two or more integers is called a *multiple* of each of them.

Thus 20 is a multiple of 2, 4, 5, and 10, because $2 \times 2 \times 5 = 4 \times 5 = 2 \times 10 = 20$. This has already been defined (§ 37). We speak of 75, for example, as a multiple of 25, and of 51 as a multiple of 17, or we speak of each as a multiple of any other of its factors.

86. Common Multiple. A multiple of two or more numbers is called a *common multiple* of the numbers.

Thus 60 is a common multiple of 6 and 10, and so are 30, 90, 120, and so on, common multiples of 6 and 10.

87. Least Common Multiple. The smallest common multiple of two or more numbers is called their *least common multiple.* It is the least number that is exactly divisible by each of them.

Thus 60 is a common multiple of 6 and 10, but the *least* common multiple is 30.

The letters L.C.M. are used for the words Least Common Multiple.

Since the L.C.M. must contain each of the numbers, it must contain all of the prime factors of each, and this is the principle upon which the finding of the L.C.M. rests.

88. Finding the L.C.M. Required the L.C.M. of 24, 30, 36.

Resolving each number into its prime factors, we have

$2\overline{)24}$	$2\overline{)30}$	$2\overline{)36}$
$2\overline{)12}$	$3\overline{)15}$	$2\overline{)18}$
$2\overline{)6}$	5	$3\overline{)9}$
3		3

The L.C.M. must contain the factors of 24, which are 2, 2, 2, and 3. These are therefore taken as part of its factors.

It must contain the factors of 30, which are 2, 3, and 5; but we have already taken 2 and 3, so we need take only 5.

It must contain the factors of 36, which are 2, 2, 3, and 3; but we have already taken 2, 2, and 3, so we need take only one more 3.

Hence the factors of the L.C.M. are 2, 2, 2, 3, 5, 3. Their product is 360, and 360 is the L.C.M.

It is often convenient to adopt another arrangement of the work, as in finding the L.C.M. of 12, 18, 30, 45.

Here we arrange the numbers in a horizontal line, and divide by the smallest prime factor common to *two* or more of them.

$$\begin{array}{r|llll} 2 & 12 & 18 & 30 & 45 \\ \hline 3 & 6 & 9 & 15 & 45 \\ \hline & 2 & & & 15 \end{array}$$

We first divide by 2, and write the quotients and *the undivided numbers* as well in a line below. In the first line of quotients we cancel the 9 and the 15 because each is a factor of 45, and therefore 45 contains all the factors of 9 and of 15. We next divide by 3, and in the last line of quotients we have the numbers 2 and 15, which are prime to each other. Hence the L.C.M. is $2 \times 3 \times 2 \times 15$, or 180.

Required the L.C.M. of 5, 15, 30, 60.

~~5~~ ~~15~~ ~~30~~ 60

Here we cancel the 5 because it is a factor of 15. We cancel the 15 because it is a factor of 30, and cancel the 30 because it is a factor of 60. Therefore 60 contains all the factors of the other numbers and is itself the L.C.M.

89. General Directions for Finding the L.C.M. From the above examples the following directions may be observed:

Separate each number into its prime factors in any convenient manner. Find the product of these factors, using each factor the greatest number of times it occurs in any one of the given numbers.

EXERCISE 31

Find the L.C.M. of the following:

1. 22, 33.
2. 17, 51.
3. 16, 24.
4. 15, 35.
5. 27, 39.
6. 33, 55.
7. 3, 21, 35.
8. 4, 12, 38.
9. 6, 15, 39.
10. 8, 20, 48.
11. 9, 42, 49.
12. 12, 24, 144.
13. 6, 9, 12, 15.
14. 7, 14, 21, 35.
15. 8, 28, 36, 52.
16. 9, 21, 39, 93.
17. 10, 15, 25, 50.
18. 16, 24, 64, 96.

19. 6, 9, 12, 33.
20. 9, 27, 36, 63.
21. 20, 24, 30, 40.
22. 24, 36, 48, 60.
23. 11, 22, 33, 66.
24. 14, 28, 77, 91.
25. 20, 28, 56, 80.
26. 30, 40, 50, 60.
27. 32, 40, 48, 80.
28. 45, 63, 72, 99.
29. 4, 8, 12, 16, 32.
30. 5, 10, 20, 35, 45.
31. 6, 18, 30, 42, 66.
32. 7, 21, 63, 70, 84.
33. 8, 32, 64, 72, 96.
34. 9, 27, 54, 63, 90.
35. 10, 11, 12, 22, 33.
36. 11, 15, 33, 45, 55.
37. 12, 16, 24, 36, 72.
38. 13, 26, 52, 78, 104.

39. Find the least number of marbles that can be divided equally among 15, 20, or 25 boys.

40. Find the shortest distance that can be exactly measured by a 3-foot rule, or a 5-foot pole, or a 10-foot pole.

41. What is the smallest quantity of liquid that can be measured exactly by 2-quart, 4-quart, 6-quart, or 8-quart measures ?

42. What is the smallest sum of money that can be counted out in 5-cent pieces, 10-cent pieces, 25-cent pieces, or 50-cent pieces ?

43. What is the capacity of the smallest water tank that can be filled in an integral number of minutes by either of two pipes, if one discharges 32 gallons a minute and the other 48 gallons a minute ?

44. Find the least number of oranges that, when arranged in groups of 6, 7, 8, or 9, has just 5 over in each case. (Find the L.C.M. and add 5. Test the result.)

45. Find the least number of marbles that, when arranged in groups of 8, 12, 15, 60, or 120, has just 7 over in each case.

90. Cancellation. The process of shortening work in division by removing or canceling equal factors from both dividend and divisor is called *cancellation.*

For example, divide $11 \times 27 \times 30$ by $3 \times 9 \times 15$.

The common factor 3 is canceled from the 3 in the divisor and the 27 in the dividend. The common factor 9 is canceled from the 9 in the divisor and the 9 that remains uncanceled in the 27 of the dividend. The common factor 15 is canceled from the divisor and the 30 in the dividend. There remains for a dividend $11 \times 1 \times 2$, and for a divisor $1 \times 1 \times 1$. Hence the quotient is 22. This is much shorter than finding two products, 8910 and 405, and then the quotient 22.

$$\frac{11 \times \overset{9}{\cancel{27}} \times \overset{2}{\cancel{30}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{9}} \times \underset{1}{\cancel{15}}} = \frac{11 \times 1 \times 2}{1 \times 1 \times 1} = 22.$$

EXERCISE 32

Find the quotient of:

1. $\dfrac{6 \times 8 \times 99}{3 \times 4 \times 11}$.
2. $\dfrac{8 \times 12 \times 51}{4 \times 9 \times 17}$.
3. $\dfrac{9 \times 30 \times 38}{6 \times 10 \times 19}$.
4. $\dfrac{12 \times 21 \times 57}{9 \times 14 \times 19}$.
5. $\dfrac{13 \times 20 \times 91}{13 \times 13 \times 14}$.
6. $\dfrac{12 \times 32 \times 78}{9 \times 16 \times 26}$.
7. $\dfrac{15 \times 25 \times 64}{10 \times 80 \times 10}$.
8. $\dfrac{18 \times 27 \times 96}{12 \times 36 \times 54}$.
9. $\dfrac{4 \times 5 \times 6 \times 8}{2 \times 3 \times 4 \times 5}$.
10. $\dfrac{8 \times 16 \times 28 \times 92}{2 \times 32 \times 23 \times 14}$.
11. $\dfrac{36 \times 19 \times 29 \times 39}{3 \times 38 \times 58 \times 13}$.
12. $\dfrac{27 \times 35 \times 42 \times 72}{5 \times 9 \times 14 \times 18}$.
13. $\dfrac{34 \times 35 \times 36 \times 37}{5 \times 17 \times 9 \times 74}$.
14. $\dfrac{24 \times 34 \times 44 \times 54}{3 \times 8 \times 17 \times 33}$.
15. $\dfrac{15 \times 18 \times 48 \times 75}{25 \times 30 \times 36 \times 24}$.
16. $\dfrac{24 \times 63 \times 82 \times 91}{41 \times 13 \times 21 \times 12}$.

17. What is the quotient of 35 times the product of 48 and 119, divided by 14 times the product of 21 and 17 ?

18. If a dividend is the product of 68, 91, and 95, and the divisor is the product of all the prime numbers between 12 and 20, what is the quotient ?

19. If we divide the product of 16, twice 16, and three times 16, by the product of the square of 2 taken three times as a factor, what is the quotient ?

20. How many times is the product of all the prime numbers between 20 and 30 contained in the product of 203 and 230 ?

21. A man worked 4 weeks, 6 days a week, for a market gardener, and received for his pay 24 crates of berries valued at $2 a crate. How much did he earn a day ?

22. How many dozen quart bottles of imported pickles at $6 a dozen must be given in exchange for 3 cases of olives, each containing 2 dozen bottles at $3 a dozen ?

23. A grocer exchanged 16 boxes of soap, each containing 60 cakes at 4 cents a cake, for a certain number of cases of preserved fruit, 2 dozen jars to a case at 40 cents a jar. How many cases did he get ?

24. A man exchanged 9 firkins of butter weighing 56 pounds each at 30 cents a pound for several rolls of matting, 40 yards to the roll worth 42 cents a yard. How many rolls did he get ?

25. If 24 men, working 9 hours a day, can do a piece of work in 12 days, how many days will it take 18 men, working 8 hours a day, to do the work ?

26. If 15 men can do a piece of work in 16 days, working 9 hours a day, how many days will it take 20 men working 12 hours a day to do the work ?

27. A clerk working 48 weeks in a store, at $12 a week, receives the same amount that another clerk receives in 36 weeks. What is the weekly wages of the other clerk?

28. A dealer sells a shipment of 36 horses for $2700. He sells another shipment of 25 horses for the same amount. What is the average price per horse in each of the two shipments?

29. We find the volume of a rectangular box by taking the product of its three dimensions expressed in the same unit of measure. The dimensions of one box, expressed as abstract numbers, are 4, 5, and 6, and two dimensions of another box of the same volume are 3 and 10. What is the other dimension of the second box?

30. Expressed as abstract numbers, the dimensions of one box are 8, 12, and 15, and two dimensions of another box of the same volume are 6 and 30. What is the other dimension of the second box?

31. Expressing the dimensions as abstract numbers, a cellar is 20 long, 14 wide, and 6 deep. How deep must another cellar be that is 16 long and 15 wide, in order that the capacity of both cellars may be the same?

32. How many times as much earth must be removed from a cellar excavation 175 feet long, 80 feet wide, and 15 feet deep, as from a cellar excavation 150 feet long, 40 feet wide, and 5 feet deep?

33. How many times as much earth must be removed from a cellar excavation 160 feet long, 40 feet wide, and 9 feet deep, as from a cellar excavation 60 feet long, 40 feet wide, and 6 feet deep?

34. How many times as much will a box contain whose dimensions are 8, 27, and 35, as one whose dimensions are 7, 12, and 18?

CHAPTER VII

COMMON FRACTIONS

91. Fractions. A number that shows what part or what number of parts of a unit is taken is called a *fraction.*

Thus $\frac{1}{2}$ shows that one of the two equal parts of a unit is taken; $\frac{3}{4}$ shows that three of the four equal parts of a unit are taken.

92. Common Fractions. A fraction expressed by two numbers, one under the other with a line between them, is called a *common fraction.*

93. Terms of a Fraction. In a common fraction the number above the line is called the *numerator;* the number below the line is called the *denominator;* the two together are called the *terms* of the fraction.

In the fraction $\frac{3}{4}$ the numerator is 3 and the denominator is 4. The denominator (namer) shows into how many equal parts the unit has been divided, and the numerator (numberer) shows how many of these parts have been taken.

94. Unit Fractions. A fraction whose numerator is one is called a *unit fraction.*

Thus $\frac{1}{4}$ is a unit fraction. Since it is *one* of the fourths, it is also called a *fractional unit.*

95. Proper Fractions. A fraction whose numerator is less than the denominator is called a *proper fraction.*

For example, $\frac{3}{4}$ is a proper fraction.

96. Improper Fractions. A fraction whose numerator is not less than the denominator is called an *improper fraction.*

For example, $\frac{4}{4}$ and $\frac{5}{3}$ are improper fractions.

EXERCISE 33

Write in words the following fractions:

1. $\frac{1}{3}$.	6. $\frac{2}{3}$.	11. $\frac{3}{4}$.	16. $\frac{4}{5}$.	21. $\frac{3}{2}$.
2. $\frac{1}{5}$.	7. $\frac{2}{5}$.	12. $\frac{3}{8}$.	17. $\frac{5}{9}$.	22. $\frac{8}{3}$.
3. $\frac{1}{7}$.	8. $\frac{2}{9}$.	13. $\frac{3}{16}$.	18. $\frac{6}{10}$.	23. $\frac{9}{4}$.
4. $\frac{1}{10}$.	9. $\frac{2}{10}$.	14. $\frac{3}{32}$.	19. $\frac{8}{15}$.	24. $\frac{10}{10}$.
5. $\frac{1}{32}$.	10. $\frac{2}{15}$.	15. $\frac{3}{64}$.	20. $\frac{15}{16}$.	25. $\frac{33}{33}$.

Express in fractional form the following:

26. Three sevenths.
27. Four ninths.
28. One twelfth.
29. Two fifteenths.
30. Seven eighths.
31. Five sixths.
32. One twentieth.
33. Eleven twelfths.
34. Nine twentieths.
35. Seven tenths.
36. Eight fifths.
37. Ten ninths.
38. Seven sevenths.
39. Twelve twelfths.
40. One one-hundredth.
41. Two three-hundredths.
42. Five one-hundred fiftieths.
43. One hundred five-hundredths.
44. Two hundred hundredths.
45. Seven thousandths.

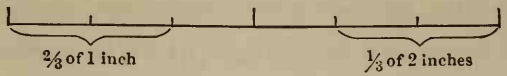
46. Draw a line 3 inches long and divide each inch into fourths.

47. Draw a line 3 inches long and mark off 5 fourths of an inch.

48. Draw a line 3 inches long and mark off $\frac{4}{4}$ of an inch from one end and $\frac{7}{4}$ of an inch from the other end.

49. Draw a line 4 inches long and divide each inch into fourths. Mark off $\frac{2}{4}$ of an inch from one end and $\frac{1}{2}$ of an inch from the other end. How do they compare?

97. Fraction as an Expressed Division. In this figure we see that $\frac{2}{3}$ of 1 inch equals $\frac{1}{3}$ of 2 inches, or the result of



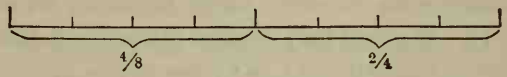
dividing 2 inches by 3; that is, $\frac{2}{3}$ is the same as $\frac{1}{3}$ of 2, or 2 divided by 3. Therefore,

A fraction is an expressed division, the numerator being the dividend and the denominator the divisor.

98. Reduction. The process of changing the unit of a number without changing the value is called *reduction.*

Thus \$2 = 200 cents. These numbers are measured by different units (\$1 and 1 cent), but they have the same value. Therefore if we reduce \$2 to cents we express it as 200 cents.

99. Reduction of Fractions to Lower Terms. In this figure we see that $\frac{4}{8}$ of the line is the same in length as $\frac{2}{4}$ of the



line, and that each equals $\frac{1}{2}$ of the line. Hence $\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$. Here $\frac{4}{8}$ has been reduced to fractions whose terms are lower than 4 and 8.

A fraction may be reduced to lower terms by dividing both numerator and denominator by the same number.

For example, required to reduce $\frac{20}{30}$ to fifteenths.

Dividing both terms by 2, so as to make the denominator 15, we have $\frac{20}{30} = \frac{10}{15}$. We could evidently reduce to still lower terms.

EXERCISE 34

Reduce as indicated:

To fourths:	1. $\frac{9}{12}$.	2. $\frac{18}{24}$.	3. $\frac{14}{28}$.	4. $\frac{72}{144}$.
To fifths:	5. $\frac{21}{35}$.	6. $\frac{15}{75}$.	7. $\frac{64}{80}$.	8. $\frac{60}{150}$.
To eighths:	9. $\frac{24}{64}$.	10. $\frac{36}{72}$.	11. $\frac{60}{96}$.	12. $\frac{84}{168}$.

100. Reduction of Fractions to Lowest Terms. Since we may divide both terms of a fraction by the same number without changing the value of the fraction, we may continue this division until the terms are prime to each other.

If the terms of a fraction are prime to each other, the fraction is expressed in *lowest terms.*

For example, $\frac{15}{17}$ is expressed in lowest terms, but $\frac{8}{12}$ is not, because it may be reduced to $\frac{2}{3}$.

Therefore, *to reduce a fraction to lowest terms, cancel all the factors common to both numerator and denominator; or divide both terms by their G.C.M.*

For example, reduce $\frac{24}{30}$ to lowest terms.

Canceling 2, we have $\frac{12}{15}$; canceling 3, we have $\frac{4}{5}$. Since 4 and 5 are prime to each other, $\frac{4}{5}$ is expressed in lowest terms.

Reduce $\frac{231}{253}$ to lowest terms.

Since 231 seems to be the more easily factored, we separate it into its factors 3, 7, 11. We find that neither 3 nor 7 is a factor of 253, but that 11 is a factor. Canceling 11, we have $\frac{231}{253} = \frac{21}{23}$.

We might find the G.C.M. by the long-division method (§ 84), and cancel it, but it is rarely necessary to do this.

EXERCISE 35

Reduce to lowest terms:

1. $\frac{24}{96}$.
2. $\frac{36}{84}$.
3. $\frac{72}{88}$.
4. $\frac{48}{84}$.
5. $\frac{54}{99}$.
6. $\frac{23}{69}$.
7. $\frac{21}{63}$.
8. $\frac{12}{38}$.
9. $\frac{32}{48}$.
10. $\frac{16}{50}$.
11. $\frac{17}{51}$.
12. $\frac{19}{57}$.
13. $\frac{23}{92}$.
14. $\frac{42}{56}$.
15. $\frac{35}{63}$.
16. $\frac{48}{64}$.
17. $\frac{36}{144}$.
18. $\frac{48}{144}$.
19. $\frac{64}{156}$.
20. $\frac{72}{156}$.
21. $\frac{84}{168}$.
22. $\frac{64}{192}$.
23. $\frac{64}{160}$.
24. $\frac{33}{144}$.
25. $\frac{96}{144}$.
26. $\frac{72}{256}$.
27. $\frac{64}{448}$.
28. $\frac{78}{273}$.
29. $\frac{111}{120}$.
30. $\frac{121}{132}$.
31. $\frac{132}{144}$.
32. $\frac{144}{156}$.

101. Reduction of Fractions to Higher Terms. By multiplying both terms of a fraction by the same number we may reduce a fraction to *higher terms.*

For example, reduce $\frac{1}{12}$ to sixtieths.

Here $60 \div 12 = 5$, so that the denominator must be multiplied by 5 to make 60. Therefore we multiply both terms by 5, and $\frac{1}{12} = \frac{5}{60}$.

Therefore, *to reduce a fraction to a fraction having a higher denominator, divide the required denominator by the given denominator and multiply both terms of the fraction by the quotient.*

102. Reduction of Integers to Improper Fractions. An integer may be expressed as an improper fraction with any required denominator.

Reduce 3 to an improper fraction with denominator 7.

We may simply think of 3 as equal to $\frac{3}{1}$. We have, then, the same case as in § 101. Multiplying both terms by 7, we have $\frac{3}{1} = \frac{21}{7}$.

Therefore, *to reduce an integer to a fraction with a given denominator, multiply it by the given denominator and under this product write the denominator.*

EXERCISE 36

Reduce as indicated:

To 60ths:	1. $\frac{1}{2}$.	2. $\frac{8}{15}$.	3. $\frac{9}{20}$.	4. $\frac{11}{30}$.
To 80ths:	5. $\frac{1}{2}$.	6. $\frac{5}{16}$.	7. $\frac{7}{20}$.	8. $\frac{9}{40}$.
To 144ths:	9. $\frac{1}{2}$.	10. $\frac{1}{12}$.	11. $\frac{5}{24}$.	12. $\frac{7}{72}$.
To 156ths:	13. $\frac{1}{2}$.	14. $\frac{2}{39}$.	15. $\frac{5}{78}$.	16. $\frac{7}{78}$.
To 256ths:	17. $\frac{1}{2}$.	18. $\frac{3}{16}$.	19. $\frac{5}{32}$.	20. $\frac{7}{64}$.
To 288ths:	21. $\frac{1}{2}$.	22. $\frac{1}{12}$.	23. $\frac{5}{24}$.	24. $\frac{7}{48}$.
To 320ths:	25. $\frac{1}{2}$.	26. $\frac{3}{16}$.	27. $\frac{5}{32}$.	28. $\frac{9}{64}$.
To 100ths:	29. $\frac{1}{2}$.	30. $\frac{3}{10}$.	31. $\frac{4}{25}$.	32. $\frac{9}{50}$.

103. Mixed Numbers. A number composed of an integer and a fraction taken together is called a *mixed number.*

Thus $2\frac{3}{4}$ is a mixed number. It is read "two and three fourths."

104. Reduction of Improper Fractions. An improper fraction may always be reduced to an integer or a mixed number.

For example, required to reduce $\frac{72}{12}$ to an integer or a mixed number.

Dividing both terms by 12, we have $\frac{6}{1}$, which equals 6.

Required to reduce $\frac{36}{16}$ to an integer or a mixed number.

Since $\frac{36}{16}$ is an expressed division, we divide 36 by 16 and the quotient is $2\frac{4}{16}$, or $2\frac{1}{4}$.

Therefore, *to reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator.*

EXERCISE 37

Reduce to integers or mixed numbers:

1. $\frac{36}{12}$.	10. $\frac{96}{16}$.	19. $\frac{84}{48}$.	28. $\frac{80}{64}$.	37. $\frac{198}{96}$.
2. $\frac{38}{12}$.	11. $\frac{36}{24}$.	20. $\frac{240}{48}$.	29. $\frac{88}{64}$.	38. $\frac{288}{96}$
3. $\frac{40}{12}$.	12. $\frac{72}{24}$.	21. $\frac{60}{50}$.	30. $\frac{448}{64}$.	39. $\frac{480}{96}$.
4. $\frac{60}{12}$.	13. $\frac{96}{24}$.	22. $\frac{75}{50}$.	31. $\frac{80}{72}$.	40. $\frac{576}{96}$.
5. $\frac{72}{12}$.	14. $\frac{72}{18}$.	23. $\frac{90}{50}$	32. $\frac{84}{72}$.	41. $\frac{250}{100}$.
6. $\frac{36}{16}$.	15. $\frac{96}{32}$.	24. $\frac{95}{50}$.	33. $\frac{96}{72}$.	42. $\frac{375}{100}$.
7. $\frac{40}{16}$.	16. $\frac{96}{48}$.	25. $\frac{350}{50}$.	34. $\frac{144}{72}$.	43. $\frac{425}{100}$.
8. $\frac{72}{16}$.	17. $\frac{72}{48}$.	26. $\frac{64}{64}$.	35. $\frac{288}{72}$.	44. $\frac{550}{100}$.
9. $\frac{80}{16}$.	18. $\frac{56}{48}$.	27. $\frac{72}{64}$.	36. $\frac{192}{96}$.	45. $\frac{750}{100}$.

46. If the numerator of a fraction is equal to the denominator, to what integer is the fraction equal?

47. If the numerator of a fraction is $2\frac{1}{2}$ times the denominator, to what mixed number is the fraction equal? To what integer is it equal if the denominator is $\frac{1}{3}$ of the numerator?

105. Reduction of Mixed Numbers. A mixed number may always be reduced to an improper fraction.

For example, required to reduce $2\frac{3}{4}$ to an improper fraction.

Since 1 equals 4 fourths ($\frac{4}{4}$), 2 equals 2×4 fourths, or 8 fourths ($\frac{8}{4}$); and 8 fourths and 3 fourths are 11 fourths ($\frac{11}{4}$).

Therefore, *to reduce a mixed number to an improper fraction, multiply the denominator by the integral part of the mixed number, and to the product add the numerator; under this sum write the denominator.*

EXERCISE 38

Reduce to improper fractions:

1. $3\frac{7}{8}$.	11. $2\frac{5}{24}$.	21. $8\frac{5}{36}$.	31. $1\frac{7}{64}$.
2. $4\frac{7}{8}$.	12. $5\frac{7}{24}$.	22. $6\frac{7}{36}$.	32. $5\frac{11}{64}$.
3. $7\frac{5}{8}$.	13. $8\frac{11}{24}$.	23. $4\frac{11}{36}$.	33. $7\frac{17}{64}$.
4. $6\frac{7}{8}$.	14. $2\frac{13}{24}$.	24. $7\frac{13}{36}$.	34. $3\frac{29}{64}$.
5. $9\frac{5}{8}$.	15. $6\frac{17}{24}$.	25. $2\frac{17}{36}$.	35. $6\frac{37}{64}$.
6. $7\frac{3}{16}$.	16. $7\frac{3}{32}$.	26. $5\frac{5}{48}$.	36. $1\frac{5}{72}$.
7. $5\frac{5}{16}$.	17. $9\frac{5}{32}$.	27. $8\frac{7}{48}$.	37. $6\frac{7}{72}$.
8. $8\frac{9}{16}$.	18. $3\frac{7}{32}$.	28. $2\frac{11}{48}$.	38. $5\frac{23}{72}$.
9. $9\frac{7}{16}$.	19. $6\frac{9}{32}$.	29. $6\frac{17}{48}$.	39. $35\frac{5}{72}$.
10. $6\frac{11}{16}$.	20. $2\frac{11}{32}$.	30. $1\frac{23}{48}$.	40. $72\frac{1}{72}$.

41. How many eighths ($\frac{1}{8}$) are there in $\frac{7}{8}$? in 3? in $3\frac{7}{8}$?

42. How many sixteenths ($\frac{1}{16}$) are there in $\frac{5}{16}$? in 5? in $5\frac{5}{16}$?

43. How many thirty-seconds ($\frac{1}{32}$) are there in $\frac{7}{32}$? in 3? in $3\frac{7}{32}$?

44. How many tins, $\frac{1}{8}$ of a pound each, can be filled from $\frac{7}{8}$ of a pound of pepper? from 2 pounds? from $2\frac{7}{8}$ pounds?

106. Common Denominator. If two or more fractions have the same denominator they are said to have a *common denominator.*

Thus $\frac{3}{8}$ and $\frac{5}{8}$ have the common denominator 8. The letters L.C.D. stand for the words Least Common Denominator.

107. Similar Fractions. Fractions that have a common denominator are called *similar fractions.* It is always possible to reduce fractions to similar fractions.

For example, required to reduce $\frac{2}{3}$ and $\frac{3}{4}$ to similar fractions.

The L.C.M. of the denominators is 12. Reducing $\frac{2}{3}$ to twelfths (§ 101) we have $\frac{8}{12}$, and reducing $\frac{3}{4}$ to twelfths we have $\frac{9}{12}$.

The L.C.D. of several fractions is the L.C.M. of the given denominators.

Hence, *to reduce fractions to similar fractions, reduce them to fractions whose common denominator is the L.C.M. of the given denominators.*

EXERCISE 39

Reduce to similar fractions, using the L.C.D.:

1. $\frac{3}{8}, \frac{5}{12}$.
2. $\frac{5}{6}, \frac{7}{8}$.
3. $\frac{5}{8}, \frac{3}{32}$.
4. $\frac{7}{8}, \frac{4}{5}$.
5. $\frac{5}{12}, \frac{7}{16}$.
6. $\frac{7}{12}, \frac{9}{16}$.
7. $\frac{11}{12}, \frac{15}{16}$.
8. $\frac{15}{16}, \frac{19}{24}$.
9. $\frac{13}{16}, \frac{29}{32}$.
10. $\frac{13}{24}, \frac{5}{32}$.
11. $\frac{7}{32}, \frac{11}{48}$.
12. $\frac{15}{32}, \frac{29}{48}$.
13. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$.
14. $\frac{1}{3}, \frac{1}{4}, \frac{5}{8}$.
15. $\frac{2}{3}, \frac{1}{4}, \frac{3}{8}$.
16. $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}$.
17. $\frac{1}{5}, \frac{1}{8}, \frac{1}{12}$.
18. $\frac{2}{5}, \frac{3}{8}, \frac{5}{12}$.
19. $\frac{3}{5}, \frac{7}{8}, \frac{7}{12}$.
20. $\frac{1}{8}, \frac{1}{10}, \frac{1}{12}$.
21. $\frac{3}{8}, \frac{7}{10}, \frac{7}{12}$.
22. $\frac{3}{8}, \frac{3}{10}, \frac{5}{12}$.
23. $\frac{5}{8}, \frac{5}{16}, \frac{7}{24}$.
24. $\frac{7}{8}, \frac{7}{16}, \frac{7}{24}$.
25. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$.
26. $\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}$.
27. $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{1}{8}$.
28. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{3}{8}$.
29. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{7}{8}$.
30. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{3}{16}$.
31. $\frac{3}{4}, \frac{7}{8}, \frac{5}{16}, \frac{3}{32}$.
32. $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{24}$.
33. $\frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{5}{24}$.
34. $\frac{5}{8}, \frac{1}{16}, \frac{3}{32}, \frac{1}{64}$.
35. $\frac{7}{8}, \frac{5}{16}, \frac{5}{32}, \frac{7}{64}$.
36. $\frac{7}{8}, \frac{1}{24}, \frac{1}{32}, \frac{5}{64}$.

108. Addition of Fractions. If we add 3 marbles and 5 pencils, we have as a sum neither marbles nor pencils, but just 8 things, of two different kinds. So if we add $\frac{3}{4}$ and $\frac{5}{6}$ we shall have as a sum neither fourths nor sixths, but just 8 parts of a unit, of two different sizes.

Therefore, to add fractions that are not similar and obtain a definite sum, we must reduce them to similar fractions.

Required the sum of $\frac{3}{4}$ and $\frac{5}{6}$.

Here the L.C.M. of 4 and 6 is 12. Therefore the L.C.D. is 12.

Then $\frac{3}{4} = \frac{9}{12}$,

and $\frac{5}{6} = \frac{10}{12}$.

Now just as \$9 + \$10 = \$19, 9 twelfths + 10 twelfths = 19 twelfths.

That is, $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12} = 1\frac{7}{12}$.

Therefore, *to add fractions, reduce them to similar fractions having the least common denominator, if they are not similar. Write the sum of the numerators over the least common denominator, and reduce the result to lowest terms.*

EXERCISE 40

Find the sum of:

1. $\frac{1}{2}, \frac{1}{4}$.
2. $\frac{1}{2}, \frac{3}{4}$.
3. $\frac{1}{2}, \frac{1}{8}$.
4. $\frac{1}{2}, \frac{5}{8}$.
5. $\frac{1}{2}, \frac{7}{8}$.
6. $\frac{1}{4}, \frac{1}{8}$.
7. $\frac{1}{4}, \frac{3}{8}$.
8. $\frac{1}{4}, \frac{5}{8}$.
9. $\frac{1}{4}, \frac{7}{8}$.
10. $\frac{3}{4}, \frac{3}{8}$.
11. $\frac{3}{4}, \frac{5}{8}$.
12. $\frac{3}{4}, \frac{7}{8}$.
13. $\frac{1}{4}, \frac{1}{16}$.
14. $\frac{3}{4}, \frac{7}{16}$.
15. $\frac{3}{8}, \frac{5}{16}$.
16. $\frac{7}{8}, \frac{11}{16}$.
17. $\frac{1}{2}, \frac{3}{32}$.
18. $\frac{3}{4}, \frac{3}{32}$.
19. $\frac{5}{8}, \frac{7}{32}$.
20. $\frac{7}{8}, \frac{13}{32}$.
21. $\frac{1}{4}, \frac{5}{24}$.
22. $\frac{3}{4}, \frac{11}{24}$.
23. $\frac{3}{8}, \frac{7}{24}$.
24. $\frac{5}{8}, \frac{11}{24}$.
25. $\frac{3}{16}, \frac{1}{24}$.
26. $\frac{5}{16}, \frac{5}{24}$.
27. $\frac{7}{16}, \frac{11}{24}$.
28. $\frac{9}{16}, \frac{13}{24}$.
29. $\frac{1}{2}, \frac{1}{48}$.
30. $\frac{3}{4}, \frac{1}{48}$.
31. $\frac{3}{4}, \frac{15}{28}$.
32. $\frac{3}{8}, \frac{13}{48}$.
33. $\frac{5}{8}, \frac{17}{48}$.
34. $\frac{7}{8}, \frac{23}{48}$.
35. $\frac{1}{16}, \frac{17}{48}$.
36. $\frac{7}{16}, \frac{31}{48}$.
37. $\frac{1}{8}, \frac{5}{64}$.
38. $\frac{5}{12}, \frac{1}{64}$.
39. $\frac{7}{12}, \frac{9}{64}$.
40. $\frac{15}{16}, \frac{63}{64}$.

109. Addition of Mixed Numbers. In adding mixed numbers we may proceed as in the following example:

Required the sum of $3\frac{4}{5}$, $5\frac{7}{8}$, and $16\frac{3}{4}$.

The L.C.D. is $2 \times 2 \times 2 \times 5$, or 40. Reducing the fractions to similar fractions with denominator 40,

we have $3\frac{4}{5} = 3\frac{32}{40}$,

$5\frac{7}{8} = 5\frac{35}{40}$,

and $16\frac{3}{4} = 16\frac{30}{40}$.

Therefore the sum $= 24\frac{97}{40} = 24 + 2\frac{17}{40} = 26\frac{17}{40}$.

Therefore, *to add mixed numbers, add separately the integers and the fractions and find the sum of the results.*

EXERCISE 41

Find the sum of:

1. $2\frac{1}{2}$, $3\frac{5}{8}$.
2. $2\frac{3}{4}$, $7\frac{7}{8}$.
3. $4\frac{1}{4}$, $6\frac{5}{8}$.
4. $3\frac{1}{2}$, $5\frac{1}{3}$.
5. $4\frac{1}{2}$, $6\frac{2}{3}$.
6. $5\frac{1}{4}$, $7\frac{1}{3}$.
7. $8\frac{3}{4}$, $5\frac{1}{3}$.
8. $6\frac{3}{4}$, $8\frac{2}{3}$.
9. $2\frac{1}{3}$, $7\frac{1}{8}$.
10. $6\frac{1}{3}$, $5\frac{3}{8}$.
11. $8\frac{2}{3}$, $9\frac{5}{8}$.
12. $7\frac{2}{3}$, $8\frac{7}{8}$.
13. $3\frac{3}{4}$, $5\frac{5}{12}$.
14. $4\frac{3}{8}$, $6\frac{5}{16}$.
15. $7\frac{1}{3}$, $8\frac{3}{16}$.
16. $8\frac{2}{3}$, $9\frac{5}{32}$.
17. $27\frac{3}{4}$, $42\frac{3}{8}$.
18. $26\frac{4}{5}$, $37\frac{1}{3}$.
19. $29\frac{3}{5}$, $32\frac{2}{3}$.
20. $46\frac{1}{8}$, $35\frac{3}{16}$.
21. $27\frac{2}{3}$, $62\frac{1}{8}$.
22. $37\frac{7}{8}$, $71\frac{5}{16}$.
23. $62\frac{5}{6}$, $29\frac{3}{5}$.
24. $75\frac{7}{8}$, $62\frac{15}{16}$.
25. $92\frac{5}{6}$, $75\frac{4}{15}$.
26. $73\frac{2}{3}$, $68\frac{7}{15}$.
27. $52\frac{4}{5}$, $75\frac{13}{15}$.
28. $48\frac{9}{10}$, $73\frac{11}{12}$.
29. $52\frac{1}{5}$, $61\frac{7}{20}$.
30. $37\frac{4}{15}$, $32\frac{11}{20}$.
31. $49\frac{3}{16}$, $21\frac{5}{48}$.
32. $62\frac{5}{16}$, $91\frac{31}{32}$.
33. $1\frac{5}{8}$, $3\frac{1}{3}$, $4\frac{3}{4}$.
34. $1\frac{2}{3}$, $3\frac{4}{5}$, $5\frac{2}{15}$.
35. $2\frac{1}{5}$, $4\frac{1}{8}$, $6\frac{3}{10}$.
36. $3\frac{1}{8}$, $6\frac{3}{4}$, $5\frac{3}{16}$.
37. $4\frac{2}{3}$, $3\frac{1}{6}$, $7\frac{1}{2}$.
38. $3\frac{7}{8}$, $4\frac{3}{4}$, $5\frac{5}{16}$.
39. $5\frac{4}{5}$, $6\frac{2}{3}$, $4\frac{2}{15}$.
40. $6\frac{7}{8}$, $3\frac{1}{2}$, $5\frac{15}{16}$.
41. $4\frac{1}{3}$, $3\frac{3}{5}$, $5\frac{1}{30}$.
42. $2\frac{7}{8}$, $5\frac{13}{16}$, $6\frac{5}{32}$.
43. $3\frac{5}{6}$, $2\frac{5}{12}$, $6\frac{7}{24}$.
44. $4\frac{7}{8}$, $3\frac{2}{3}$, $5\frac{11}{24}$.
45. $3\frac{1}{16}$, $5\frac{3}{32}$, $7\frac{11}{64}$.
46. $9\frac{2}{15}$, $13\frac{13}{30}$, $15\frac{17}{20}$.
47. $8\frac{7}{16}$, $15\frac{15}{32}$, $73\frac{51}{64}$.
48. $4\frac{7}{12}$, $19\frac{11}{72}$, $36\frac{13}{144}$.

EXERCISE 42

1. Add $152\frac{1}{2}$ pounds, $16\frac{3}{4}$ pounds, and $37\frac{5}{8}$ pounds.

2. Add $16\frac{3}{8}$ yards, $27\frac{1}{4}$ yards, 6 yards, and $48\frac{1}{2}$ yards.

3. A teacher's desk is $4\frac{1}{3}$ feet long and $3\frac{1}{12}$ feet wide. What is the perimeter (distance around)?

4. Some boys built a hut $7\frac{3}{4}$ feet long and $5\frac{3}{8}$ feet wide. What was the perimeter?

5. The three sides of a triangle are $3\frac{2}{3}$ inches, $4\frac{1}{2}$ inches, and $4\frac{5}{16}$ inches. What is the perimeter?

6. If a boy weighs $56\frac{7}{8}$ pounds, his books $1\frac{3}{4}$ pounds, and his dog $20\frac{1}{2}$ pounds, how much do all together weigh?

7. A lady bought three remnants of $17\frac{3}{8}$ yards, $16\frac{3}{4}$ yards, and $23\frac{1}{3}$ yards respectively. How many yards did she buy?

8. One pole is $10\frac{1}{2}$ feet long, another $10\frac{7}{8}$ feet, and a third $10\frac{5}{16}$ feet. What is their total length when placed end to end?

9. A man has $168\frac{3}{4}$ acres of land. He buys $3\frac{3}{8}$ acres of one neighbor and $49\frac{5}{16}$ acres of another. How many acres does he then own?

10. In a pile of four books the first is $\frac{7}{8}$ of an inch thick, the second $1\frac{1}{16}$ inches, the third $\frac{3}{4}$ of an inch, and the fourth $2\frac{1}{3}$ inches. What is the height of the pile?

11. A kite string was broken, and four parts were saved. The first was $75\frac{1}{4}$ feet long, the second $127\frac{5}{8}$ feet, the third 261 feet, and the fourth $89\frac{1}{3}$ feet. Allowing 1 foot for tying, how long was the string when all four were tied together?

12. Five cans of sirup were measured very carefully and the first was found to contain $2\frac{1}{4}$ quarts, the second $2\frac{3}{8}$ quarts, the third $2\frac{7}{16}$ quarts, the fourth $2\frac{5}{32}$ quarts, and the fifth $2\frac{2}{3}$ quarts. How much sirup was there in all?

110. Subtraction of Fractions. In subtraction we reduce the fractions to similar fractions with the L.C.D. as in addition

For example, required to take $\frac{3}{8}$ from $\frac{2}{3}$.

The L.C.D. is 24. Reducing to twenty-fourths,

we have $\frac{2}{3} = \frac{16}{24}$,

and $\frac{3}{8} = \frac{9}{24}$.

Therefore the difference is $\frac{16}{24} - \frac{9}{24}$, or $\frac{7}{24}$.

Therefore, *to subtract one fraction from another, reduce them to similar fractions having the least common denominator, if they are not similar. Write the difference of the numerators over the least common denominator, and reduce the result to lowest terms.*

EXERCISE 43

Subtract as indicated:

1. $\frac{1}{2} - \frac{3}{8}$.
2. $\frac{3}{4} - \frac{3}{8}$.
3. $\frac{7}{8} - \frac{3}{4}$.
4. $\frac{7}{8} - \frac{1}{2}$.
5. $\frac{5}{8} - \frac{1}{4}$.
6. $\frac{1}{2} - \frac{5}{16}$.
7. $\frac{1}{2} - \frac{3}{16}$.
8. $\frac{1}{4} - \frac{1}{16}$.
9. $\frac{3}{4} - \frac{5}{16}$.
10. $\frac{5}{8} - \frac{5}{16}$.
11. $\frac{7}{8} - \frac{9}{16}$.
12. $\frac{1}{2} - \frac{3}{32}$.
13. $\frac{1}{4} - \frac{5}{32}$.
14. $\frac{29}{32} - \frac{3}{4}$.
15. $\frac{29}{32} - \frac{7}{8}$.
16. $\frac{1}{2} - \frac{1}{64}$.
17. $\frac{1}{2} - \frac{15}{64}$.
18. $\frac{1}{2} - \frac{23}{64}$.
19. $\frac{1}{4} - \frac{3}{64}$.
20. $\frac{3}{4} - \frac{31}{64}$.
21. $\frac{3}{4} - \frac{35}{64}$.
22. $\frac{57}{64} - \frac{3}{4}$.
23. $\frac{1}{8} - \frac{3}{64}$.
24. $\frac{3}{8} - \frac{5}{64}$.
25. $\frac{5}{8} - \frac{21}{64}$.
26. $\frac{7}{8} - \frac{51}{64}$.
27. $\frac{3}{16} - \frac{3}{64}$.
28. $\frac{5}{16} - \frac{7}{64}$.
29. $\frac{7}{16} - \frac{13}{48}$.
30. $\frac{9}{16} - \frac{17}{48}$.
31. $\frac{1}{2} - \frac{5}{72}$.
32. $\frac{1}{2} - \frac{11}{72}$.
33. $\frac{1}{4} - \frac{7}{72}$.
34. $\frac{3}{4} - \frac{35}{72}$.
35. $\frac{1}{8} - \frac{1}{72}$.
36. $\frac{3}{8} - \frac{23}{72}$.
37. $\frac{5}{8} - \frac{41}{72}$.
38. $\frac{7}{8} - \frac{61}{72}$.
39. $\frac{1}{16} - \frac{1}{72}$.
40. $\frac{3}{16} - \frac{5}{72}$.
41. $\frac{5}{16} - \frac{7}{72}$.
42. $\frac{7}{12} - \frac{1}{144}$.
43. $\frac{11}{12} - \frac{37}{144}$.
44. $\frac{7}{10} - \frac{37}{100}$.
45. $\frac{9}{10} - \frac{113}{1000}$.

111. Subtraction of Mixed Numbers. In subtracting mixed numbers we proceed as in the following examples:

(1) Required to subtract $4\frac{2}{3}$ from $7\frac{7}{8}$.

The L.C.D. is 3×8, or 24. Reducing to twenty-fourths,

we have $7\frac{7}{8} = 7\frac{21}{24}$,

and $4\frac{2}{3} = 4\frac{16}{24}$.

Therefore the difference is $7\frac{21}{24} - 4\frac{16}{24}$, or $3\frac{5}{24}$.

(2) Required to subtract $5\frac{3}{4}$ from $8\frac{1}{6}$.

The L.C.D. is $2 \times 2 \times 3$, or 12. Reducing to twelfths,

we have $8\frac{1}{6} = 8\frac{2}{12}$,

and $5\frac{3}{4} = 5\frac{9}{12}$.

Since we cannot take 9 twelfths from 2 twelfths, we change $8\frac{2}{12}$ to $7 + 1\frac{2}{12}$, or $7\frac{14}{12}$. Then $7\frac{14}{12} - 5\frac{9}{12} = 2\frac{5}{12}$.

Therefore, *to subtract one mixed number from another, reduce the fractions to similar fractions; then subtract the integers and fractions separately.*

EXERCISE 44

Subtract as indicated:

1. $7\frac{7}{8} - 2\frac{1}{4}$.
2. $9\frac{7}{8} - 3\frac{3}{4}$.
3. $6\frac{5}{8} - 2\frac{1}{2}$.
4. $8\frac{5}{8} - 2\frac{1}{4}$.
5. $7\frac{3}{16} - 5\frac{1}{8}$.
6. $9\frac{7}{16} - 7\frac{3}{8}$.
7. $8\frac{9}{16} - 3\frac{3}{8}$.
8. $6\frac{11}{16} - 4\frac{5}{8}$.
9. $7\frac{15}{16} - 3\frac{7}{8}$.
10. $9\frac{31}{32} - 6\frac{3}{4}$.
11. $8\frac{29}{32} - 5\frac{3}{8}$.
12. $11\frac{1}{8} - 2\frac{1}{4}$.
13. $10\frac{3}{8} - 3\frac{3}{4}$.
14. $15\frac{5}{8} - 7\frac{3}{4}$.
15. $12\frac{3}{4} - 6\frac{7}{8}$.
16. $14\frac{1}{4} - 9\frac{3}{8}$.
17. $23\frac{1}{8} - 8\frac{3}{16}$.
18. $25\frac{3}{8} - 9\frac{7}{16}$.
19. $28\frac{5}{8} - 7\frac{25}{32}$.
20. $32\frac{7}{8} - 9\frac{31}{32}$.
21. $35\frac{1}{16} - 6\frac{3}{32}$.
22. $39\frac{3}{16} - 7\frac{11}{32}$.
23. $12\frac{1}{3} - 8\frac{1}{4}$.
24. $15\frac{1}{4} - 7\frac{1}{3}$.
25. $16\frac{1}{3} - 2\frac{3}{5}$.
26. $19\frac{2}{3} - 3\frac{4}{5}$.
27. $31\frac{5}{6} - 7\frac{7}{12}$.
28. $42\frac{5}{6} - 8\frac{5}{12}$.
29. $39\frac{3}{5} - 2\frac{1}{12}$.
30. $48\frac{4}{5} - 6\frac{5}{12}$.
31. $73\frac{1}{3} - 7\frac{7}{12}$.
32. $69\frac{2}{3} - 8\frac{3}{16}$.
33. $75\frac{1}{16} - 9\frac{3}{32}$.

EXERCISE 45

Find the value of:

1. $32 - 4\frac{7}{8}$.
2. $63 - 5\frac{3}{4}$.
3. $72 - 8\frac{9}{10}$.
4. $48 - 3\frac{7}{32}$.
5. $67 - 5\frac{3}{16}$.
6. $71 - 6\frac{5}{24}$.
7. $80 - 9\frac{5}{48}$.
8. $52 - 7\frac{13}{36}$.
9. $46 - 9\frac{9}{64}$.
10. $53 - 6\frac{3}{50}$.
11. $\frac{1}{2} + \frac{3}{4} - \frac{1}{3}$.
12. $\frac{2}{3} + \frac{3}{5} - \frac{1}{2}$.
13. $\frac{3}{4} + \frac{7}{8} - \frac{2}{3}$.
14. $\frac{4}{5} + \frac{5}{8} - \frac{3}{4}$.
15. $\frac{3}{16} + \frac{7}{8} - \frac{3}{4}$.
16. $\frac{1}{2} - \frac{1}{4} + \frac{7}{8} - 1$.
17. $\frac{7}{8} - \frac{1}{2} + \frac{3}{4} - 1$.
18. $\frac{3}{5} + \frac{1}{15} - \frac{1}{3} + 2$.
19. $\frac{2}{3} + \frac{1}{6} - \frac{1}{2} + 5\frac{1}{2}$.
20. $20 - 2\frac{1}{3} + 1\frac{3}{5} - 6$.

21. If we add $12\frac{1}{2}$ to $16\frac{2}{3}$, and then subtract $9\frac{1}{6}$ from the sum, what is the result?

22. If I make some purchases costing $\$1\frac{1}{2}$, $\$3\frac{1}{4}$, $\$2\frac{3}{4}$, $\$5\frac{3}{4}$, and $\$1\frac{1}{2}$, and give the dealer \$15 in bills, how much change is due me?

23. From a piece of goods containing $48\frac{1}{4}$ yards there is sold to one man $6\frac{1}{2}$ yards, to another $5\frac{7}{8}$ yards, and to a third $7\frac{3}{4}$ yards. How much is left?

24. A man had a farm of $168\frac{1}{4}$ acres. He sold $2\frac{3}{8}$ acres to one man, $3\frac{1}{2}$ acres to another, and $7\frac{3}{4}$ acres to a third. How much had he left?

25. A train that reached Chicago at $8\frac{3}{4}$ o'clock A.M. left Detroit $7\frac{7}{12}$ hours earlier. What time did it leave Detroit?

26. What number must be subtracted from $301\frac{1}{4}$ to leave $175\frac{7}{8}$?

27. From what number must $268\frac{7}{16}$ be subtracted to leave $107\frac{3}{32}$?

28. What number must be added to $172\frac{3}{16}$ to make $341\frac{1}{8}$?

29. If a piece of flannel measuring $32\frac{1}{8}$ yards shrinks $2\frac{3}{4}$ yards in dyeing, what is the length after dyeing?

30. It was found that $\frac{3}{16}$ of a well was through gravel, $\frac{1}{4}\frac{1}{8}$ through rock, and the rest through soil. What part of it was through soil? (Take $\frac{3}{16} + \frac{1}{4}\frac{1}{8}$ from 1.)

31. A man spent $\frac{1}{5}$ of his income for rent, $\frac{3}{16}$ for food, and $\frac{5}{24}$ for sundries. What part of his income was left?

32. A boy lost $\frac{1}{4}$ of his kite string in a tree, $\frac{1}{3}$ in some wires, and $\frac{1}{5}$ in a hedge. What part of the string was left?

33. A manufacturer of rings had an ingot of gold, of which he used $\frac{1}{3}$ at one time, $\frac{1}{6}$ at another, $\frac{1}{12}$ at another, and $\frac{3}{8}$ at another. What part of the ingot remained?

34. In building a battle ship it was reported at one time that $\frac{5}{96}$ of the work was finished. During the following month $\frac{1}{24}$ of the work was done, during the next month $\frac{3}{48}$, and during the next month $\frac{5}{192}$. What part of the work remained unfinished?

35. Of some bridge construction, $\frac{3}{32}$ was done the first month, $\frac{1}{8}$ the second, $\frac{3}{16}$ the third, $\frac{3}{64}$ the fourth, and $\frac{5}{128}$ the fifth. What part of the work then remained to be accomplished?

36. A farmer has $\frac{3}{8}$ of his land given up to pasture, $\frac{5}{16}$ to meadow, $\frac{1}{32}$ to a garden, and $\frac{1}{64}$ to his house and grounds. How much of the farm is used for other purposes?

37. A farmer had been using $\frac{7}{16}$ of his land for a pasture, when he decided to increase it by $\frac{3}{32}$ of his farm. The next year he took from the pasture $\frac{3}{64}$ of his farm and used it for meadow. What part of his land then remained for the pasture?

38. What number must be added to the sum of $2\frac{3}{4}$, $5\frac{7}{8}$, $4\frac{3}{16}$, and $9\frac{2}{3}$ to make the sum of $5\frac{7}{12}$, $9\frac{5}{8}$, $12\frac{1}{3}$, and $8\frac{1}{24}$?

112. Multiplication of a Fraction by an Integer. Just as

2 times 2 feet are 4 feet,

so 2 times 2 fifths ($\frac{2}{5}$) are 4 fifths ($\frac{4}{5}$).

In the same way, $3 \times \frac{1}{8} = \frac{3}{8}$, $3 \times \frac{2}{7} = \frac{6}{7}$, $5 \times \frac{3}{8} = \frac{15}{8} = 1\frac{7}{8}$.

If we have to find $6 \times \frac{3}{4}$, it is better to indicate the multiplication first and shorten the actual work by cancellation, thus:

$$6 \times \frac{3}{4} = \frac{\overset{3}{\cancel{6}} \times 3}{\underset{2}{\cancel{4}}} = \frac{9}{2} = 4\frac{1}{2}.$$

Therefore, *to multiply a fraction by an integer, multiply the numerator by the integer and write the product over the denominator.*

Before actually multiplying, indicate the multiplication and cancel if possible. Reduce the result to lowest terms.

EXERCISE 46

Multiply:

1. $6 \times \frac{3}{5}$.
2. $8 \times \frac{3}{4}$.
3. $15 \times \frac{2}{3}$.
4. $18 \times \frac{3}{4}$.
5. $22 \times \frac{3}{8}$.
6. $26 \times \frac{5}{8}$.
7. $32 \times \frac{7}{8}$.
8. $35 \times \frac{3}{16}$.
9. $42 \times \frac{5}{16}$.
10. $56 \times \frac{7}{16}$.
11. $120 \times \frac{9}{16}$.
12. $128 \times \frac{11}{16}$.
13. $160 \times \frac{3}{32}$.
14. $176 \times \frac{5}{32}$.
15. $192 \times \frac{7}{32}$.
16. $208 \times \frac{9}{32}$.
17. $224 \times \frac{11}{32}$.
18. $240 \times \frac{15}{32}$.
19. $256 \times \frac{27}{32}$.
20. $272 \times \frac{31}{32}$.
21. $126 \times \frac{5}{12}$.
22. $140 \times \frac{7}{12}$.
23. $150 \times \frac{11}{12}$.
24. $216 \times \frac{5}{24}$.
25. $228 \times \frac{7}{24}$.
26. $252 \times \frac{11}{24}$.
27. $264 \times \frac{13}{24}$.
28. $325 \times \frac{9}{10}$.
29. $432 \times \frac{7}{48}$.
30. $500 \times \frac{11}{48}$.

31. If a book costs $\$\frac{3}{4}$, how much will 17 books cost?

32. If a train travels $\frac{5}{6}$ of a mile in a minute, how far will it travel in 132 minutes?

113. Multiplication of an Integer by a Fraction. To take a fractional part of a number is called *multiplication by a fraction.*

That is, to take $\frac{1}{3}$ of 6 is called the multiplication of 6 by $\frac{1}{3}$. We indicate the operation by the expression: $\frac{1}{3}$ of $6 = 2$, or $\frac{1}{3} \times 6 = 2$, the symbol $\times$ being read *of* in the latter case.

Therefore, *to multiply an integer by a fraction, multiply the integer by the numerator of the fraction and write the product over the denominator.*

It is advisable to cancel before the actual multiplication, if possible. Thus, in taking $\frac{4}{15}$ of 12, the work should be arranged as follows:

$$\frac{4}{15} \text{ of } 12 = \frac{4 \times \overset{4}{\cancel{12}}}{\underset{5}{\cancel{15}}} = \frac{16}{5} = 3\tfrac{1}{5}.$$

EXERCISE 47

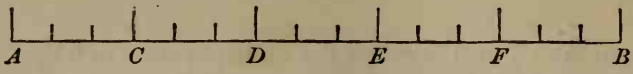
Multiply:

1. $\frac{2}{3}$ of 18.	11. $\frac{5}{12}$ of 72.	21. $\frac{5}{24}$ of 72.
2. $\frac{3}{4}$ of 24.	12. $\frac{7}{12}$ of 80.	22. $\frac{7}{24}$ of 80.
3. $\frac{2}{5}$ of 35.	13. $\frac{11}{12}$ of 96.	23. $\frac{11}{24}$ of 84.
4. $\frac{3}{5}$ of 48.	14. $\frac{3}{16}$ of 72.	24. $\frac{13}{24}$ of 96.
5. $\frac{4}{5}$ of 75.	15. $\frac{5}{16}$ of 80.	25. $\frac{17}{24}$ of 100.
6. $\frac{5}{6}$ of 93.	16. $\frac{7}{16}$ of 96.	26. $\frac{5}{32}$ of 100.
7. $\frac{3}{8}$ of 24.	17. $\frac{9}{16}$ of 100.	27. $\frac{7}{32}$ of 200.
8. $\frac{5}{8}$ of 68.	18. $\frac{11}{16}$ of 110.	28. $\frac{5}{64}$ of 144.
9. $\frac{7}{8}$ of 96.	19. $\frac{13}{16}$ of 120.	29. $\frac{7}{64}$ of 128.
10. $\frac{5}{8}$ of 90.	20. $\frac{15}{16}$ of 240.	30. $\frac{15}{64}$ of 192.

31. At 72 cents a yard, how much will $\frac{5}{8}$ of a yard of cloth cost?

32. At 96 cents a yard, how much will $\frac{7}{8}$ of a yard of cloth cost?

114. Multiplication of a Fraction by a Fraction. Suppose we are required to multiply $\frac{4}{5}$ by $\frac{2}{3}$.



In this figure let the line AB be divided into five equal parts at the points C, D, E, and F. Then AF is $\frac{4}{5}$ of AB.

Let each of the 5 parts be divided into 3 equal parts. Then each part evidently is $\frac{1}{15}$ of AB.

That is, $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ of the whole.
Then $\frac{1}{3}$ of $\frac{4}{5}$ must be 4 times as much, or $\frac{4}{15}$.
Then $\frac{2}{3}$ of $\frac{4}{5}$ must be twice $\frac{4}{15}$, or $\frac{8}{15}$.

Therefore, *to multiply a fraction by a fraction, find the product of the numerators for the required numerator and the product of the denominators for the required denominator.*

Always cancel as far as possible before the actual multiplication. For example,

$$\frac{11}{12} \text{ of } \frac{15}{22} = \frac{11 \times \overset{5}{\cancel{15}}}{\underset{4}{\cancel{12}} \times \underset{2}{\cancel{22}}} = \frac{5}{8}.$$

EXERCISE 48

Multiply:

1. $\frac{3}{4}$ of $\frac{8}{9}$.
2. $\frac{2}{3}$ of $\frac{9}{10}$.
3. $\frac{2}{3}$ of $\frac{15}{16}$.
4. $\frac{2}{3}$ of $\frac{11}{24}$.
5. $\frac{2}{3}$ of $\frac{25}{32}$.
6. $\frac{3}{4}$ of $\frac{5}{24}$.
7. $\frac{1}{8}$ of $\frac{4}{5}$.
8. $\frac{3}{8}$ of $\frac{7}{8}$.
9. $\frac{7}{8}$ of $\frac{4}{7}$.
10. $\frac{5}{8}$ of $\frac{3}{10}$.
11. $\frac{3}{16}$ of $\frac{8}{9}$.
12. $\frac{7}{16}$ of $\frac{8}{21}$.
13. $\frac{1}{32}$ of $\frac{4}{5}$.
14. $\frac{1}{48}$ of $\frac{24}{25}$.
15. $\frac{7}{48}$ of $\frac{16}{21}$.
16. $\frac{3}{5}$ of $\frac{5}{48}$.
17. $\frac{4}{5}$ of $\frac{25}{48}$.
18. $\frac{81}{100}$ of $\frac{5}{9}$.
19. How much is half of three quarters of a yard?
20. How much is $\frac{2}{3}$ of $\frac{5}{8}$ of an acre?
21. How much is a quarter of $\frac{7}{8}$ of a mile?
22. How much is an eighth of $\frac{5}{16}$ of an inch?

Find the products of the following:

23. $\frac{5}{8} \times \frac{24}{25}$.
24. $\frac{3}{8} \times \frac{16}{21}$.
25. $\frac{7}{8} \times \frac{32}{35}$.
26. $\frac{9}{16} \times \frac{4}{9}$.
27. $\frac{7}{16} \times \frac{5}{7}$.
28. $\frac{5}{16} \times \frac{32}{45}$.
29. $1\frac{3}{16} \times \frac{2}{3}$.
30. $1\frac{5}{16} \times \frac{2}{5}$.
31. $\frac{7}{12} \times \frac{4}{9}$.
32. $\frac{5}{12} \times \frac{3}{5}$.
33. $1\frac{1}{12} \times \frac{4}{5}$.
34. $1\frac{1}{16} \times \frac{4}{11}$.
35. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.
36. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$.
37. $\frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}$.
38. $\frac{3}{5} \times \frac{3}{4} \times \frac{1}{8}$.
39. $\frac{3}{4} \times \frac{3}{8} \times \frac{4}{5}$.
40. $\frac{5}{8} \times \frac{7}{8} \times \frac{3}{8}$.

41. The perimeter of (distance around) a square being $\frac{7}{8}$ of an inch, what is the length of each side?

42. The perimeter of a triangle being $\frac{7}{8}$ of an inch and the sides being equal, what is the length of each side?

43. If the distance to a certain house is $\frac{3}{8}$ of a mile, what fraction of a mile has a boy gone when he has covered $\frac{5}{16}$ of the distance?

44. If it takes a train $\frac{3}{4}$ of an hour to reach a certain station, what fraction of an hour will it take to cover $\frac{3}{5}$ of the distance?

45. A remnant of lace cost $\$\frac{5}{8}$. How much did $\frac{4}{5}$ of the remnant cost? How much did $\frac{3}{5}$ of it cost?

46. If $\frac{3}{4}$ of a yard of lace is needed to trim two neck bands, how much lace will be needed to trim one neck band?

47. If an automobile travels a mile in $\frac{52}{60}$ of a minute in a certain race, how long would it take to travel $\frac{3}{4}$ of a mile at the same rate?

48. If a woman pays $\$\frac{3}{4}$ for a yard of ribbon, how much would she pay for $\frac{2}{3}$ of a yard? for $\frac{1}{3}$ of a yard? for $\frac{1}{2}$ of a yard?

49. A milliner used $\frac{7}{8}$ of a yard of velvet for the upper brim and the crown of a hat, and $\frac{2}{5}$ as much for the lower brim. How much velvet did she use for the hat?

50. A piece of goods $\frac{7}{8}$ of a yard long is $\frac{2}{3}$ as long after tucking. How long is it after tucking?

51. A piece of cloth $\frac{5}{8}$ of a yard long loses $\frac{2}{5}$ of its length in tucking. How long is it then?

52. A woman has cut some goods $\frac{1}{4}$ of a yard wide for a ruffle. She loses $\frac{1}{16}$ of this width in hemming and $\frac{2}{3}$ of the rest in tucking. How wide is the ruffle when finished?

53. If I pay \$2 for $\frac{3}{4}$ of a yard of a certain kind of velvet, what part of a yard will I buy with \$1? What part with \$1.50, which is $\frac{3}{4}$ of \$2?

54. A girl in a shop makes $\frac{5}{6}$ of a yard of a certain kind of chiffon appliqué in an hour. How much does she make in 20 minutes? in 45 minutes?

55. If it takes $\frac{3}{4}$ of an hour to solve a page of examples, how long would it take to solve $\frac{3}{5}$ of them at the same rate? How many minutes?

56. A man who owned $\frac{5}{9}$ of the stock in a certain company sold $\frac{3}{5}$ of what he owned. What part of the stock of the company did he then own?

57. In making the plans for a house an architect wishes to draw a line that shall be $\frac{2}{3}$ as long as one that is $\frac{5}{16}$ of an inch long. How long shall he draw it?

58. In 40 minutes the minute hand of a clock makes $\frac{2}{3}$ of a revolution. What part of a revolution will it make in 32 minutes? (32 minutes $= \frac{32}{40}$ of 40 minutes.)

59. In 55 minutes the minute hand of a clock makes $\frac{11}{2}$ of a revolution. What part of a revolution will it make in 33 minutes?

60. A boy finds that when the front wheel of his cart has made one revolution the cart has moved $\frac{5}{6}$ of a yard. How far has it gone when the wheel has made $\frac{3}{4}$ of a revolution?

115. Multiplication of a Mixed Number by an Integer. Since $3 \times 5 = 15$, and $3 \times \frac{5}{6} = \frac{5}{2} = 2\frac{1}{2}$, therefore $3 \times 5\frac{5}{6}$ equals the sum of these products, that is, $15 + 2\frac{1}{2}$, or $17\frac{1}{2}$.

Therefore, *to multiply a mixed number by an integer, multiply the whole part and the fractional part separately by the integer and add the products.*

EXERCISE 49

Multiply:

1. $2 \times 17\frac{2}{3}$.
2. $3 \times 26\frac{1}{4}$.
3. $3 \times 18\frac{3}{4}$.
4. $4 \times 35\frac{3}{4}$.
5. $2 \times 43\frac{3}{8}$.
6. $3 \times 25\frac{5}{8}$.
7. $6 \times 56\frac{7}{8}$.
8. $4 \times 33\frac{5}{12}$.
9. $3 \times 25\frac{7}{12}$.
10. $5 \times 48\frac{11}{12}$.
11. $2 \times 16\frac{3}{16}$.
12. $3 \times 17\frac{5}{16}$.
13. $4 \times 25\frac{7}{16}$.
14. $8 \times 39\frac{11}{16}$.
15. $8 \times 58\frac{15}{16}$.
16. $2 \times 45\frac{3}{32}$.
17. $4 \times 74\frac{5}{32}$.
18. $9 \times 86\frac{7}{32}$.
19. $8 \times 69\frac{9}{32}$.
20. $6 \times 97\frac{11}{32}$.
21. $4 \times \$32\frac{3}{4}$.
22. $7 \times \$21\frac{1}{4}$.
23. $6 \times \$35\frac{1}{2}$.
24. $8 \times \$27\frac{3}{4}$.
25. $12 \times 117\frac{1}{2}$.
26. $15 \times 222\frac{1}{5}$.
27. $12 \times 305\frac{1}{8}$.
28. $16 \times 211\frac{7}{8}$.
29. $25 \times 107\frac{1}{5}$.
30. $35 \times 121\frac{3}{10}$.

31. At $\$2\frac{1}{4}$ a yard, how much will 9 yards of cloth cost?

32. How many feet in the perimeter of a square $26\frac{3}{16}$ feet on a side?

33. A dealer sells 16 notebooks at $3\frac{3}{4}$ cents each. How much does he receive?

34. A rod being $16\frac{1}{2}$ feet long, how many feet in the perimeter of a lot that is 26 rods long and 16 rods wide?

35. In a motor race one of the cars averaged $1\frac{3}{8}$ miles a minute. How many miles did it travel in 7 minutes?

36. If a boy lives $1\frac{3}{4}$ miles from this school, and has a cousin living 25 times as far from here, how far from here does the cousin live?

116. Multiplication of an Integer by a Mixed Number. Since $2 \times 9 = 18$, and $\frac{3}{4}$ of $9 = \frac{27}{4} = 6\frac{3}{4}$, therefore $2\frac{3}{4} \times 9$ equals the sum of these products, or $18 + 6\frac{3}{4}$, or $24\frac{3}{4}$.

The symbol $\times$ after a mixed number is read *times*, as with integers.

Therefore, *to multiply an integer by a mixed number, multiply the integer by the whole part and the fractional part separately and add the products.*

In practice, to take the product of an integer and a mixed number we use as the multiplier whichever is the more convenient. For example, it is shorter to multiply $249\frac{3}{4}$ by 2 than to multiply 2 by $249\frac{3}{4}$. But if it is required to multiply $4\frac{7}{8}$ by 648, it is easier to change the order and multiply 648 by $4\frac{7}{8}$. These two examples may be arranged thus:

$$\begin{array}{r} 249\frac{3}{4} \\ \underline{\quad 2 \quad} \\ 498\frac{3}{2} = 499\frac{1}{2} \end{array} \qquad \begin{array}{rl} 648 & \\ \underline{4\frac{7}{8}} & \\ 81 & (\tfrac{1}{8} \text{ of } 648) \\ \underline{7} & \\ 567 & (\tfrac{7}{8} \text{ of } 648) \\ 2592 & (4 \times 648) \\ \hline 3159 & \end{array}$$

EXERCISE 50

Multiply:

1. $3\frac{4}{5} \times 75$.
2. $4\frac{2}{3} \times 69$.
3. $5\frac{1}{4} \times 72$.
4. $7\frac{3}{4} \times 84$.
5. $6\frac{1}{4} \times 31$.
6. $8\frac{3}{4} \times 39$.
7. $7\frac{3}{8} \times 48$.
8. $9\frac{3}{8} \times 50$.
9. $5\frac{5}{8} \times 64$.
10. $7\frac{5}{8} \times 70$.
11. $3\frac{7}{8} \times 96$.
12. $5\frac{7}{8} \times 91$.
13. $2\frac{1}{16} \times 32$.
14. $3\frac{3}{16} \times 64$.
15. $5\frac{5}{16} \times 80$.
16. $9\frac{7}{16} \times 96$.
17. $5\frac{3}{16} \times 38$.
18. $4\frac{5}{16} \times 60$.
19. $9\frac{7}{16} \times 88$.
20. $7\frac{9}{16} \times 62$.
21. $3\frac{3}{32} \times 160$.
22. $4\frac{5}{32} \times 640$.
23. $7\frac{7}{32} \times 176$.
24. $8\frac{9}{32} \times 656$.
25. $9\frac{11}{32} \times 352$.
26. $12\frac{13}{32} \times 288$.
27. $15\frac{15}{32} \times 256$.
28. $20\frac{5}{48} \times 240$.
29. $300\frac{7}{48} \times 528$.
30. $500\frac{9}{64} \times 384$.

117. Multiplication of Two Mixed Numbers. We may multiply one mixed number by another by the method explained in § 116 or by that in § 114.

$$\begin{array}{r} 9\frac{3}{4} \\ 2\frac{1}{3} \\ \hline 3\frac{1}{4} \quad (\frac{1}{3} \text{ of } 9\frac{3}{4}) \\ 19\frac{2}{4} \quad (2 \times 9\frac{3}{4}) \\ \hline 22\frac{3}{4} \end{array}$$

For example, we may multiply $9\frac{3}{4}$ by $2\frac{1}{3}$ as here shown. But it is usually easier to reduce both to improper fractions and multiply as in § 114. Thus in the case of $6\frac{2}{5} \times 4\frac{11}{16}$ we have $6\frac{2}{5} = \frac{32}{5}$, and $4\frac{11}{16} = \frac{75}{16}$. Then

$$6\frac{2}{5} \times 4\frac{11}{16} = \frac{\overset{2}{\cancel{32}} \times \overset{15}{\cancel{75}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{16}}} = 30.$$

Therefore, *to multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply.*

In the case of a fraction and a mixed number, reduce the mixed number to an improper fraction.

EXERCISE 51

Multiply:

1. $2\frac{1}{2} \times 3\frac{3}{8}$.
2. $3\frac{1}{3} \times 4\frac{3}{4}$.
3. $2\frac{2}{3} \times 3\frac{1}{4}$.
4. $5\frac{3}{5} \times 7\frac{3}{4}$.
5. $4\frac{1}{3} \times 6\frac{3}{8}$.
6. $5\frac{3}{5} \times 6\frac{7}{8}$.
7. $6\frac{2}{7} \times 7\frac{7}{8}$.
8. $6\frac{1}{8} \times 9\frac{5}{7}$.
9. $7\frac{3}{5} \times 7\frac{7}{8}$.
10. $9\frac{2}{3} \times 8\frac{5}{8}$.
11. $2\frac{1}{2} \times 31\frac{1}{2}$.
12. $3\frac{1}{2} \times 25\frac{1}{3}$.
13. $7\frac{1}{2} \times 33\frac{1}{3}$.
14. $3\frac{3}{4} \times 66\frac{2}{3}$.
15. $5\frac{1}{5} \times 29\frac{3}{8}$.
16. $7\frac{3}{4} \times 63\frac{7}{8}$.
17. $4\frac{2}{9} \times 28\frac{1}{8}$.
18. $8\frac{5}{7} \times 31\frac{1}{16}$.
19. $5\frac{3}{7} \times 48\frac{11}{16}$.
20. $4\frac{4}{5} \times 51\frac{9}{16}$.
21. $33\frac{1}{3} \times 66\frac{2}{3}$.
22. $33\frac{1}{3} \times 120\frac{3}{5}$.
23. $16\frac{2}{3} \times 132\frac{3}{5}$.
24. $16\frac{2}{3} \times 336\frac{6}{7}$.
25. $21\frac{3}{4} \times 441\frac{1}{3}$.
26. $35\frac{1}{5} \times 228\frac{9}{32}$.
27. $63\frac{1}{5} \times 127\frac{1}{32}$.
28. $48\frac{2}{3} \times 48\frac{3}{16}$.
29. $61\frac{1}{24} \times 240\frac{24}{25}$.
30. $37\frac{7}{24} \times 480\frac{48}{100}$.

31. At $\$66\frac{2}{3}$ a month, how much rent must a man pay for a city apartment for 6 months?

32. If a train travels at the rate of $48\frac{3}{4}$ miles per hour, how far will it travel in $2\frac{1}{2}$ hours?

EXERCISE 52

1. At $\$12\frac{3}{4}$ a ton, how much will $10\frac{1}{2}$ tons of hay cost?

2. How much will 54 plates cost at $\$3\frac{1}{2}$ a dozen?

3. At $\$6\frac{3}{4}$ each, how much will 16 dictionary holders cost? How much will 24 cost?

4. At $\$1\frac{1}{4}$ a dozen, how much will $3\frac{1}{2}$ dozen pin cushions cost? How much will $7\frac{1}{2}$ dozen cost?

5 There being $16\frac{1}{2}$ feet in a rod, how many feet in $15\frac{2}{3}$ rods? How many feet in $17\frac{1}{3}$ rods?

6. There being $5\frac{1}{2}$ yards in a rod, how many yards in $17\frac{2}{3}$ rods? How many yards in 32 rods?

7. How much will $8\frac{3}{4}$ dozen pencils cost at the rate of $\$\frac{1}{4}$ for a half dozen?

8. At the rate of $3\frac{3}{4}$ pounds a week, how much sugar will a family use in the $4\frac{3}{7}$ weeks of January?

9. At the rate of $14\frac{7}{8}$ pounds of meat a week, how much meat will a family use in April?

10. How far will a commercial salesman travel in 6 days at an average of $42\frac{5}{12}$ miles a day?

11. An ocean steamer averaged $16\frac{7}{8}$ miles an hour for $23\frac{1}{2}$ hours. What was the distance made?

12. Some blotting paper is $\frac{3}{64}$ of an inch thick. What will be the height of a pile of $2\frac{3}{4}$ dozen sheets?

13. If an arithmetic is $1\frac{1}{8}$ inches thick and a reader is $\frac{5}{16}$ of an inch thick, what is the height of a pile of 7 arithmetics and 9 readers?

14. Two farmers buy a mowing machine for $72, each contributing according to the land he mows. The first paid $\frac{3}{8}$ of the price and the second paid the rest. How much did the second pay?

118. Division of a Fraction by an Integer. There are two cases to be considered:

(1) When the numerator of the fraction is divisible by the divisor.

In this case, just as 8 feet $\div$ 2 = 4 feet, and \$8 $\div$ 2 = \$4, so 8 ninths ($\frac{8}{9}$) $\div$ 2 = 4 ninths ($\frac{4}{9}$). In the same way, $\frac{9}{10} \div 3 = \frac{3}{10}$, and $\frac{15}{16} \div 5 = \frac{3}{16}$.

(2) When the numerator of the fraction is not divisible by the divisor.

Take the case of $\frac{3}{5} \div 2$. This is the same as $\frac{1}{2}$ of $\frac{3}{5}$, which equals $\frac{3}{10}$. In the same way, $\frac{5}{8} \div 3 = \frac{1}{3}$ of $\frac{5}{8} = \frac{5}{24}$.

Therefore, *to divide a fraction by an integer, divide the numerator by the integer if possible, and write the quotient over the denominator.*

If it is impossible to divide the numerator by the integer, multiply the denominator by the integer and write the product under the numerator.

It is easier to indicate this multiplication and then cancel if possible. Thus

$$\frac{15}{16} \div 6 = \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{6}} \times 16} = \frac{5}{32}.$$

EXERCISE 53

Find the quotient of:

1. $\frac{3}{4}\frac{5}{8} \div 7$.
2. $\frac{2}{3}\frac{7}{2} \div 9$.
3. $\frac{5}{6}\frac{1}{4} \div 17$.
4. $\frac{3}{16} \div 6$.
5. $\frac{1}{1}\frac{5}{6} \div 10$.
6. $\frac{2}{3}\frac{5}{2} \div 15$.
7. $\frac{1}{1}\frac{5}{6} \div 25$.
8. $\frac{1}{3}\frac{5}{2} \div 20$.
9. $\frac{1}{2}\frac{9}{4} \div 57$.
10. $\frac{1}{4}\frac{7}{8} \div 51$.
11. $\frac{34}{100} \div 51$.
12. $\frac{38}{100} \div 57$.
13. $\frac{2}{3}\frac{5}{2} \div 100$.
14. $\frac{2}{6}\frac{5}{4} \div 125$.
15. $\frac{2}{2}\frac{1}{4} \div 126$.
16. $\frac{2}{2}\frac{3}{4} \div 161$.
17. $\frac{27}{100} \div 170$.
18. $\frac{37}{100} \div 333$.
19. $\frac{33}{1000} \div 77$.
20. $\frac{51}{1000} \div 170$.
21. $\frac{143}{1000} \div 130$.

119. Division of a Mixed Number by an Integer. A mixed number may be divided by an integer by first reducing it to an improper fraction and then proceeding as in § 118.

For example, $3\frac{5}{8} \div 7 = \frac{29}{8} \div 7 = \frac{29}{56}$.

In case the mixed number is long, however, it is easier to divide as with integers, continuing the division until a remainder is found that is smaller than the divisor.

For example, $177\frac{1}{2} \div 5 = 35$ with remainder $2\frac{1}{2}$. Dividing the remainder, we have $2\frac{1}{2} \div 5 = \frac{5}{2} \div 5 = \frac{1}{2}$. Hence the entire quotient is $35\frac{1}{2}$.

Therefore, *to divide a mixed number by an integer, divide as with integers until a remainder is found that is less than the divisor; then divide this remainder by the divisor.*

EXERCISE 54

Find the quotient of:

1. $3\frac{3}{4} \div 9$.
2. $4\frac{3}{5} \div 7$.
3. $5\frac{2}{3} \div 6$.
4. $6\frac{7}{8} \div 5$.
5. $8\frac{2}{3} \div 4$.
6. $7\frac{3}{8} \div 59$.
7. $23\frac{2}{3} \div 8$.
8. $28\frac{3}{4} \div 7$.
9. $32\frac{5}{8} \div 9$.
10. $41\frac{5}{8} \div 6$.
11. $34\frac{1}{16} \div 7$.
12. $78\frac{3}{16} \div 9$.
13. $142\frac{1}{2} \div 5$.
14. $166\frac{2}{3} \div 10$.
15. $133\frac{1}{3} \div 20$.
16. $275\frac{3}{4} \div 12$.
17. $266\frac{7}{8} \div 15$.
18. $345\frac{5}{16} \div 25$.

19. If 9 books cost $\$6\frac{3}{4}$, what is the average cost?

20. If 7 yards of arras cloth cost $\$8\frac{3}{4}$, what does it cost per yard?

21. If 19 yards of basket weave burlap cost $\$21\frac{3}{8}$, what does it cost per yard?

22. If 17 reams of tissue paper cost $\$20\frac{2}{5}$, what does it cost per ream?

23. If 15 dozen pairs of buttonhole scissors cost $\$67\frac{1}{2}$, what do they cost per dozen?

120. Dividing by a Unit Fraction. Since there are 4 fourths of an inch in 1 inch, therefore 1 inch $\div \frac{1}{4}$ of an inch $= 4$, and 2 inches $\div \frac{1}{4}$ of an inch $= 8$. In the same way, $5 \div \frac{1}{4} = 20$, $7 \div \frac{1}{4} = 28$, and so on.

And if $5 \div \frac{1}{4} = 20$, then $\frac{5}{8} \div \frac{1}{4}$ must equal $\frac{1}{8}$ as much, or $\frac{20}{8}$, or $2\frac{1}{2}$.

Therefore, *to divide by a unit fraction, multiply by the denominator.*

That is,

$$15 \div \tfrac{1}{8} = 8 \times 15 = 120,$$
$$27 \div \tfrac{1}{4} = 4 \times 27 = 108,$$
$$\tfrac{3}{8} \div \tfrac{1}{16} = 16 \times \tfrac{3}{8} = 6,$$
$$2\tfrac{1}{2} \div \tfrac{1}{32} = 32 \times 2\tfrac{1}{2} = 80.$$

EXERCISE 55

Find the quotient of:

1. $28 \div \frac{1}{2}$.
2. $37 \div \frac{1}{3}$.
3. $49 \div \frac{1}{4}$.
4. $56 \div \frac{1}{5}$.
5. $93 \div \frac{1}{7}$.
6. $\frac{2}{3} \div \frac{1}{9}$.
7. $\frac{3}{4} \div \frac{1}{16}$.
8. $\frac{4}{5} \div \frac{1}{35}$.
9. $\frac{5}{6} \div \frac{1}{72}$.
10. $\frac{7}{8} \div \frac{1}{25}$.
11. $2\frac{1}{2} \div \frac{1}{2}$.
12. $3\frac{1}{3} \div \frac{1}{6}$.
13. $5\frac{1}{4} \div \frac{1}{8}$.
14. $6\frac{3}{4} \div \frac{1}{12}$.
15. $25\frac{7}{8} \div \frac{1}{32}$.

16. If a sheet of veneer is $\frac{1}{16}$ of an inch thick, how many sheets pressed together will have a thickness of $1\frac{1}{8}$ inches?

17. If a sheet of blotting paper is $\frac{1}{32}$ of an inch thick, how many sheets are there in a pile that is $15\frac{5}{8}$ inches thick?

18. If each book is $\frac{1}{12}$ of a foot thick, how many books are there in a single pile of books $8\frac{1}{3}$ feet high?

19. If a bicycle is traveling at the rate of a mile in $\frac{1}{8}$ of an hour, how many miles will it travel in $3\frac{3}{4}$ hours?

20. How many weights of $\frac{1}{16}$ of a pound (or 1 ounce) each will it take to weigh $17\frac{3}{4}$ pounds?

21. If a wagon wheel makes $\frac{1}{12}$ of a revolution in going 1 foot, how many feet will it go in making $17\frac{3}{4}$ revolutions?

121. Dividing by a Fraction. We have found (§ 120) that when we have to divide by $\frac{1}{3}$ we may multiply by 3. We have also found (§ 66) that when we multiply the divisor by 2 we divide the quotient by 2.

That is, $6 \div \frac{1}{3} = 3 \times 6$ (§ 120),

and $6 \div \frac{2}{3} =$ half as much (§ 66) $= \frac{3 \times 6}{2} = 9.$

That is, $6 \div \frac{2}{3} = \frac{3}{2}$ of 6.

Therefore, *to divide by a common fraction, multiply by the inverted divisor.*

That is, $15 \div \frac{3}{5} = \frac{5}{\cancel{3}}$ of $\overset{5}{\cancel{15}} = 25$. Similarly, $\frac{3}{8} \div \frac{5}{6} = \frac{\overset{3}{\cancel{6}}}{5}$ of $\frac{3}{\underset{4}{\cancel{8}}} = \frac{9}{20}$.

An inverted fraction is also called the *reciprocal* of the fraction. That is, the reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$. The product of a fraction and its reciprocal is always one; thus, $\frac{5}{6}$ of $\frac{6}{5} = 1$.

EXERCISE 56

Find the quotient of:

1. $3 \div \frac{2}{3}$.
2. $5 \div \frac{3}{4}$.
3. $7 \div \frac{4}{5}$.
4. $9 \div \frac{3}{5}$.
5. $8 \div \frac{3}{8}$.
6. $5 \div \frac{5}{8}$.
7. $8 \div \frac{7}{8}$.
8. $9 \div \frac{5}{6}$.
9. $8 \div \frac{2}{5}$.
10. $9 \div \frac{3}{7}$.
11. $15 \div \frac{2}{3}$.
12. $15 \div \frac{3}{4}$.
13. $16 \div \frac{4}{5}$.
14. $21 \div \frac{3}{5}$.
15. $24 \div \frac{2}{5}$.
16. $30 \div \frac{3}{8}$.
17. $40 \div \frac{5}{8}$.
18. $20 \div \frac{7}{8}$.
19. $32 \div \frac{5}{6}$.
20. $75 \div \frac{5}{9}$.
21. $\frac{2}{3} \div \frac{3}{4}$.
22. $\frac{3}{4} \div \frac{2}{3}$.
23. $\frac{3}{8} \div \frac{5}{8}$.
24. $\frac{7}{8} \div \frac{5}{8}$.
25. $\frac{3}{8} \div \frac{3}{4}$.
26. $\frac{3}{4} \div \frac{7}{8}$.
27. $\frac{5}{12} \div \frac{3}{4}$.
28. $\frac{7}{12} \div \frac{2}{3}$.
29. $\frac{3}{8} \div \frac{7}{12}$.
30. $\frac{5}{8} \div 1\frac{1}{2}$.
31. $2\frac{1}{3} \div \frac{3}{4}$.
32. $3\frac{2}{3} \div \frac{3}{4}$.
33. $5\frac{1}{3} \div \frac{1}{2}$.
34. $5\frac{3}{8} \div \frac{3}{4}$.
35. $7\frac{5}{8} \div \frac{3}{8}$.
36. $8\frac{5}{8} \div \frac{5}{8}$.
37. $6\frac{7}{8} \div \frac{5}{8}$.
38. $7\frac{3}{16} \div \frac{7}{8}$.
39. $9\frac{5}{16} \div \frac{3}{4}$.
40. $8\frac{5}{32} \div \frac{7}{16}$.

122. Dividing by a Mixed Number. To divide by a mixed number it is generally best to reduce the divisor to an improper fraction and proceed as in § 121.

For example, to divide 10 by $3\frac{1}{3}$ is the same as to divide 10 by $\frac{10}{3}$, or to take $\frac{3}{10}$ of 10. But $\frac{3}{10}$ of $10 = 3$; hence $10 \div 3\frac{1}{3} = 3$.

Therefore, *to divide by a mixed number, reduce the divisor to an improper fraction and multiply the dividend by the reciprocal of the divisor.*

Thus $17\frac{1}{2} \div 3\frac{1}{4} = \frac{35}{2} \div \frac{13}{4} = \frac{\overset{2}{\cancel{4}}}{13}$ of $\frac{35}{\cancel{2}} = \frac{70}{13} = 5\frac{5}{13}$.

EXERCISE 57

Find the quotient of:

1. $3 \div 7\frac{1}{4}$.
2. $5 \div 4\frac{3}{4}$.
3. $9 \div 6\frac{1}{2}$.
4. $7 \div 5\frac{1}{8}$.
5. $5 \div 4\frac{3}{8}$.
6. $5 \div 6\frac{7}{8}$.
7. $8 \div 3\frac{4}{5}$.
8. $25 \div 3\frac{1}{3}$.
9. $62 \div 6\frac{2}{3}$.
10. $48 \div 12\frac{1}{2}$.
11. $72 \div 16\frac{2}{3}$.
12. $26 \div 33\frac{1}{3}$.
13. $48 \div 66\frac{2}{3}$.
14. $50 \div 37\frac{1}{2}$.
15. $\frac{3}{4} \div 2\frac{1}{2}$.
16. $\frac{3}{8} \div 3\frac{1}{3}$.
17. $\frac{4}{5} \div 2\frac{2}{3}$.
18. $\frac{3}{8} \div 5\frac{1}{4}$.
19. $\frac{5}{8} \div 7\frac{3}{4}$.
20. $\frac{3}{16} \div 6\frac{2}{3}$.
21. $\frac{5}{16} \div 16\frac{2}{3}$.
22. $12\frac{1}{2} \div 6\frac{1}{4}$.
23. $12\frac{1}{2} \div 3\frac{1}{3}$.
24. $12\frac{1}{2} \div 6\frac{2}{3}$.
25. $16\frac{2}{3} \div 12\frac{1}{2}$.
26. $33\frac{1}{3} \div 66\frac{2}{3}$.
27. $37\frac{1}{2} \div 6\frac{1}{4}$.

28. Find the cost per ton if $3\frac{1}{4}$ tons of hay cost $\$37\frac{3}{8}$.

29. How many strips of cloth, each $2\frac{1}{2}$ yards long, can be cut from a piece that is 45 yards long?

30. A dealer paid $\$247\frac{1}{2}$ for $7\frac{1}{2}$ dozen pairs of shoes. How much did they cost per dozen? How much per pair?

31. A dealer paid $\$224\frac{1}{4}$ for $3\frac{1}{4}$ dozen pairs of shears. How much did they cost per dozen? How much per pair?

123. To find the Whole when a Fractional Part is Given. For example, if $\frac{3}{8}$ of a yard of cloth costs 30 cents, what does 1 yard cost?

Since $\frac{3}{8}$ of a yard costs 30 cents,
then $\frac{1}{8}$ of a yard costs $\frac{1}{3}$ of 30 cents, or 10 cents,
and $\frac{8}{8}$ of a yard, or a whole yard, costs 8×10 cents, or 80 cents.

A short way to obtain this result is evidently to multiply 30 cents by $\frac{8}{3}$, that is, by the reciprocal (§ 121) of the fraction.

Therefore, *to find the whole when a fractional part is given, multiply the given part by the reciprocal of the fraction.*

EXERCISE 58

1. \$16 is $\frac{4}{7}$ of what sum of money?
2. \$45 is $\frac{5}{8}$ of what sum of money?
3. 84 miles is $\frac{7}{9}$ of how many miles?
4. $17\frac{1}{5}$ feet is $\frac{4}{5}$ of how many feet?
5. $243\frac{2}{5}$ pounds is $\frac{4}{5}$ of how many pounds?
6. If $\frac{5}{8}$ of an acre of land costs \$35, what is the cost of an acre?
7. If $\frac{7}{8}$ of a roll of carpeting costs \$84, what is the cost of the whole roll?
8. A dealer sold 12 yards of cloth, which was $\frac{4}{15}$ of the whole piece. How many yards were there in the piece?
9. A dealer sold $28\frac{1}{2}$ yards of cloth, which was $\frac{2}{3}$ of the whole piece. How many yards were there in the piece?
10. \$75 is $\frac{1}{4}$ more than (that is, $\frac{5}{4}$ of) what sum of money?
11. \$75 is $\frac{1}{4}$ less than (that is, $\frac{3}{4}$ of) what sum of money?
12. 119 feet is $\frac{1}{16}$ more than what length?
13. 155 feet is $\frac{1}{32}$ less than what length?
14. \$1107 is $\frac{1}{8}$ more than what sum of money?
15. \$1001 is $\frac{1}{8}$ less than what sum of money?

124. Complex Fractions. A fraction that has a fraction in one or both of its terms is called a *complex fraction.*

Thus $\frac{\frac{2}{3}}{3}, \frac{4}{\frac{2}{3}}, \frac{\frac{3}{4}}{\frac{5}{8}}, \frac{2\frac{1}{2}}{5}, \frac{7}{3\frac{1}{2}}, \frac{5\frac{1}{2}}{7\frac{1}{3}}$ are complex fractions.

To distinguish it from a complex fraction, a fraction whose terms are both integers is called a *simple fraction.*

For example, $\frac{3}{4}$ is a simple fraction.

A complex fraction indicates merely the division of its numerator by its denominator.

Therefore, *to reduce a complex fraction to simplest form divide the numerator by the denominator.*

Thus $\frac{\frac{2}{3}}{3} = \frac{1}{3}$ of $\frac{2}{3} = \frac{2}{9}$.

Likewise, $\frac{8\frac{3}{4}}{12\frac{5}{6}} = \frac{35}{4} \div \frac{77}{6} = \frac{\overset{3}{\cancel{6}} \times \overset{5}{\cancel{35}}}{\underset{11}{\cancel{77}} \times \underset{2}{\cancel{4}}} = \frac{15}{22}$.

EXERCISE 59

Reduce to simplest form:

1. $\frac{\frac{2}{3}}{8}$.
2. $\frac{\frac{4}{5}}{16}$.
3. $\frac{\frac{5}{8}}{15}$.
4. $\frac{1\frac{5}{16}}{10}$.
5. $\frac{1\frac{5}{24}}{30}$.
6. $\frac{1\frac{9}{32}}{57}$.
7. $\frac{5}{\frac{3}{8}}$.
8. $\frac{16}{\frac{4}{6}}$.
9. $\frac{18}{\frac{2}{3}}$.
10. $\frac{45}{1\frac{5}{16}}$.
11. $\frac{13}{\frac{7}{8}}$.
12. $\frac{27}{\frac{9}{32}}$.
13. $\frac{\frac{2}{3}}{\frac{3}{4}}$.
14. $\frac{\frac{5}{8}}{\frac{7}{16}}$.
15. $\frac{\frac{5}{16}}{\frac{7}{8}}$.
16. $\frac{\frac{7}{32}}{\frac{5}{8}}$.
17. $\frac{\frac{5}{16}}{\frac{3}{4}}$.
18. $\frac{\frac{7}{8}}{\frac{5}{24}}$.
19. $\frac{31\frac{1}{3}}{10}$.
20. $\frac{331\frac{1}{3}}{100}$.
21. $\frac{66\frac{2}{3}}{100}$.
22. $\frac{66\frac{2}{3}}{33\frac{1}{3}}$.
23. $\frac{12\frac{1}{2}}{75}$.
24. $\frac{16\frac{2}{3}}{100}$.

125. To Find what Fraction One Number is of Another. For example, 3 is what fraction of 4?

Since $1 = \frac{1}{4}$ of 4,
therefore $3 = 3 \times \frac{1}{4}$ of 4, or $\frac{3}{4}$ of 4.

The result therefore equals $3 \div 4$.

As a further example, $\frac{3}{7}$ is what part of $\frac{3}{2}$?

Since $1 = \frac{2}{3}$ of $\frac{3}{2}$,
therefore $\frac{3}{7} = \frac{3}{7}$ of $\frac{2}{3}$ of the second fraction
$= \frac{2}{7}$ of the second fraction.

The result therefore equals $\frac{3}{7} \div \frac{3}{2}$.

Therefore, *to find what fraction one number is of a second, divide the first number by the second.*

The result may be a mixed number. It may also be integral, as when we find what fraction 33 is of $16\frac{1}{2}$, the answer being 2.

EXERCISE 60

Find what fraction the first number is of the second:

1. 5, 15.
2. 19, 57.
3. 15, 35.
4. 29, 87.
5. 17, 85.
6. $16\frac{2}{3}$, 50.
7. 17, $8\frac{1}{2}$.
8. 39, $9\frac{3}{4}$.
9. $9\frac{2}{5}$, 47.
10. $7\frac{3}{8}$, 59.
11. $5\frac{11}{16}$, 91.
12. $3\frac{3}{32}$, 99.
13. $\frac{2}{3}$, $\frac{3}{5}$.
14. $\frac{3}{4}$, $\frac{5}{8}$.
15. $\frac{7}{8}$, $\frac{5}{16}$.
16. $\frac{5}{12}$, $\frac{3}{16}$.
17. $\frac{7}{16}$, $\frac{5}{32}$.
18. $\frac{9}{32}$, $\frac{5}{48}$.
19. $3\frac{2}{3}$, $5\frac{1}{2}$.
20. $4\frac{1}{4}$, $5\frac{3}{8}$.
21. $6\frac{2}{3}$, $3\frac{1}{3}$.
22. $8\frac{1}{3}$, $16\frac{2}{3}$.
23. $33\frac{1}{3}$, 100.
24. $66\frac{2}{3}$, 100.

Find what part of a dollar is represented by the following:

25. 10 cents.
26. 20 cents.
27. 50 cents.
28. 25 cents.
29. 75 cents.
30. $12\frac{1}{2}$ cents.
31. $33\frac{1}{3}$ cents.
32. $66\frac{2}{3}$ cents.
33. $6\frac{1}{4}$ cents.
34. $37\frac{1}{2}$ cents.
35. $62\frac{1}{2}$ cents.
36. $87\frac{1}{2}$ cents.

37. If a man has to drive $17\frac{1}{2}$ miles, what part of the distance has he gone when he has driven $3\frac{1}{2}$ miles?

EXERCISE 61

Review Problems

1. At $\$1\frac{3}{4}$ each, what will 35 chairs cost?

2. Find the cost of $9\frac{3}{4}$ tons of hay at $\$12\frac{1}{2}$ a ton.

3. Find the cost of $8\frac{1}{2}$ yards of cloth at $12\frac{1}{2}$ cents a yard.

4. If 1 yard of silk costs $\$2\frac{1}{2}$, what will $7\frac{3}{8}$ yards cost?

5. At $12\frac{1}{2}$ cents a dozen, what will $6\frac{3}{4}$ dozen buttons cost?

6. If 1 yard of calico costs $\$\frac{1}{8}$, how much will 23 yards cost?

7. At $12\frac{1}{2}$ cents a yard, what will $15\frac{1}{2}$ yards of cambric cost?

8. Find the cost of $6\frac{1}{4}$ pounds of mutton at 18 cents a pound.

9. Find the wages due for $17\frac{1}{2}$ hours of work at $37\frac{1}{2}$ cents an hour.

10. If 1 yard of silk costs $\$2\frac{1}{2}$, how many yards can be bought for $\$17\frac{1}{2}$?

11. If a bottle holds $\frac{4}{5}$ of a quart, how many quarts will a dozen such bottles hold?

12. If a boy rides $23\frac{3}{8}$ miles on a bicycle in $2\frac{3}{4}$ hours, what is his rate per hour?

13. When plain gingham costs $\$\frac{1}{4}$ a yard, how many yards can be bought for $\$7\frac{3}{4}$?

14. When gas costs $\$1\frac{1}{4}$ per thousand cubic feet, what is the cost of $3\frac{1}{8}$ thousand cubic feet?

15. If a boy rides on a bicycle at the rate of $8\frac{1}{4}$ miles an hour, how many miles will he ride in $2\frac{1}{3}$ hours?

16. If a man is walking at the rate of $\frac{3}{4}$ of a mile in 15 minutes, at what rate per hour is he walking?

17. What part of a barrel ($31\frac{1}{2}$ gallons) is $1\frac{23}{100}$ gallons?

18. Of $\frac{2}{13}$, $\frac{7}{45}$, $\frac{3}{20}$, which is the greatest, and which is the least?

19. The perimeter of a square is $71\frac{1}{2}$ inches. What is the length of a side?

20. A dealer paid $\$86\frac{2}{3}$ for some horse blankets at $\$3\frac{1}{3}$ each. How many did he buy?

21. If $\frac{3}{5}$ of the weight of beefsteak is water, what is the weight of water in $6\frac{7}{8}$ pounds of beefsteak?

22. If a train is traveling at the rate of $\frac{5}{6}$ of a mile in a minute, what is the rate per second?

23. A knot, used in measuring distances at sea, being $1\frac{3}{20}$ miles (nearly), how many knots in $11\frac{1}{2}$ miles?

24. What length must be added to the sum of $2\frac{3}{4}$ inches, $1\frac{3}{8}$ inches, $3\frac{2}{3}$ inches, and $\frac{7}{12}$ of an inch to make 12 inches?

25. How much does the sum of $\frac{1}{8}$ of an inch, $\frac{3}{16}$ of an inch, and $\frac{5}{32}$ of an inch lack of being $\frac{1}{2}$ of an inch?

26. A room is $11\frac{1}{4}$ feet wide. How many widths of carpet will it take to cover it, the carpet being $2\frac{1}{4}$ feet wide?

27. If a man by working 8 hours can finish $\frac{3}{8}$ of a piece of work, what part of the work can he finish in 3 hours?

28. How many boards, each $\frac{7}{8}$ of an inch thick, must be piled one on top of another to make a pile 28 inches high?

29. From a barrel containing 30 gallons of oil $1\frac{7}{8}$ gallons leaked out. What fractional part of all the oil remained?

30. If a boy is making a 25-mile trip on his bicycle, what fractional part of his journey has been completed when he has gone $18\frac{3}{4}$ miles?

31. If a cubic foot of water weighs $62\frac{1}{2}$ pounds, how many cubic feet in a tank that contains, when full, 50,000 pounds of water?

32. At \$80 an acre, how much will $\frac{7}{16}$ of an acre of land cost?

33. At $18\frac{3}{4}$ pounds to a jar, how many jars can be filled from 675 pounds of butter?

34. If a freight train travels $133\frac{1}{2}$ miles in 6 hours, what is its rate per hour?

35. If 45 quarts will fill 48 bottles, each bottle requires what part of a quart?

36. If a man earns $\$2\frac{1}{3}$ a day, how many days will it take him to earn \$35?

37. If it takes $\frac{7}{8}$ of a quart to fill a bottle, how many bottles can be filled from $8\frac{3}{4}$ quarts?

38. If it takes $\frac{7}{8}$ of a quart to fill a bottle, how many quarts will it take to fill a dozen bottles?

39. If a yard of cloth costs a quarter of a dollar, what part of a dollar will $\frac{5}{8}$ of a yard cost?

40. If an express train runs a mile in $\frac{3}{4}$ of a minute, how many minutes will it take to run $\frac{3}{4}$ of a mile?

41. If 25 dozen sheets of gold foil make a pile $\frac{1}{1000}$ of an inch high, how thick is each sheet?

42. If a piece of bronze weighing $7\frac{1}{2}$ pounds contains $5\frac{1}{4}$ pounds of copper, what part of the bronze is copper?

43. If a mass of lead and zinc weighing $16\frac{1}{8}$ pounds contains $10\frac{3}{4}$ pounds of lead, what fractional part of the whole mass is lead?

44. If a barrel of oil contains 42 gallons, how many gallons are there in $2\frac{1}{4}$ barrels? What is the value of this oil at $\$\frac{1}{10}$ a gallon?

45. From a piece of cloth containing $42\frac{1}{8}$ yards there were sold $5\frac{1}{4}$ yards, $6\frac{7}{8}$ yards, $2\frac{1}{2}$ yards, and $\frac{5}{8}$ of a yard. What was the rest worth at $\$\frac{3}{4}$ a yard?

46. Of what number is $26\frac{1}{4}$ exactly the $\frac{3}{4}$ part?

47. Of what number is $47\frac{2}{3}$ exactly the $\frac{2}{3}$ part?

48. What number decreased by $\frac{2}{5}$ of itself equals 45?

49. What number increased by $\frac{2}{5}$ of itself equals 45?

50. How much are 354 eggs worth at $22\frac{1}{2}$ cents a dozen?

51. If 24 dozen pairs of shears cost $\$134\frac{2}{5}$, how much do they cost per dozen?

52. How many times is a length of $\frac{7}{8}$ of an inch contained in a length of 14 inches?

53. How much are $2\frac{1}{4}$ thousand feet of lumber worth at $\$33\frac{1}{3}$ a thousand feet?

54. If a cup holds $\frac{3}{16}$ of a quart, how many cupfuls are there in $9\frac{3}{4}$ quarts?

55. If an express train travels at the rate of $48\frac{3}{4}$ miles an hour, what is its rate per minute?

56. If a freight train travels at the rate of $\frac{2}{5}$ of a mile in a minute, how far will it travel in $\frac{3}{4}$ of an hour?

57. If 7 is added to both terms of the fraction $\frac{2}{3}$, is the resulting fraction greater than or less than $\frac{2}{3}$? How much?

58. If 7 is added to both terms of the fraction $\frac{3}{2}$, is the resulting fraction greater than or less than $\frac{3}{2}$? How much?

59. If 7 is added to both terms of the fraction $\frac{5}{5}$, is the resulting fraction greater than or less than $\frac{5}{5}$? How much?

60. If a man can row $4\frac{3}{4}$ miles an hour in still water, how many miles an hour can he row up a river that flows $1\frac{5}{8}$ miles an hour? How many miles can he row downstream in an hour?

61. In the preceding example, how long will it take him to row 6 miles up the river? 6 miles down the river?

62. Two men bought a boat for $63. The first paid twice as much as the second. How much did each pay?

63. If steel is worth $1\frac{3}{8}$ cents a pound, what will a ton (2000 pounds) of steel be worth?

64. If a horse eats $2\frac{1}{8}$ bushels of oats in a week, how long will $53\frac{1}{8}$ bushels last him?

65. If a cubic foot of water weighs $62\frac{1}{2}$ pounds, how many pounds will $7\frac{3}{4}$ cubic feet of water weigh?

66. How many lots of $\frac{3}{8}$ of an acre each can be cut from a piece of land containing $7\frac{1}{2}$ acres?

67. If it takes $2\frac{7}{8}$ yards of cloth for a boy's suit, how many suits can be made from $31\frac{5}{8}$ yards?

68. A man owning $\frac{5}{8}$ of a building that is worth \$16,000 sells $\frac{3}{4}$ of his share. How much should he receive?

69. What will it cost to ship $28\frac{3}{5}$ tons of grain 120 miles, if the charge for carrying 1 ton 1 mile is $1\frac{1}{2}$ cents?

70. If a certain quality of wheat produces $\frac{17}{20}$ of its weight in flour, how much flour will be produced from 240 pounds of wheat?

71. If a certain quality of wheat produces $\frac{17}{20}$ of its weight in flour, what is the weight of wheat necessary to produce 255 pounds of flour?

72. At $9\frac{1}{4}$ cents a yard, how many yards of muslin can be bought for $462\frac{1}{2}$ cents? (That is, for \$4.62$\frac{1}{2}$; the dealer would really charge \$4.63.)

73. A man receives \$$3\frac{1}{5}$ for every 8 hours of work, and works during six days the following number of hours: 8, $7\frac{3}{4}$, $7\frac{1}{2}$, $8\frac{1}{4}$, $8\frac{3}{4}$, 8. How much does he receive in all?

74. The distance from New York to Albany is 143 miles. A freight train traveling $22\frac{1}{4}$ miles an hour starts from Albany at the same time that an express traveling $48\frac{3}{4}$ miles an hour starts from New York. How far apart are the trains at the end of $1\frac{1}{2}$ hours?

75. Two men bought a mill for $6560. The first paid $1\frac{2}{3}$ times as much as the second. How much did each pay?

76. If a freight train travels at the rate of $22\frac{1}{8}$ miles an hour, how long will it take to travel $199\frac{1}{8}$ miles?

77. Three building lots of $\frac{2}{5}$ of an acre, $\frac{3}{4}$ of an acre, and $\frac{3}{8}$ of an acre, respectively, are sold at the rate of $1440 an acre. Required the price of each lot.

78. If cork is $\frac{6}{25}$ times and lead $11\frac{3}{10}$ times as heavy as water, lead is how many times as heavy as cork? Cork is what part as heavy as lead?

79. If a cubic foot of water weighs $62\frac{1}{2}$ pounds, and steel is $7\frac{4}{5}$ times as heavy as water, how much will a cubic foot of steel weigh?

80. If gold is $19\frac{3}{10}$ times and steel $7\frac{4}{5}$ times as heavy as water, gold is how many times as heavy as steel? What part as heavy as gold is steel?

81. A cubic foot of a certain kind of pine weighs $25\frac{5}{8}$ pounds. How many cubic feet are there in a beam of that pine if it weighs $153\frac{3}{4}$ pounds?

82. A man divided 250 acres of land between his two sons, giving $\frac{5}{8}$ to the elder and $\frac{3}{8}$ to the younger. How many acres were given to each?

83. A man divided 275 acres of land between his two sons, giving $\frac{3}{5}$ to the elder and the remainder to the younger. How many acres did the younger receive?

84. If a man's income for a year is $1650 and his expenses are $1320, his expenses are what part of his income? He saves what part of his income?

85. What part of the weight of a 196-pound barrel of flour is a 49-pound bag of flour? If a barrel of flour is worth $7\frac{1}{2}$, what is a bag of flour worth at the same rate?

86. If $207\frac{1}{2}$ pounds of flour are produced from 250 pounds of wheat, the flour is what part of the weight of the wheat?

87. A cubic inch of copper weighs $5\frac{3}{20}$ ounces. How many cubic inches in a copper wire weighing $108\frac{3}{20}$ ounces?

88. If a sheet of paper and an envelope together weigh $\frac{3}{8}$ of an ounce, how much will a dozen such sheets of paper and a dozen such envelopes together weigh?

89. A dealer bought $7256\frac{1}{2}$ bushels of wheat at $83\frac{1}{2}$ cents a bushel and sold it at a profit of $435.39. What was the selling price per bushel?

90. Two men are working on the same job for the same wages. One works $4\frac{7}{8}$ days and the other $5\frac{3}{4}$ days. What part of the total wages should each receive?

91. The diameter of a bicycle wheel is $2\frac{1}{3}$ feet and the circumference is $3\frac{1}{7}$ times the diameter. How many times will the wheel turn in going 1 mile (5280 feet)?

92. A grocer bought a barrel of sugar weighing 432 pounds for $4\frac{7}{8}$ cents a pound and sold it for $5\frac{1}{4}$ cents a pound. How much did he gain by the transaction?

93. Two men are working on the same job for the same wages. One works 15 days and the other works 21 days. If the total wages is $99, how much does each receive?

94. Two men are working on the same job for the same wages. One works 22 days and receives $55. If the total wages amounts to $100, how many days does the second man work?

95. From a bin containing $32\frac{5}{8}$ bushels of wheat there were taken out at various times $5\frac{1}{4}$ bushels, $3\frac{7}{8}$ bushels, and $6\frac{1}{2}$ bushels. There were added during this time $3\frac{3}{4}$ bushels and $7\frac{1}{4}$ bushels. How many bushels were there then in the bin?

126. Multiplying and Dividing the Terms of a Fraction. We have now discovered a number of principles in connection with the terms of a fraction.

For example, we have found (§ 112) that to multiply the numerator by any number multiplies the fraction by that number.

That is, $$\frac{2 \times 4}{15} = 2 \times \frac{4}{15}.$$

We have also found (§ 118) that to divide the numerator by any number divides the fraction by that number.

That is, $$\frac{4 \div 2}{5} = \frac{4}{5} \div 2.$$

We have also found (§ 118) that to multiply the denominator by any number divides the fraction by that number.

That is, $$\frac{3}{2 \times 5} = \frac{3}{5} \div 2.$$

Now, $\frac{3}{8 \div 2} = \frac{3}{4} = 2 \times \frac{3}{8}$, and $\frac{5}{11 \div 2} = \frac{5}{5\frac{1}{2}} = \frac{2 \times 5}{11} = 2 \times \frac{5}{11}$.

Therefore we may see that to divide the denominator by any number multiplies the fraction by that number.

We have also found (§ 101, § 100) that multiplying or dividing both terms by any number does not change the value of the fraction.

That is, $\frac{2}{3} = \frac{5 \times 2}{5 \times 3} = \frac{10}{15}$; $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$.

We may summarize these important principles as follows:

Multiplying the numerator by any number multiplies the fraction by that number.

Dividing the numerator by any number divides the fraction by that number.

Multiplying the denominator by any number divides the fraction by that number.

Dividing the denominator by any number multiplies the fraction by that number.

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

EXERCISE 62

Review Questions

1. What are factors of a number? Illustrate.

2. What are prime numbers? prime factors?

3. What are composite numbers? Illustrate.

4. What are even numbers? odd numbers?

5. What is meant by exact divisors? Illustrate.

6. What is meant by the greatest common measure? How many numbers must there be to have a G.C.M?

7. What factors must the G.C.M. contain?

8. What do you mean by a multiple of a number? by a common multiple of two or more numbers?

9. What is meant by the least common multiple? How many numbers must there be to have a L.C.M.?

10. What factors must the L.C.M. contain? Illustrate by the L.C.M. of 18 and 48.

11. What is cancellation? What are its advantages? On what principle does it depend?

12. What are common fractions?

13. What are the terms of a fraction called? What does the lower term indicate? What does the upper term indicate?

14. What is a proper fraction? an improper fraction?

15. How is a fraction reduced to lowest terms? When is it in lowest terms? Is $\frac{2}{3}\frac{8}{2}\frac{9}{3}$ in lowest terms?

16. How do you reduce fractions for the purpose of adding them? Illustrate by the case of $\frac{2}{3} + \frac{7}{8}$.

17. How do you proceed to subtract one fraction from another? Illustrate by the case of $\frac{7}{8} - \frac{2}{3}$.

18. How do you multiply a fraction by an integer?

19. How do you multiply a mixed number by an integer?

20. How do you multiply an integer by a fraction?

21. How do you multiply an integer by a mixed number?

22. How do you multiply a fraction by a fraction? Illustrate.

23. How do you multiply a mixed number by a mixed number? Illustrate.

24. How do you divide a fraction by an integer? Illustrate by $\frac{15}{16} \div 5$, $\frac{2}{3} \div 4$, and $\frac{3}{4} \div 4$.

25. How do you divide by a unit fraction? Illustrate by $7 \div \frac{1}{3}$.

26. Draw a line and divide it in such a way as to show that $\frac{1}{4}$ is contained four times in 1.

27. How do you divide by any common fraction? Illustrate by $\frac{2}{3} \div \frac{3}{5}$, $\frac{3}{4} \div \frac{3}{5}$, and $\frac{1}{2} \div \frac{1}{8}$.

28. What two methods do you know for dividing a mixed number by an integer? Illustrate by $2\frac{1}{2} \div 9$, and $182\frac{1}{2} \div 9$.

29. How do you find a number when a fractional part of it is given? Illustrate by supposing that 16 is $\frac{4}{5}$ of some number.

30. A fraction is multiplied by a given number when you perform what operation on the numerator? what operation on the denominator? Illustrate both cases.

31. A fraction is divided by a given number when you perform what operation on the numerator? what operation on the denominator? Illustrate both cases.

32. Is the value of a fraction changed by multiplying both terms by the same number? by dividing both by the same number? by adding the same number to each? Illustrate each answer.

CHAPTER VIII

DECIMAL FRACTIONS

127. Meaning of Decimals. We have found that \$2.25 means \$2 + 25 cents, which is the same as $\$2\frac{25}{100}$ or $\$2\frac{1}{4}$.

In the same way, 2.25 miles means $2\frac{25}{100}$ miles, or $2\frac{1}{4}$ miles; 1.50 means $1\frac{50}{100}$, or $1\frac{1}{2}$; 1.75 means $1\frac{75}{100}$, or $1\frac{3}{4}$; 1.5 means $1\frac{5}{10}$, or $1\frac{1}{2}$; 2.125 means $2\frac{125}{1000}$, or $2\frac{1}{8}$; and 0.125 means $\frac{125}{1000}$, or $\frac{1}{8}$.

That is, 0.5 or .5 is a fraction whose denominator is not written, it being understood to be 10 from the fact that 5 occupies the first place to the right of the decimal point.

We therefore have the following:

0.5 means $\frac{5}{10}$, for the 5 extends to the 10th's place;
0.25 means $\frac{25}{100}$, for the 25 extends to the 100th's place;
0.125 means $\frac{125}{1000}$, for the 125 extends to the 1000th's place.

The names of the places are, in part, as follows:

Thousands	Hundreds	Tens	Units	(Decimal point)	Tenths	Hundredths	Thousandths	Ten-thousandths
1	3	4	5	.	2	7	6	5

This number is read "one thousand three hundred forty-five and two thousand seven hundred sixty-five ten-thousandths." The orders beyond ten-thousandths are hundred-thousandths, millionths, ten-millionths, and so on.

128. Decimal Fractions. A fraction whose denominator is not written, but is some power of 10, is called a *decimal fraction,* or more often simply a *decimal.*

Thus 0.25, which has the same value as $\frac{25}{100}$, is a decimal fraction.

An integer and a decimal together form a *mixed decimal;* for example, 2.25.

129. Decimal Point. The period written at the left of the tenths is called the *decimal point.*

For example, $0.7 = \frac{7}{10}$; $0.08 = \frac{8}{100}$, or $\frac{2}{25}$; $0.005 = \frac{5}{1000}$, or $\frac{1}{200}$.

It was not necessary to write a zero at the left of the decimal point in the above cases, for 0.5 means the same as .5. The zero is often written there to call attention more quickly to the decimal point.

130. Reading Decimals. We read a decimal precisely as if it were a whole number, and then give it the name of the lowest decimal place.

It is best to pronounce the word *and* at the decimal point only. Thus 100.023 is read "one hundred and twenty-three thousandths," while 0.123 is read "one hundred twenty-three thousandths."

EXERCISE 63

Read aloud the following:

1. 0.2.	**9.** 1.6.	**17.** 6.125.	**25.** 0.1225.
2. 0.7.	**10.** 2.9.	**18.** 7.335.	**26.** 0.3025.
3. 0.9.	**11.** 3.5.	**19.** 0.475.	**27.** 2.0025.
4. 0.12.	**12.** 4.75.	**20.** 5.500.	**28.** 0.0007.
5. 0.65.	**13.** 5.35.	**21.** 5.005.	**29.** 9.0009.
6. 0.70.	**14.** 6.40.	**22.** 6.075.	**30.** 200.025.
7. 0.85.	**15.** 7.08.	**23.** 6.750.	**31.** 0.225.
8. 0.09.	**16.** 7.10.	**24.** 0.875.	**32.** 220.005.

Read aloud as dollars and decimals of a dollar:

33. \$1.25. **34.** \$25.05. **35.** \$26.70. **36.** \$125.50.

131. Writing Decimals. Although we write a decimal fraction without its denominator, the denominator is shown by the decimal point and is indicated in reading.

Thus 0.9 has no printed denominator, but when we say "nine tenths" we indicate the fact that the denominator is ten as well as state that the numerator is nine.

Therefore, *write only the numerator of the decimal, inserting such zeros as are necessary to indicate the denominator, and place the decimal point before the tenths.*

Thus, to write in figures two hundred one hundred-thousandths, we write 201, and prefix two zeros, preceded by a decimal point, so that the last figure is in hundred-thousandths' place. We then have 0.00201.

EXERCISE 64

Write in figures:

1. Seven tenths; one tenth; five tenths; nine tenths.

2. Twelve hundredths; fifty-five hundredths; nine hundredths; one hundredth; twenty hundredths.

3. One hundred twenty-five thousandths; forty-seven thousandths; ten thousandths; five thousandths.

4. One thousand two hundred fifty ten-thousandths; seven hundred sixty-five ten-thousandths; seventy-five ten-thousandths; eight ten-thousandths.

5. Seventeen thousand six hundred fifty-two hundred-thousandths; two thousand four hundred-thousandths; two hundred six hundred-thousandths; thirty-five hundred-thousandths; seven hundred-thousandths.

6. One hundred twenty-five and sixty-two thousandths; one hundred and seventy-eight thousandths; one hundred seventy-eight thousandths; one hundred and one ten-thousandth; one thousand and two ten-thousandths; one thousand two ten-thousandths.

7. 492 ten-thousandths.
8. 243725 millionths.
9. 2643 hundred-thousandths.
10. 751 hundred-thousandths.
11. 63 hundred-thousandths.
12. 227 and 46 ten-thousandths.
13. 1352 and 1352 ten-thousandths.
14. 1000 and 1 thousandth.
15. 29,876 and 61 millionths.

Express as decimal fractions:

16. $\frac{4}{10}$.
17. $\frac{5}{10}$.
18. $\frac{7}{10}$.
19. $\frac{62}{100}$.
20. $\frac{75}{100}$.
21. $\frac{232}{1000}$.
22. $\frac{348}{1000}$.
23. $\frac{29}{1000}$.
24. $\frac{37}{1000}$.
25. $\frac{6}{1000}$.
26. $\frac{7231}{10000}$.
27. $\frac{4182}{10000}$.
28. $\frac{328}{10000}$.
29. $\frac{25}{10000}$.
30. $\frac{7}{10000}$.
31. $\frac{3}{10}$.
32. $\frac{30}{100}$.
33. $\frac{300}{1000}$.
34. $\frac{3000}{10000}$.
35. $\frac{30000}{100000}$.

Express as integers or mixed decimals:

36. $\frac{12}{10}$.
37. $\frac{15}{10}$.
38. $\frac{27}{10}$.
39. $\frac{38}{10}$.
40. $\frac{43}{10}$.
41. $\frac{123}{100}$.
42. $\frac{225}{100}$.
43. $\frac{625}{100}$.
44. $\frac{450}{100}$.
45. $\frac{400}{100}$.
46. $\frac{6225}{1000}$.
47. $\frac{3007}{1000}$.
48. $\frac{4040}{1000}$.
49. $\frac{4500}{1000}$.
50. $\frac{8000}{1000}$.
51. $\frac{72255}{10000}$.
52. $\frac{28005}{10000}$.
53. $\frac{78050}{10000}$.
54. $\frac{62000}{10000}$.
55. $\frac{90000}{10000}$.

Express in figures as dollars and cents:

56. Two and seventy-five hundredths dollars.
57. Seventy-one and two hundredths dollars.
58. One hundred and eight hundredths dollars.
59. One thousand three and three hundredths dollars.

132. Reduction of Decimals. Since decimals are only a special form of common fractions, we may reduce them as we reduce common fractions.

For example, since $\frac{5}{10} = \frac{50}{100}$ (§ 101), therefore $0.5 = 0.50$.

133. Annexing Zeros. Because, as just shown, $0.5 = 0.50$, therefore

Annexing zeros to a decimal does not change its value.

134. Similar Decimals. Decimals that have the same number of decimal places are called *similar decimals.*

Thus 0.75 and 0.25 are similar decimals, and so are 0.150 and 0.275; but 0.15 and 0.275 are dissimilar decimals.

135. Reduction of Dissimilar Decimals to Similar Decimals. Since we may give any decimals the same denominators (understood, not written) by annexing or cutting off zeros,

Therefore, *to reduce dissimilar decimals to similar decimals, give them the same number of decimal places by annexing or cutting off zeros.*

Thus 0.4, 0.72, and 0.125 may all be reduced to thousandths as follows: 0.400, 0.720, and 0.125. Similarly, 0.40 and 0.600 are equal to 0.4 and 0.6.

EXERCISE 65

Reduce to similar decimals:

1. 0.2, 0.17.
2. 0.8, 0.08.
3. 0.7, 0.002.
4. 0.9, 0.130.
5. 0.6, 0.123.
6. 0.5, 0.700.
7. 0.4, 0.050.
8. 0.1, 0.357.
9. 0.12, 0.103.
10. 0.72, 0.008.
11. 0.69, 0.070.
12. 0.30, 0.700.
13. 0.99, 0.999.
14. 0.75, 0.630.
15. 0.50, 0.400.
16. 0.85, 0.392.
17. 0.001, 0.1100.
18. 0.207, 0.2376.
19. 0.412, 0.9320.
20. 0.681, 0.6881.
21. 0.900, 0.7000.
22. 0.620, 0.5700.
23. 0.090, 0.6550.
24. 0.584, 0.5834.

136. Reduction of Decimals to Common Fractions. We have $0.75 = \frac{75}{100}$, which may be reduced to $\frac{3}{4}$. In a similar way any decimal may be expressed as a common fraction.

Therefore, *to reduce a decimal to a common fraction, omit the decimal point, write the denominator of the decimal, and then reduce the common fraction to lowest terms.*

In a case like that of $0.33\frac{1}{3}$ we proceed in the same way, thus,

$$0.33\tfrac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{1}{3}.$$

EXERCISE 66

Reduce to common fractions in lowest terms or to mixed numbers:

1. .2.	11. .125.	21. .0125.	31. 1.25.
2. .4.	12. .625.	22. .0250.	32. 2.50.
3. .6.	13. .250.	23. .0375.	33. 3.75.
4. .8.	14. .500.	24. .0500.	34. 6.25.
5. .10.	15. .375.	25. .0625.	35. 1.875.
6. .15.	16. .735.	26. .7500.	36. 18.75.
7. .25.	17. .875.	27. .1875.	37. 187.5.
8. .12$\frac{1}{2}$.	18. .333$\frac{1}{3}$.	28. .3333$\frac{1}{3}$.	38. 3.37$\frac{1}{2}$.
9. .16$\frac{2}{3}$.	19. .166$\frac{2}{3}$.	29. .6666$\frac{2}{3}$.	39. 3.375.
10. .66$\frac{2}{3}$.	20. .666$\frac{2}{3}$.	30. .1666$\frac{2}{3}$.	40. 3.3$\frac{3}{4}$.

Reduce to sixteenths:

41. $\frac{3}{4}$.	42. 0.75.	43. 0.25.	44. 0.500.

Express as mixed numbers:

45. 1.50.	47. 3.20.	49. 2.40.	51. 4.65.
46. 2.25.	48. 5.75.	50. 6.80.	52. 5.35.

137. Reduction of Common Fractions to Decimals. Since a common fraction indicates division, we may divide the numerator by the denominator and thus reduce the fraction to the decimal form. For example, consider the fraction $\frac{3}{4}$.

We have found that we may annex zeros after the decimal point without changing the value; that is, $3 = 3.00$. We may now divide exactly as in United States money, and the result is 0.75.

$$4\overline{)3.00}\quad 0.75$$

That this is allowable may be seen from the following: 3.0 means 30 tenths, and 30 tenths $\div 4 = 7$ tenths with remainder 2 tenths. But 2 tenths $= 20$ hundredths ($\frac{2}{10} = \frac{20}{100}$), and 20 hundredths $\div 4 = 5$ hundredths. Therefore $\frac{3}{4} = 0.75$.

In the same way, $0.1\frac{3}{4} = 0.175$.

Therefore, *to reduce a common fraction to a decimal, place a decimal point after the numerator, and divide by the denominator as in United States money.*

A common fraction cannot always be reduced to an exact decimal. Thus $\frac{1}{3} = 0.3\frac{1}{3} = 0.33\frac{1}{3} = 0.333\frac{1}{3}$. It is customary to carry the division as far as necessary and then write a common fraction or the plus sign to indicate that the work is not complete; thus $\frac{1}{3} = 0.333\frac{1}{3}$, or $0.333+$. Often this is written 0.333, using as many places as necessary.

No common fraction equals an exact decimal if the denominator, when the fraction is reduced to lowest terms, contains other prime factors than 2 and 5.

EXERCISE 67

Reduce to decimals, carrying the reduction only to thousandths in case of inexact decimals:

1. $\frac{1}{2}$.	7. $\frac{7}{8}$.	13. $\frac{3}{16}$.	19. $\frac{1}{3}$.	25. $1\frac{9}{25}$.
2. $\frac{1}{4}$.	8. $\frac{1}{5}$.	14. $\frac{5}{16}$.	20. $\frac{2}{3}$.	26. $3\frac{4}{75}$.
3. $\frac{1}{8}$.	9. $\frac{2}{5}$.	15. $\frac{7}{16}$.	21. $\frac{1}{6}$.	27. $0.1\frac{3}{8}$.
4. $\frac{3}{4}$.	10. $\frac{3}{5}$.	16. $\frac{15}{16}$.	22. $\frac{5}{6}$.	28. $6.2\frac{7}{8}$.
5. $\frac{3}{8}$.	11. $\frac{4}{5}$.	17. $\frac{1}{32}$.	23. $\frac{3}{25}$.	29. $1.3\frac{3}{5}$.
6. $\frac{5}{8}$.	12. $\frac{1}{16}$.	18. $\frac{3}{32}$.	24. $\frac{7}{25}$.	30. $6.1\frac{1}{4}$.

138. United States Money. The system of United States money is arranged on the decimal plan. Thus,

10 mills (m.) = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes, or 100 cents = 1 dollar ($)

This table, like the common operations with United States money, is already familiar to the pupil, and is inserted only for reference.

The unit of United States money is the dollar, and the dimes and cents are written as decimal parts of the dollar.

Thus $2.48 means 2 dollars and 48 hundredths of a dollar, or 2 dollars and 48 cents, the cent being one hundredth of a dollar. Similarly, $2.08 means 2 dollars and 8 hundredths of a dollar, or 2 dollars and 8 cents.

139. Coins. The coins of the United States, as issued at present, are as follows:

Gold		*Silver*	
Double eagle,	$20.00	Dollar,	$1.00
Eagle,	10.00	Half dollar,	50
Half eagle,	5.00	Quarter dollar,	.25
Quarter eagle,	2.50	Dime,	.10

Nickel	*Bronze*
5-cent piece (the "nickel")	1-cent piece

140. Paper Money. Bank bills, United States treasury notes, and gold and silver certificates are largely used in place of coins. These are issued in denominations of $1, $2, $5, $10, $20, $50, $100, $500, $1000, $5000, and $10,000.

Formerly paper money of smaller denominations was issued, as well as the gold dollar, the silver 20-cent, 5-cent, and 3-cent pieces, the nickel 3-cent piece, the bronze 2-cent piece, and the copper half cent. These are no longer issued. The mill was never coined.

141. Reduction of Dollars to Cents. Required to reduce \$3.75 to cents.

Since \$1 = 100 cents, therefore \$3 = 3 × 100 cents, or 300 cents; then 300 cents + 75 cents = 375 cents.

Therefore, *to reduce dollars to cents, move the decimal point two places to the right.*

142. Reduction of Cents to Dollars. Required to reduce 1425 cents to dollars, or dollars and cents.

Since 1 cent = $\$\frac{1}{100}$, therefore 1425 cents = $\$\frac{1425}{100}$ = \$14.25.

Therefore, *to reduce cents to dollars, or to dollars and cents, move the decimal point two places to the left.*

A whole number may always have a decimal point placed after it without changing its value, so if there is no decimal point given it may be placed at the right and then moved two places to the left.

EXERCISE 68

Reduce to cents:

1. \$7.	5. \$2.40.	9. \$3.45.	13. \$141.75.
2. \$29.	6. \$3.60.	10. \$9.65.	14. \$243.85.
3. \$35.	7. \$9.70.	11. \$8.05.	15. \$692.05.
4. \$175.	8. \$12.80.	12. \$29.63.	16. \$900.09.

Reduce to dollars, or to dollars and cents:

17. 300¢.	21. 732¢.	25. 2642¢.	29. 2¢.
18. 450¢.	22. $522\frac{1}{2}$¢.	26. 4900¢.	30. 5¢.
19. 575¢.	23. $633\frac{1}{3}$¢.	27. 7275¢.	31. 55¢.
20. 606¢.	24. $166\frac{2}{3}$¢.	28. 9654¢.	32. 75¢.

33. Reduce \$27.63 to mills.

34. Reduce 5240 mills to dollars and cents.

35. Reduce 2760 dimes to dollars; to cents.

143. Addition of Decimals. We add numbers containing decimals just as we add United States money.

For example, required the sum of 14.7, 3.62, 5.25, and 26.

Arranging the addends so that units of the same order are in the same column, which is easily done by placing the decimal points under one another, we add exactly as with integers, placing the decimal point in the sum directly under the decimal points in the addends.

14.7
3.62
5.25
26.
49.57

Therefore, *to add numbers containing decimals, write like orders under one another and add as with integers.*

If there are common fractions as well as decimals, these either may be reduced to decimals and added, or else may be added separately as is the more convenient.

EXERCISE 69

Add the following:

1.	2.	3.	4.	5.
22.05	19.26	3.	$5.27\frac{1}{2}$	$4.33\frac{1}{3}$
3.75	3.54	34.17	$6.42\frac{1}{4}$	$1.66\frac{2}{3}$
16.23	7.08	16.89	$7.13\frac{1}{8}$	$3.12\frac{1}{2}$
4.78	16.	17.32	$5.16\frac{3}{4}$	$5.37\frac{1}{2}$
9.6	32.89	4.08	$4.29\frac{1}{4}$	$4.25\frac{3}{4}$
3.09	1.96	10.	$2.53\frac{3}{8}$	$9.36\frac{1}{4}$

6.	7.	8.	9.	10.
23.41	14.02	38.92	$42.17\frac{1}{2}$	$33.33\frac{1}{3}$
36.98	148.36	64.87	$68.13\frac{1}{2}$	$16.66\frac{2}{3}$
124.75	49.79	49.81	$7.14\frac{1}{4}$	$66.66\frac{2}{3}$
4.92	31.64	324.06	$18.21\frac{1}{4}$	$9.95\frac{1}{3}$
6.07	5.50	4.27	$16.82\frac{3}{4}$	$3.37\frac{1}{2}$
13.53	20.00	82.46	$7.16\frac{1}{2}$	$16.52\frac{1}{2}$
4.81	1.97	73.20	8.23	$5.23\frac{1}{3}$

11. $29.82 + 15.44\frac{3}{5} + 9.87\frac{1}{5} + 2.782$.

12. $96.45 + 88.34\frac{3}{4} + 2.91\frac{1}{8} + 6.33\frac{1}{8}$.

13. $72.63 + 19.81\frac{1}{3} + 4.66\frac{2}{3} + 2.87\frac{3}{5}$.

14. $24.75 + 16.37\frac{1}{2} + 6.33\frac{1}{3} + 5.44\frac{1}{6}$.

15. $\$75\frac{1}{6} + \$25.16\frac{2}{3} + \$17\frac{2}{3} + \$250$.

16. $\$42\frac{1}{5} + \$67.80 + \$19.75 + \$135\frac{1}{4}$.

17. $\$33\frac{1}{3} + \$25\frac{1}{2} + \$17.66\frac{2}{3} + \$128.50$.

18. $\$82.75 + \$14\frac{1}{4} + \$19\frac{1}{2} + \$16.50 + \$67$.

19. The distance from Chicago to Jackson is 209.31 miles, and from Jackson to Detroit 75.43 miles. How far is it from Chicago to Detroit by way of Jackson?

20. The distance from Detroit to St. Thomas is 112.01 miles, and from St. Thomas to Buffalo 139.13 miles. How far is it from Detroit to Buffalo by way of St. Thomas?

21. Taking the distances as given in Exs. 19 and 20, find how far it is from Chicago to Buffalo by this route.

22. The distance from Buffalo to Albany is 296.64 miles, and from Albany to Boston 202 miles. How far is it from Buffalo to Boston by way of Albany?

23. Taking the distances as given in Exs. 19, 20, and 22, find how far it is from Chicago to Boston by this route.

24. If a man has 172.86 acres of land and buys $49\frac{3}{4}$ acres more, how many acres does he then have?

25. If a man has 168.41 acres of land and buys $22\frac{1}{2}$ acres more of one man, and $16\frac{2}{3}$ acres of another, how many acres does he then have?

26. A man has a deed to a farm that states the area to be $198\frac{3}{4}$ acres. He has the five fields in the farm measured and finds that their areas are 21.25 acres, 33.4 acres, 82.6 acres, 58 acres, and $3\frac{1}{2}$ acres. Is the area stated in the deed correct?

144. Subtraction of Decimals. We subtract numbers containing decimals just as we subtract United States money.

Required the difference between 26.32 and 17.754.

$$\begin{array}{r} 26.320 \\ 17.754 \\ \hline 8.566 \end{array}$$

We first reduce the numbers to similar decimals by annexing zeros as may be necessary. Then arranging the numbers so that units of the same order are in the same column, which is easily done by placing the decimal points under one another, we subtract exactly as with integers.

Therefore, *to subtract one number from another when either contains or both contain decimals, reduce to similar decimals, write like orders under one another, and subtract as with integers.*

Any common fractions may be subtracted separately if they cannot easily be reduced to decimals. Thus $4.6\frac{2}{3} - 1.3\frac{1}{3} = 3.3\frac{1}{3}$.

EXERCISE 70

Subtract:

1.	5.	9.	13.	17.
26.75	71.81	27.6	36.91	29.433
3.96	4.68	13.01	17.935	17.5
2.	**6.**	**10.**	**14.**	**18.**
43.15	24.8	61.3	82.	48.224
7.23	4.32	27.25	16.273	9.0005
3.	**7.**	**11.**	**15.**	**19.**
62.81	34.9	34.75	48.007	72.2
9.75	6.43	16.08	23.25	65.025
4.	**8.**	**12.**	**16.**	**20.**
63.42	48.7	42.27	83.422	0.72834
4.73	4.08	17.752	16.307	0.1976

21. $7 - 0.2698$.

22. $3 - 0.2709$.

23. $6 - 1.0273$.

24. $6.2 - 1.33\frac{1}{3}$.

25. $8.4 - 1.66\frac{2}{3}$.

26. $9.5 - 4.12\frac{1}{2}$.

27. A man having a farm of 175.25 acres sells a lot of $12\frac{1}{2}$ acres. How much has he left?

28. The English sovereign is worth \$4.8665. How much does this lack of \$5?

29. If $629.33\frac{1}{3}$ barrels of water are in a tank that holds 948.25 barrels, how much does it lack of being full?

30. The United States \$5 gold piece contains 116.1 grains of pure gold, and the English sovereign 113.0003 grains. Find the difference in the weight of the pure gold.

31. A man has three adjacent lots with a frontage of 148.5 feet. One of the lots has a frontage of 52.75 feet and another of 49.5 feet. What is the frontage of the third lot?

32. A man who had a farm of 215.8 acres bought an adjacent farm of $192\frac{3}{4}$ acres. He sold a field of $72\frac{1}{3}$ acres and another of 68.25 acres. How much land had he left?

33. The perimeter of a triangle is 27.25 feet, and two of the sides are $8.33\frac{1}{3}$ feet and $9.75\frac{3}{3}$ feet. What is the length of the third side?

34. The distance from New Orleans to Baton Rouge is 89.4 miles, and from New Orleans to Natchez 214.1 miles. How far is it from Baton Rouge to Natchez? In this and the following examples the cities are on the same line.

35. The distance from Chicago to Lincoln is 561.4 miles, and from Chicago to Colorado Springs 1072.2 miles. How far is it from Lincoln to Colorado Springs?

36. The distance from Salt Lake City to Riverside, California, is 723.7 miles, and to Pasadena by way of Riverside 791.2 miles. How far is it from Riverside to Pasadena?

37. The distance from Louisville to Greencastle is 146.5 miles, and from Louisville to Chicago 324.3 miles. How far is it from Greencastle to Chicago?

145. Multiplication of a Decimal by an Integer. In multiplying a decimal by an integer we proceed exactly as in multiplying United States money.

For example, required to multiply 2.375 by 7.

$$\begin{array}{r} 2.375 \\ 7 \\ \hline 16.625 \end{array}$$

Since 5 thousandths multiplied by 7 is 35 thousandths, we know that the 5 will go in the thousandths' place. Proceeding with the multiplication in the ordinary way, we finally place the decimal point before the 625 so as to have the 5 come in the thousandths' place as above stated.

146. Multiplication of a Decimal by a Mixed Number. Multiplication of a decimal by a mixed number may be illustrated by the case of $3\frac{1}{2} \times 2.44$.

$$\begin{array}{r} 2.44 \\ 3\frac{1}{2} \\ \hline 1.22 \\ 7.32 \\ \hline 8.54 \end{array}$$

First taking $\frac{1}{2}$ of 2.44, we know that $\frac{1}{2}$ of $\frac{244}{100} = \frac{122}{100}$, or 1.22. Then 3×4 hundredths $= 12$ hundredths, so we write the 2 in the hundredths' place. Proceeding in the ordinary way, the sum of the partial products is 8.54.

EXERCISE 71

Multiply:

1. 2×6.7.
2. 4×9.8.
3. 7×0.9.
4. 8×0.6.
5. 5×3.25.
6. 7×2.75.
7. $6 \times 3.3\frac{1}{3}$.
8. $9 \times 6.6\frac{2}{3}$.
9. 8×2.125.
10. 4×6.375.
11. 7×5.025.
12. 8×7.009.
13. 5×6.285.
14. $3 \times 3.33\frac{1}{3}$.
15. $6 \times 1.66\frac{2}{3}$.
16. $6 \times 6.66\frac{2}{3}$.
17. 8×4.125.
18. 9×3.627.
19. 8×5.328.
20. 7×6.1328.
21. 8×2.3125.
22. $9 \times 0.0333\frac{1}{3}$.
23. $7 \times 0.2717\frac{1}{7}$.
24. $5 \times 0.0444\frac{2}{5}$.
25. $9 \times 6.6666\frac{2}{3}$.
26. $7 \times 1.4285\frac{5}{7}$.
27. 25×0.125.
28. $24 \times 0.166\frac{2}{3}$.
29. $63 \times 3.333\frac{1}{3}$.
30. 70×1.43143.
31. 143×7.0007.
32. 125×1.0202.
33. 225×3.1231.
34. $16\frac{1}{2} \times 3.0250$.
35. $35\frac{1}{4} \times 2.376$.
36. $64\frac{1}{3} \times 2.0325$.
37. $48\frac{3}{4} \times 2.372$.
38. $75\frac{1}{2} \times 4.21624$.
39. $12\frac{1}{2} \times 8.88888$.

40. What is the perimeter of a triangle if each side is 19.25 inches? if each side is $16.33\frac{1}{3}$ inches?

41. What is the perimeter of a square if each side is $6.12\frac{1}{2}$ inches? if each side is $5.31\frac{1}{8}$ inches?

42. At $\$0.33\frac{1}{3}$ a yard, how much will 15 yards of cloth cost? How much will 27 yards cost?

43. If a meter is 39.37 inches, how many inches in 12 meters? in 25 meters? in 37 meters?

44. If a 5-cent piece weighs 77.65 grains, how much will half a dozen 5-cent pieces weigh?

45. If a French franc is worth \$0.193, what are 125 francs worth in our money?

46. If a square foot is 0.0929 of a square meter, how many square meters in 35 square feet?

47. If an English sovereign is equivalent to \$4.8665, what is the value in our money of 27 sovereigns?

48. At the rate of 49.74 miles an hour, how far will an express train travel in $6\frac{2}{3}$ hours?

49. At the rate of $52.66\frac{2}{3}$ miles an hour, how far will an express train travel in 3 hours?

50. If the thickness of a book is 1.15 inches, how many inches will 2 dozen such books occupy on a dealer's shelf?

51. If two sides of a rectangular field are respectively 96.8 feet and 137.6 feet, what is the perimeter of the field?

52. If a man cuts a piece of land into 17 building lots, each having an area of 0.72 of an acre, what is the area of the whole property?

53. It was estimated recently that the average cost of the actual necessities of life for each person in this country was \$108.955 a year. At this rate what would be the cost for 75 persons?

147. Multiplication of a Decimal by Multiples of 10. In multiplying 3.1416 by 300 we may proceed as in § 145, but there is a shorter process that may be used when the multiplier is any multiple of 10.

Since moving the decimal point one place to the right changes tenths into units, hundredths into tenths, and so on, it multiplies the number by 10. Similarly, moving the decimal point two places to the right multiplies by 100, and so on.

3.1416
300
942.48

Hence, to multiply by 300 we need only to multiply by 3 and move the decimal point two places to the right.

When necessary, zeros must be annexed to the product, as in the case of 300 × 3141.6.

EXERCISE 72

Multiply:

1. 200 × 3.42.	**6.** 700 × 4.062.	**11.** 400 × 0.1250.
2. 300 × 6.28.	**7.** 800 × 0.317.	**12.** 600 × $0.166\frac{2}{3}$.
3. 500 × 9.36.	**8.** 600 × 5.214.	**13.** 800 × $0.012\frac{1}{2}$.
4. 200 × 2.48.	**9.** 600 × 52.14.	**14.** 700 × 7.9775.
5. 200 × 24.8.	**10.** 600 × 521.4.	**15.** 8000 × 674.3.

16. A meter being 39.37 inches, how many inches are there in 4000 meters?

17. At $37.75 a front foot, what will a piece of land of 300 feet front cost?

18. If a cubic foot of water weighs 62.5 pounds, what will 2500 cubic feet weigh?

19. If the thickness of each board is 0.875 of an inch, what is the height of a pile of 80 boards?

20. The circumference of a circle being 3.1416 times the diameter, what is the circumference of a circular water tank 40 feet in diameter?

148. Multiplication by a Decimal. Let us first consider the case of 0.5 of 0.7, or 0.5×0.7.

$$\begin{array}{r} 0.7 \\ 0.5 \\ \hline 0.35 \end{array}$$

Writing these decimals as common fractions, we have

$$\tfrac{5}{10} \times \tfrac{7}{10} = \tfrac{35}{100}.$$

Therefore $0.5 \times 0.7 = 0.35$.

This we see without using common fractions, for $5 \times 7 = 35$, and a tenth of a tenth is a hundredth, so that we have 35 hundredths.

Next consider the case of 0.75×1.255.

We may reduce these decimals to common fractions as before, having $\tfrac{75}{100} \times \tfrac{1255}{1000} = \tfrac{94125}{1000000}$, or 0.94125.

It is, however, easier to multiply in the ordinary way, and to notice that 5 (hundredths) × 5 (thousandths) = 25 (hundred-thousandths), and that the 5 of the product must therefore go in the hundred-thousandths' place.

$$\begin{array}{r} 1.255 \\ 0.75 \\ \hline 6275 \\ 8785 \\ \hline 0.94125 \end{array}$$

Therefore, *to multiply by a decimal, multiply as if by an integer, and point off as many decimal places in the product as there are decimal places in both factors together.*

Zeros should be prefixed as necessary to give the required number of places; that is, $0.07 \times 0.069 = 0.00483$.

EXERCISE 73

Multiply:

1. 0.7×0.7.
2. 0.9×0.9.
3. 0.8×1.7.
4. 0.6×3.9.
5. 0.5×7.5.
6. 0.8×6.7.
7. 2.3×4.1.
8. 3.2×5.7.
9. 8.7×9.6.
10. 9.8×8.3.
11. 0.63×0.31.
12. 0.82×0.72.
13. 0.34×0.88.
14. 0.65×1.29.
15. 0.48×3.21.
16. 3.17×4.22.
17. 7.39×5.81.
18. 9.75×6.92.
19. 3.82×9.89.
20. 9.99×9.99.
21. 0.78×0.127.
22. 0.84×0.125.
23. $0.36 \times 0.33\frac{1}{3}$.
24. $0.69 \times 0.66\frac{2}{3}$.
25. 0.96×2.735.
26. 4.35×5.006.
27. 8.25×9.901.
28. 7.37×0.073.
29. 8.24×0.0009.
30. 7.62×3.1416.

31. A cubic foot of water weighs 62.5 pounds. Ice is 0.92 as heavy as water. What does a cubic foot of ice weigh?

32. Taking the weight of water as in Ex. 31, and knowing that cork is 0.24 as heavy as water, find the weight of a cubic foot of cork.

33. Taking the weight of water as in Ex. 31, and knowing that a certain kind of granite is 2.78 times as heavy as water, find the weight of a cubic foot of the granite.

34. Taking the weight of water as in Ex. 31, and knowing that copper is 8.95 times as heavy as water, what is the weight of a cubic foot of copper?

35. The circumference of a circle is 3.1416 times the diameter. What is the circumference of a wheel whose diameter is 42 inches?

36. From the statement made in Ex. 35, find the circumference of the earth, assuming its diameter to be 8000 miles.

37. From the statement made in Ex. 35, find the circumference of the sun, assuming its diameter to be 866,500 miles.

38. In a year when the average yield of oats in the United States was 31.2 bushels per acre, and the average price on the farm was 31.7 cents per bushel, what was the average value on the farm of the oat crop per acre?

39. In a year when the average yield of corn in the United States was 30.3 bushels per acre, and the average price on the farm was 39.9 cents per bushel, what was the average value on the farm of the corn crop per acre?

40. In a year when the average yield of wheat in the United States was 15.5 bushels per acre, and the average price on the farm was 66.7 cents per bushel, what was the average value on the farm of the wheat crop per acre?

149. Division of a Decimal by an Integer. In dividing a decimal by an integer we proceed exactly as in dividing United States money.

For example, required to divide 1.561 by 7.

$$\begin{array}{r|l} 7 & 1.561 \\ \hline & 0.223 \end{array}$$

Here $1 \div 7$ gives no units in the quotient, so we write 0 in the units' place; 15 tenths $\div 7 = 2$ tenths, and we write 2 in the tenths' place, considering the remainder (1 tenth) as 10 hundredths. Then 16 hundredths $\div 7 = 2$ hundredths, and we write this 2 in the hundredths' place, considering the remainder as 20 thousandths. Then 21 thousandths $\div 7 =$ 3 thousandths. Hence the quotient is 0.223.

Zeros may be annexed as needed, to the right of the decimal point, to enable the division to proceed. Thus $6.1 \div 5 = 6.10 \div 5 = 1.22$.

Therefore, *to divide a decimal by an integer divide as with integers, writing the decimal point in the quotient as soon as the decimal point in the dividend is reached.*

EXERCISE 74

Divide:

1. $4.52 \div 2$.	**6.** $7.3425 \div 25$.	**11.** $32.4714 \div 46$.
2. $5.58 \div 9$.	**7.** $81.675 \div 75$.	**12.** $8838.80 \div 19$.
3. $3.15 \div 7$.	**8.** $92.100 \div 25$.	**13.** $2.94996 \div 39$.
4. $31.5 \div 7$.	**9.** $211.428 \div 36$.	**14.** $9.65496 \div 98$.
5. $0.315 \div 7$.	**10.** $215.016 \div 34$.	**15.** $0.40715 \div 85$.

16. If 12 dozen eggs cost \$2.76, what does 1 dozen cost?

17. A train goes from Chicago to Battle Creek, 164.18 miles, in 4 hours. What is the average rate per hour?

18. A train leaves Grand Rapids at 4 P.M. and reaches Jackson, 94.5 miles distant, at 7 P.M. What is the average rate per hour?

19. There being 231 cubic inches in a gallon, how many gallons in a tank containing 14,437.5 cubic inches?

150. Division by a Decimal. Since we may multiply both divisor and dividend by the same number without affecting the quotient (§ 66), we may, if the divisor contains decimal places, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer.

For example, the quotient of $1.25 \div 0.5$ is the same as that of 12.5 by 5, and this we may find as in § 149.

Therefore, *to divide by a divisor containing decimal places, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer. Then divide as with integers, writing the decimal point in the quotient as soon as the decimal point in the dividend is reached.*

Thus, in dividing 17.28 by 1.2, we multiply both divisor and dividend by 10, and then have $172.8 \div 12$. The divisor is now an integer and we proceed as shown, finding the quotient to be 14.4. It should be noted that the decimal point in the quotient is placed exactly above that in the dividend.

```
  14.4
12)172.8
   12
   52
   48
   48
   48
```

151. Division with a Remainder. If the divisor is not contained in the dividend without a remainder, zeros may be annexed to the dividend and the division continued as many more places as desired.

```
   21.9708
24)527.3000
   48
   47
   24
   233
   216
   170
   168
   200
   192
   8
```

Required to divide 5.273 by 0.24 to four decimal places.

Multiplying both divisor and dividend by 100, we have $527.3 \div 24$. Dividing, the quotient is 21.9708 with remainder 8, and $\frac{8}{24} = \frac{1}{3}$. Therefore the quotient is $21.9708\frac{1}{3}$, or simply 21.9708 to four decimal places.

If the next figure of the quotient had been 5 or more, we should call the quotient 21.9709 to four decimal places, this being nearer the correct result than 21.9708. But since the fraction is only $\frac{1}{3}$, and we are required to go only to four decimal places, we drop the fraction and take 21.9708 as the result.

152. Division by Multiples of 10. If the divisor is an integer and ends in one zero or more than one zero, we may shorten the work by dividing both divisor and dividend by such a power of 10 as shall cut off the zeros.

For example, $42.08 \div 8000$ gives the same quotient as $0.04208 \div 8$. That is, we have divided both numbers by 1000 by cutting off three zeros from the divisor and moving the decimal point three places to the left in the dividend.

EXERCISE 75

Divide:

1. $1.12 \div 0.4$.	11. $2.72 \div 0.02$.	21. $36.08 \div 3.28$.
2. $6.25 \div 0.5$.	12. $5.34 \div 0.03$.	22. $1.036 \div 0.56$.
3. $5.04 \div 0.7$.	13. $3.64 \div 0.04$.	23. $62.32 \div 0.82$.
4. $5.84 \div 0.8$.	14. $7.15 \div 0.05$.	24. $115.6 \div 0.34$.
5. $7.02 \div 0.9$.	15. $27.6 \div 0.06$.	25. $3.509 \div 12.1$.
6. $7.26 \div 0.6$.	16. $36.4 \div 0.07$.	26. $1.9044 \div 9.2$.
7. $8.44 \div 200$.	17. $0.425 \div 2.5$.	27. $1.0989 \div 3.3$.
8. $96.4 \div 400$.	18. $0.288 \div 1.2$.	28. $0.8118 \div 1.23$.
9. $70.2 \div 300$.	19. $0.057 \div 1.9$.	29. $19.536 \div 44.4$.
10. $0.85 \div 500$.	20. $8.12 \div 0.029$.	30. $17.139 \div 59.1$.

31. A man pays \$113 for coal at \$5.65 a ton. How many tons does he buy?

32. The product of two numbers is 156.25 and one of the numbers is 5000. What is the other number?

33. The product of two numbers is 6.51021 and one of the numbers is 3.207. What is the other number?

34. If wind blows in a hurricane at the rate of 8796.6 feet per minute, what is the rate per second?

35. A knot, used at sea, is 1.152 miles. When a steamship is traveling 23.04 miles an hour, what is its rate in knots?

36. A contractor bought a shipment of bricks for $176.40, paying $7.20 a thousand. How many bricks did he buy?

37. A dealer paid $54.40 freight on some oil at $0.68 per thousand pounds. How many pounds were there?

38. A contractor bought a lot of shingles for $717.75, paying $4.12½ a thousand. How many thousand did he buy?

39. Wheat that cost 98 cents a bushel is sold for $1.04 a bushel. How much must be sold to realize a profit of $132?

40. How many times will a steam roller 4.25 feet in circumference turn in rolling the length of a lawn 136 feet long?

41. How many lengths of fence, each 12.75 feet long, will be required to inclose a square field that is 522.75 feet on a side?

42. A dealer receives a shipment of stoves, the total weight being 9.75 thousand pounds. He pays $70.20 freight. What is the freight per thousand pounds?

43. A wheel is 28 inches in diameter, and the circumference is 3.1416 times as much. Find the number of times the wheel turns in going a mile, 63,360 inches.

44. A cubic foot of a certain kind of granite weighs 173.75 pounds, and a cubic foot of water weighs 62.5 pounds. The granite is how many times as heavy as water?

45. Cedar is 0.61 as heavy as water, and oak is 1.17 times as heavy as water. Oak is how many times as heavy as cedar? Answer to the nearest hundredth.

46. Find to the nearest hundredth the weight of a cubic foot of air, water being 830 times as heavy as air, and a cubic foot of water weighing 1000 ounces.

47. A gallon contains 231 cubic inches. Alcohol is 0.84 as heavy as water. Find the number of cubic inches of alcohol that will weigh as much as a gallon of water.

EXERCISE 76

Review Problems

1. What will 320 barrels of apples cost at $\$2.12\frac{1}{2}$ a barrel? at $\$2.33\frac{1}{3}$ a barrel?

2. What will 640 pounds of sugar cost at $\$0.04\frac{1}{16}$ a pound? at $\$0.04\frac{5}{8}$ a pound?

3. A merchant bought 320 yards of silk for \$790.40, and sold it at \$2.85 a yard. How much did he gain?

4. A man bought 6250 (6.25 thousand) tiles at \$7.20 a thousand. What was the total cost?

5. A man bought 13,750 shingles for \$49.50. What did they cost per thousand?

6. A dealer paid \$0.74 per hundred pounds on a shipment of 4800 pounds of salt. How much did he pay in all?

7. A grocer bought 5 barrels of sugar containing respectively 313, 311, 307, 316, and 317 pounds, at $5\frac{5}{16}$ cents a pound. How much did the sugar cost?

8. A man owned 0.60 of a certain manufacturing plant, and sold $0.33\frac{1}{3}$ of his share for \$3000. At this rate what was the whole plant worth?

9. The sides of a field are respectively 141.25 feet, 136.75 feet, 158.6 feet, and 149.8 feet. What is the perimeter of the field?

10. If the sides of a triangle are 148.3 feet, 134.75 feet, and 179.6 feet respectively, how much less is the sum of the first two of these sides than the sum of the second two?

11. A manufacturer pays 35 men at the rate of \$1.50 a day, 48 at the rate of \$1.75, 30 at the rate of $\$2.66\frac{2}{3}$, and 10 at the rate of \$3.30. How much is the pay roll for a week of 6 days?

12. Divide the product of 22.50 and 37.50 by the product of 36.25 and 21.25, first canceling as much as possible.

13. Divide the product of $1.33\frac{1}{3}$ and 87.5 by the product of $\frac{1}{3}$ and $3\frac{3}{4}$, first canceling as much as possible.

14. Divide the product of 36.25 and 21.25 by the product of 22.5 and 37.5, by first reducing to improper fractions and then canceling as much as possible.

15. If we know, in an example in division, that both the divisor and the quotient are 25.25, what is the dividend?

16. If we know, in an example in division, that the divisor is 29.75, and that the quotient to two decimal places is also 29.75, and that there is a remainder of 0.0006, what is the dividend?

17. If the area of a triangle is represented by half the product of the base and altitude, and the base is represented by 17.25 and the altitude by 8.4, what number represents the area?

18. Mr. A owned 0.75 of a manufacturing plant and sold $0.33\frac{1}{3}$ of his share to Mr. B, who in turn sold 0.2 of what he bought to Mr. C. What part of the plant did Mr. C buy?

19. A man bought three barrels of sugar containing respectively 312.75 pounds, 314.75 pounds, and 316.5 pounds. He paid 5.375 cents a pound for the sugar. How much did it all cost him?

20. A man has to make a railway journey of 254.7 miles. After he has gone $0.33\frac{1}{3}$ of the distance he receives a telegram calling him back to a place half the distance he has already gone. After he gets back there, how many miles is it to the end of his journey?

EXERCISE 77

Review Questions

1. What is the unit of United States money, and what sign is used to indicate it?

2. How many places of decimals do cents (including dimes) occupy? How many places do dimes occupy?

3. Write two decimals showing the part of a dollar a cent is worth, and the part of a dollar a mill is worth.

4. Write three decimals showing the part of a dollar a dime is worth, the part of a dime a cent is worth, and the part of a cent a mill is worth.

5. How are dollars reduced to cents? How are dollars reduced to mills?

6. How are cents reduced to dollars? How are mills reduced to dollars?

7. How are dollars and cents arranged for addition or subtraction? Illustrate each operation.

8. In multiplying dollars and cents by an integer, how do you know where to place the decimal point in the product?

9. In dividing dollars and cents by an integer, how do you know where to place the decimal point in the quotient?

10. A unit consists of how many tenths? of how many hundredths? of how many thousandths?

11. A tenth consists of how many hundredths? of how many thousandths? of how many ten-thousandths?

12. How is the value of a figure in a number changed if it is moved one place to the left? one place to the right?

13. What decimal place is occupied by tenths? by hundredths? by thousandths? by ten-thousandths? by hundred-thousandths? by millionths?

14. How should you read a number that has an integral part and a decimal part? Illustrate by reading 200.007.

15. What effect does it have on a decimal to annex zeros to it? Explain this in the case of 0.5 and 0.50.

16. In adding and subtracting decimals, how do you arrange the decimal points? Why is this?

17. In subtraction of decimals, if the minuend has a smaller number of decimal places than the subtrahend how do you proceed? What right have you to do this?

18. What effect does it have on the value of a number if the decimal point is moved one place to the right? one place to the left?

19. What effect does it have on the value of a number if the decimal point is moved two places to the right? two places to the left?

20. In the multiplication of decimals, how many decimal places must the product have? Illustrate by the case of 0.7 and 0.25.

21. Explain why the product of 0.6 and 0.4 should have two decimal places, by reducing both to common fractions and then multiplying.

22. In division, if the divisor is not an integer, how may you make it such without altering the quotient?

23. If the divisor is an integer, how can you tell where to put the decimal point in the quotient? Illustrate by dividing 41.44 by 4.

24. What is the reason for writing the quotient over the dividend? Give an example to assist in your explanation.

25. Why do you not have to write the denominators in decimal fractions, while you do have to write them in common fractions?

CHAPTER IX

SHORT METHODS

153. Short Methods of Operation. Having considered the four fundamental operations of addition, subtraction, multiplication, and division, with integers, common fractions, and decimal fractions, we shall now see what short methods are really practical.

There are many short methods, but most of them are useful only to those who do a great deal of computing. It is also very common at present for professional computers to use machines for all the operations. Most large banks have adding machines, and there are also machines that subtract, multiply, and divide.

154. Short Methods of Addition. The most helpful suggestion in rapid addition is to add by groups instead of by single figures.

For example, in this column the pupil should see 19 when he glances at the three lowest figures; he should see two eights, or 16, in the next three, and 9 in the two top figures. He should therefore think "19, 35, 44," or some similar adding of groups.

Probably he will at first find it easier to group by twos and think "15, 24, 35, 44."

Professional computers often add two columns at once, but this is not recommended for pupils in school, where absolute accuracy is more important than speed.

7
2
8
3
5
4
6
9

When there are two or more columns of figures in the addends, professional computers usually write down the sum of each column. In this way an error may be found more easily when the work is checked by adding in the direction opposite to that of the original addition. The immediate discovery of such errors is the most important means of saving time.

EXERCISE 78

Add as rapidly as possible, trying always to add by groups of two or three figures. Check each result.

1.	4.	7.	10.	13.	16.
26	97	29	428	$34.78	$121.42
43	60	91	607	62.48	137.60
28	89	72	320	71.92	89.48
91	34	30	556	38.46	125.50
72	42	47	707	50.09	38.75
54	80	66	62	77.05	192.50
81	75	55	491	82.91	65.82
24	66	43	377	4.80	48.75

2.	5.	8.	11.	14.	17.
47	84	22	572	$75.50	$263.08
63	60	89	681	26.75	148.75
96	32	72	708	8.93	192.68
34	98	35	320	14.98	635.35
81	77	60	572	6.37	870.08
29	63	81	647	24.42	428.70
88	49	92	998	69.70	642.35
72	92	37	123	8.84	408.77

3.	6.	9.	12.	15.	18.
29	35	29	547	$18.92	$221.62
68	42	68	629	17.86	342.98
73	87	73	372	23.48	476.35
40	98	52	588	52.96	192.09
96	63	31	793	78.32	778.38
85	47	48	874	91.46	658.02
77	72	77	645	87.00	523.65
42	31	65	880	9.85	807.91

155. Short Methods of Subtraction. There are no special suggestions to be given for subtraction beyond those set forth in § 32 and § 33.

3	9	9	9	10
1	7	6	5	2
2	2	3	4	8

The one given in § 33 is particularly helpful. In a case like 40,000 – 17,652 we may think of the places of the minuend as 3, 9, 9, 9, 10 and subtract from left to right.

EXERCISE 79

Subtract as rapidly as possible, beginning at the left when this can easily be done:

1.	7.	13.	19.
7000	50,000	$100.00	$250.
2763	26,492	75.62	75.65
2.	**8.**	**14.**	**20.**
6000	60,000	$300.00	$350.
4296	31,429	128.64	82.50
3.	**9.**	**15.**	**21.**
5000	40,000	$500.00	$640.
1297	17,792	275.55	121.75
4.	**10.**	**16.**	**22.**
8000	70,000	$700.00	$480.
2776	20,078	428.50	135.25
5.	**11.**	**17.**	**23.**
9000	65,000	$900.00	$530.
4326	21,572	230.20	192.25
6.	**12.**	**18.**	**24.**
8000	80,000	$800.00	$320.
7080	27,600	206.00	175.75

156. Short Methods of Multiplication. There are several short methods of multiplication that are valuable, particularly because certain multipliers are so often used.

Such multipliers are 10, 100, 1000, and other powers of 10; 5, 25, 125, and $12\frac{1}{2}$; $16\frac{2}{3}$, $33\frac{1}{3}$, and $66\frac{2}{3}$; 9, 11, and 75.

157. Aliquot Parts. A number that will exactly divide another number is called an *aliquot part* of that number.

Thus 5, 10, 25, $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, and so on, are aliquot parts of 100, and they include some of the most important multipliers mentioned above.

The most important aliquot parts of 100 are as follows:

$50 = \frac{1}{2}$ of 100	$33\frac{1}{3} = \frac{1}{3}$ of 100	$20 = \frac{1}{5}$ of 100
$25 = \frac{1}{4}$ of 100	$16\frac{2}{3} = \frac{1}{6}$ of 100	$10 = \frac{1}{10}$ of 100
$12\frac{1}{2} = \frac{1}{8}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100	$5 = \frac{1}{20}$ of 100
$6\frac{1}{4} = \frac{1}{16}$ of 100	$4\frac{1}{6} = \frac{1}{24}$ of 100	$2 = \frac{1}{50}$ of 100

Other important fractional parts of 100, but not aliquot parts, are as follows:

$75 = \frac{3}{4}$ of 100	$62\frac{1}{2} = \frac{5}{8}$ of 100	$66\frac{2}{3} = \frac{2}{3}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100	$87\frac{1}{2} = \frac{7}{8}$ of 100	$83\frac{1}{3} = \frac{5}{6}$ of 100

158. Multiplication by Aliquot Parts. We may simplify multiplication by 10, 100, and 1000, and by their aliquot parts, as follows:

(1) *To multiply by 10, move the decimal point one place to the right, annexing zero if necessary.*

Thus $10 \times 46.75 = 467.5$, and $10 \times 75\frac{1}{2} = 10 \times 75.5 = 755$.

(2) *To multiply by 100, move the decimal point two places to the right, annexing zeros if necessary.*

Thus $100 \times 0.275 = 27.5$, $100 \times 6.5 = 650$, and $100 \times 72 = 7200$.

(3) *To multiply by 1000, move the decimal point three places to the right, annexing zeros if necessary.*

Thus $1000 \times 0.02365 = 23.65$, and $1000 \times 147 = 147{,}000$.

(4) *To multiply by 50, multiply by 100 and divide by 2.*

For $50 = \frac{100}{2}$; therefore $50 \times 84 = \frac{100}{2} \times 84 = \frac{8400}{2} = 4200$.

(5) *To multiply by 25, multiply by 100 and divide by 4.*

For $25 = \frac{100}{4}$; therefore $25 \times 84 = \frac{100}{4} \times 84 = \frac{8400}{4} = 2100$.

(6) *To multiply by $12\frac{1}{2}$, multiply by 100 and divide by 8.*

For $12\frac{1}{2} = \frac{100}{8}$; therefore $12\frac{1}{2} \times 64 = \frac{100}{8} \times 64 = 800$.

(7) *To multiply by 125, multiply by 1000 and divide by 8.*

For $125 = \frac{1000}{8}$; therefore $125 \times 72 = \frac{1000}{8} \times 72 = 9000$.

(8) *To multiply by $33\frac{1}{3}$, multiply by 100 and divide by 3.*

For $33\frac{1}{3} = \frac{100}{3}$; therefore $33\frac{1}{3} \times 69 = \frac{6900}{3} = 2300$.

(9) *To multiply by $16\frac{2}{3}$, multiply by 100 and divide by 6.*

For $16\frac{2}{3} = \frac{100}{6}$; therefore $16\frac{2}{3} \times 84 = \frac{8400}{6} = 1400$.

(10) *To multiply by 5, multiply by 10 and divide by 2.*

For $5 = \frac{10}{2}$; therefore $5 \times 0.42 = \frac{4.2}{2} = 2.1$.

EXERCISE 80

Multiply:

1. $10 \times \$0.25$.
2. $10 \times \$8251$.
3. $10 \times \$6.25$.
4. $10 \times \$5.50$.
5. $100 \times \$0.37$.
6. $100 \times \$2.87\frac{1}{2}$.
7. $1000 \times \$0.25$.
8. $1000 \times \$6.37\frac{1}{2}$.
9. $50 \times \$64{,}224$.
10. $50 \times \$752.20$.
11. $50 \times \$861.55$.
12. $25 \times \$144$.
13. $25 \times \$640$.
14. $25 \times \$325$.
15. $675 \times \$25$.
16. $25 \times \$25.50$.
17. $25 \times \$75.25$.
18. $12\frac{1}{2} \times \$6400$.
19. $12\frac{1}{2} \times \$4880$.
20. $125 \times \$640$.
21. $125 \times \$6400$.
22. $125 \times \$2800$.
23. $33\frac{1}{3} \times 36$.
24. $33\frac{1}{3} \times 48$.
25. $33\frac{1}{3} \times 144$.
26. $33\frac{1}{3} \times \$312.30$.
27. $16\frac{2}{3} \times 12{,}630$.
28. $16\frac{2}{3} \times \$636.30$.
29. $5 \times 47{,}664.80$.
30. $5 \times \$210{,}650$.
31. $125 \times \$84{,}480$.
32. $125 \times \$844.80$.
33. $16\frac{2}{3} \times 4{,}800{,}000$.

34. What will 25 books cost at $1.20 each?

35. What will 25 wagons cost at $116 each?

36. What will 50 cows cost at $54 a head?

37. What will 100 books cost at $62\frac{1}{2}¢$ each?

38. What will 25 tons of hay cost at $12.40 a ton?

39. What will $12\frac{1}{2}$ yards of cloth cost at $35¢$ a yard?

40. What will 25 dozen pencils cost at $0.48 a dozen?

41. What will 48 office desks cost at $25 each?

42. At $5 an acre, what is a tract of 14,264 acres of mountain land worth?

43. How much will a man earn in 36 weeks at $25 a week? at $33\frac{1}{3}$ a week?

44. How much will a dozen tables weigh at 25 pounds each? at 50 pounds each?

45. How much will 615 men earn in a week if each earns $12\frac{1}{2}$? if each earns $16\frac{2}{3}$?

46. How much must a dealer pay for 336 easy-chairs at $12\frac{1}{2}$ each? at $16\frac{2}{3}$ each? at $6\frac{1}{4}$ each?

47. A dealer buys 144 boxes of soap, each weighing $12\frac{1}{2}$ pounds. What is the total weight?

48. At $33\frac{1}{3}$ a month, how much rent will a man pay for his house in a year? in six months?

49. At $33\frac{1}{3}$ a week, how much will a salesman receive if he works 48 weeks? if he works 52 weeks?

50. If a man earns $16\frac{2}{3}$ a week, how much will he earn in 36 weeks? in 48 weeks? in 52 weeks?

51. Multiply 1728 by $1\frac{1}{4}$; 172.8 by $12\frac{1}{2}$; 17.28 by 125. How do the results differ? Write the reason for this.

52. Multiply 1728 by $3\frac{1}{3}$; 172.8 by $33\frac{1}{3}$; 17.28 by $333\frac{1}{3}$. Compare the results and explain, as in Ex. 51.

159. Multiplication by Fractional Parts of 100. We may simplify multiplication by fractional parts of 100 as follows:

(1) *To multiply by 75, multiply by 100 and take $\frac{3}{4}$ of the product.*

For $75 = \frac{3}{4}$ of 100; therefore $75 \times \$64.40 = \frac{3}{4}$ of $\$6440 = \$4830$.

(2) *To multiply by $37\frac{1}{2}$, multiply by 100 and take $\frac{3}{8}$ of the product.*

For $37\frac{1}{2}$ is $\frac{3}{8}$ of 100; therefore $37\frac{1}{2} \times \$72 = \frac{3}{8}$ of $\$7200 = \$2700$.

(3) *To multiply by $62\frac{1}{2}$, multiply by 100 and take $\frac{5}{8}$ of the product.*

For $62\frac{1}{2} = \frac{5}{8}$ of 100; therefore $62\frac{1}{2} \times 640 = \frac{5}{8}$ of $64{,}000 = 40{,}000$.

(4) *To multiply by $87\frac{1}{2}$, multiply by 100 and take $\frac{7}{8}$ of the product.*

For $87\frac{1}{2} = \frac{7}{8}$ of 100; therefore $87\frac{1}{2} \times 96 = \frac{7}{8}$ of $9600 = 8400$.

(5) *To multiply by $66\frac{2}{3}$, multiply by 100 and take $\frac{2}{3}$ of the product.*

For $66\frac{2}{3} = \frac{2}{3}$ of 100; therefore $66\frac{2}{3} \times 144 = \frac{2}{3}$ of $14{,}400 = 9600$.

160. Multiplication by Certain Decimals. From § 158 and § 159 the following short methods are evident:

(1) *To multiply by 0.1, move the decimal point one place to the left; by 0.01, move the decimal point two places to the left; by 0.001, move the decimal point three places to the left.*

(2) *To multiply by 0.5, take half of the multiplicand.*

(3) *To multiply by 0.25, take a fourth of the multiplicand.*

(4) *To multiply by $0.12\frac{1}{2}$, take an eighth of the multiplicand.*

(5) *To multiply by $0.33\frac{1}{3}$, take a third of the multiplicand.*

(6) *To multiply by $0.16\frac{2}{3}$, take a sixth of the multiplicand.*

(7) *To multiply by 0.75, take three fourths of the multiplicand.*

Other similar rules will easily be found by the student.

EXERCISE 81

Multiply:

1. 75×684.
2. 75×816.
3. 75×936.
4. $75 \times \$1.48$.
5. $75 \times \$3.20$.
6. $75 \times 24{,}616$.
7. $248 \times \$0.75$.
8. $432 \times \$0.75$.
9. 0.75 of 2436.
10. 3574×0.05.
11. 0.25 of 536.
12. 0.25 of 5.36.
13. 0.25 of 0.536.
14. $704 \times \$0.25$.
15. $172 \times \$0.25$.
16. $776 \times \$0.12\frac{1}{2}$.
17. 0.75 of 6884.
18. $37\frac{1}{2} \times 424$.
19. $37\frac{1}{2} \times 1776$.
20. $736 \times \$0.37\frac{1}{2}$.
21. $656 \times \$0.37\frac{1}{2}$.
22. $568 \times \$0.62\frac{1}{2}$.
23. $736 \times \$0.62\frac{1}{2}$.
24. $824 \times \$0.87\frac{1}{2}$.
25. $592 \times \$0.87\frac{1}{2}$.
26. $368 \times \$0.12\frac{1}{2}$.
27. $336 \times \$0.33\frac{1}{3}$.
28. $297 \times \$0.33\frac{1}{3}$.
29. $372 \times \$0.66\frac{2}{3}$.
30. $492 \times \$0.66\frac{2}{3}$.
31. $558 \times \$0.16\frac{2}{3}$.
32. $174 \times \$0.16\frac{2}{3}$.
33. $744 \times 0.87\frac{1}{2}$.
34. $66\frac{2}{3} \times 3366$.
35. $66\frac{2}{3} \times 2886$.
36. $66\frac{2}{3} \times 288.6$.
37. $66\frac{2}{3} \times 28.86$.
38. 0.5 of 6432.
39. 0.5 of 1338.
40. 0.5 of 7952.
41. 2952×0.05.
42. 248×0.05.
43. 24.8×0.5.
44. 2.48×0.5.
45. 24×7.5.
46. 2.4×7.5.
47. 2.4×0.75.
48. $0.12\frac{1}{2} \times 0.16$.
49. How much will 75 cows cost at \$48 each?
50. How much will 75 cows cost at \$55 each?
51. How much will 72 horses cost at \$75 each?
52. How much will 88 horses cost at \$125 each?
53. How much will 288 whalebones cost at 25¢ each?
54. How much will 275 packing boxes cost at $62\frac{1}{2}$¢ each?
55. How much will 484 yards of silk cost at 75¢ a yard?
56. How much will 364 yards of cloth cost at 75¢ a yard?
57. How much will 624 yards of jute burlap cost at $37\frac{1}{2}$¢ a yard?

161. Multiplication by Numbers near 10, 100, and so on. Since $9 = 10 - 1$, we may multiply easily by 9 by annexing a zero, which multiplies by 10, and then subtracting the given multiplicand from the product by 10.

$10 \times 7286 = 72860$
$1 \times 7286 = \quad 7286$
$9 \times 7286 = 65574$

Thus 9×7286 is evidently the same as $10 \times 7286 - 7286$, as here shown. It is evidently shorter to subtract 7286 from 72,860 than to multiply 7286 by 9.

From this reasoning and similar courses of reasoning we have the following short processes:

(1) *To multiply by 9, multiply by 10 and subtract the given multiplicand.*

Thus $9 \times \$374.62 = \$3746.20 - \$374.62 = \$3371.58$.

(2) *To multiply by 11, multiply by 10 and add the given multiplicand.*

Thus $11 \times \$374.62 = \$3746.20 + \$374.62 = \$4120.82$.

(3) *To multiply by 99, multiply by 100 and subtract the given multiplicand.*

Thus $99 \times 4368 = 436{,}800 - 4368 = 432{,}432$.

(4) *To multiply by 98, multiply by 100 and subtract twice the given multiplicand.*

Thus $98 \times \$435.75 = \$43{,}575 - 2 \times \$435.75 = \$43{,}575 - \$871.50 = \$42{,}703.50$.

To multiply by 999, 998, and like numbers, proceed in a manner similar to the above.

EXERCISE 82

Multiply:

1. $9 \times \$724.56$.
2. $9 \times \$875.93$.
3. $11 \times \$682.97$.
4. $11 \times \$493.82$.
5. $99 \times \$128.72$.
6. $99 \times \$426.45$.
7. $98 \times \$327.47$.
8. $98 \times \$462.09$.
9. $999 \times \$43.75$.
10. $998 \times \$62.86$.
11. $1001 \times \$27.63$.
12. $1002 \times \$324.75$.

162. Short Methods of Division. There are certain divisors that are used so often that it becomes convenient to know short methods of dividing by them.

The principal ones are 10, 100, 1000, and other powers of 10; 5, 25, 125, $12\frac{1}{2}$, and $33\frac{1}{3}$.

Since division is the inverse of multiplication, we easily derive the following from the cases already considered:

(1) *To divide by 10, move the decimal point one place to the left; by 100, two places; by 1000, three places; and so on.*

Thus $4936 \div 10 = 493.6$; $\$4876 \div 100 = \$48.76$; $29.5 \div 1000 = 0.0295$. This has already been explained in studying decimal fractions.

(2) *To divide by 5, multiply by 2 and divide by 10.*

For to divide by 5 is to multiply by $\frac{1}{5}$ or $\frac{2}{10}$. To divide 32,305 by 5 we may multiply by 2, obtaining 64,610, and cut off the zero, obtaining 6461, a process somewhat easier than actual division.

(3) *To divide by 25, multiply by 4 and divide by 100.*

For $\frac{1}{25} = \frac{4}{100}$. It is much easier to multiply by 4 and move the decimal point two places than to divide by 25.

(4) *To divide by 125, multiply by 8 and divide by 1000.*

Why is this true?

(5) *To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100.*

Why is this true?

EXERCISE 83

Divide:

1. $\$7463 \div 10$.
2. $\$2896 \div 100$.
3. $\$4960 \div 1000$.
4. $\$2215.50 \div 5$.
5. $\$3122.35 \div 5$.
6. $\$172,232 \div \$5$.
7. $\$9125 \div \$25$.
8. $12,325 \div 25$.
9. $31,325 \div 125$.
10. $62,750 \div 125$.
11. $\$1525 \div 33\frac{1}{3}$.
12. $\$3275 \div 33\frac{1}{3}$.

13. At 25¢ a yard, how many yards of gingham can a dealer buy for $124?

14. At 25¢ a yard, how many yards of percaline can a dealer buy for $160?

15. At 25¢ a dozen, how many dozen spools of darning cotton can a dealer buy for $180?

16. At $33\frac{1}{3}$¢ a yard, how many yards of canvas can a dealer buy for $135?

17. At $33\frac{1}{3}$¢ a yard, how many yards of jute burlap can a dealer buy for $165?

18. At $\$3.33\frac{1}{3}$ a day, how many days must a man work in order to earn $130?

19. At $3\frac{1}{3}$¢ a yard, how many yards of cheese cloth can a dealer buy for $115?

20. At $2\frac{1}{2}$¢ a skein, how many skeins of cotton must a manufacturer sell in order to receive $125.75?

21. At $1.25 a box, how many boxes of fancy candies can a dealer buy for $180?

22. At $125 each, how many horses can a dealer buy for $1750? for $1375?

23. At $5 each, how many rocking-chairs must a factory sell in order to receive $3375?

24. A manufacturer sells $3750 worth of typewriters at $50 each. How many does he sell?

25. How long will it take an express train to go 362.5 miles at the rate of 50 miles an hour?

26. How long will it take a freight train to go $166\frac{2}{3}$ miles at the rate of 25 miles an hour?

27. A manufacturer sells $1810 worth of silk at $1.25 a yard. How many yards does he sell?

EXERCISE 84

Review Problems

1. How much will 144 boxes cost at $37\frac{1}{2}¢$ each?

2. How much will 22 typewriters cost at $\$62\frac{1}{2}$ each?

3. How much will 16 acres of land cost at $\$87\frac{1}{2}$ an acre?

4. How much will 375 mules cost at \$125 each?

5. If 25 tons of coal cost \$187.50, what will be the cost of 99 tons?

6. How much will 288 dozen sewing-machine needles cost at $37\frac{1}{2}¢$ a dozen?

7. How much will 250 dozen yards of millinery braid cost at $62\frac{1}{2}¢$ a dozen?

8. There are 44,000 yards in 25 miles. How many yards are there in one mile?

9. How much will 18 thousand feet of oak flooring cost at \$125 a thousand feet?

10. How many bushels of oats will 81 acres produce, if an average acre produces $33\frac{1}{3}$ bushels?

11. How many bushels of potatoes will 32 acres produce, if an average acre produces 125 bushels?

12. A tailor paid \$210 for some broadcloth at $\$3.33\frac{1}{3}$ a yard. How many yards did he buy?

13. A dealer sold some overcoats for \$2900, at the rate of $\$33\frac{1}{3}$ apiece. How many overcoats did he sell?

14. How much will 16 thousand feet of mixed red and white birch flooring cost at \$50 a thousand feet?

15. A farmer owns 78 acres of land covered with a growth of white pine which he values at $\$166\frac{2}{3}$ an acre. At what amount does he value his wood lot?

CHAPTER X

DENOMINATE NUMBERS

163. Denominate Numbers. A concrete number that expresses measure is called a *denominate number.*

For example, \$7 or 2 feet 3 inches, but not 5 chairs or 8 rivers.

164. Compound Numbers. Denominate numbers involving two or more different units are called *compound numbers.*

For example, 2 feet 5 inches, or 3 hours 20 minutes.

165. Reduction. The process of changing the unit of a number without changing the value is called *reduction* (§ 98).

Thus, 2 feet 6 inches = 30 inches = 2.5 feet.

166. Reduction Descending. Reduction from a higher to a lower denomination is called *reduction descending.*

For example, required to reduce 2 hours 10 minutes 20 seconds to seconds.

hr.	min.	sec.
2	10	20
60		
120		
10		
130		
60		
7800		
20		
7820		

Since 1 hour = 60 minutes,
therefore 2 hours = 2 × 60 minutes
= 120 minutes.
120 minutes + 10 minutes = 130 minutes.
Since 1 minute = 60 seconds,
therefore 130 minutes = 130 × 60 seconds
= 7800 seconds.
7800 seconds + 20 seconds = 7820 seconds.

For convenience the actual multiplications and additions are carried out as shown at the left.

In practical life we seldom have reductions involving more than two different units (denominations), and in this arithmetic other cases will not in general be considered.

Therefore, *in reduction descending, multiply the number of units of the next lower denomination required to make one unit of the highest denomination given by the given number of units of this highest denomination. To the product add the given number of units of this lower denomination.*

Proceed thus with each successive result until the required denomination is reached.

167. Measures of Length. The table of common measures of length is as follows:

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

A *hand* (4 in.) is used in measuring the height of horses; a *fathom* (6 ft.) and *cable length* (120 fathoms) in measuring depths of water; a *knot* (nautical mile, 1.152 common or statute miles, or 6080.27 ft.) in measuring distances at sea. Carpenters, mechanics, and others usually write 2′ 6″ for 2 ft. 6 in.

EXERCISE 85

Reduce:

1. 54 ft. 7 in. to inches.
2. 72 ft. 9 in. to inches.
3. 28 yd. to feet; to inches.
4. $5\frac{1}{2}$ yd. to feet; to inches.
5. 26 rd. to yards; to feet.
6. 320 rd. to yards; to feet.
7. 41.8 mi. to miles and feet.
8. 25.75 mi. to miles and feet.
9. 126.7 mi. to miles and feet.
10. 160 rd. to yards.
11. 5 mi. to rods.
12. 7 mi. to yards.
13. $22\frac{1}{2}$ mi. to rods.
14. 3.25 mi. to feet.
15. 0.5 mi. to feet.
16. 0.6 mi. to feet.
17. 0.65 mi. to feet.
18. 0.01 mi. to yards.
19. A knot being 1.152 miles, how many miles does a ship sail in going 480 knots?

168. Reduction Ascending. Reduction from a lower to a higher denomination is called *reduction ascending.*

For example, required to reduce 104 in. to higher units.

For practical purposes the work may be arranged as shown at the left, but the explanation is better understood from the following:

$$\begin{array}{r|l} 12 & 104 \text{ (in.)} \\ 3 & 8 \text{ (ft.)} \cdots 8 \text{ in.} \\ & 2 \text{ (yd.)} \cdots 2 \text{ ft.} \end{array}$$

Since 1 in. $= \frac{1}{12}$ ft.,

104 in. $= 104 \times \frac{1}{12}$ ft. $= 8\frac{8}{12}$ ft. $= 8$ ft. 8 in.

Since 1 ft. $= \frac{1}{3}$ yd.,

8 ft. $= 8 \times \frac{1}{3}$ yd. $= 2\frac{2}{3}$ yd. $= 2$ yd. 2 ft.

Therefore the answer is 2 yd. 2 ft. 8 in.

Therefore, in reduction ascending,

Divide the given number of units by the number of units required to make one of the next higher denomination.

Divide this quotient and each successive quotient in like manner until the required denomination is reached.

The last quotient with the several remainders arranged in order is the answer sought.

EXERCISE 86

Reduce to higher units:

1. 96 in.	4. 5 ft.	7. 11 yd.	10. 700 rd.
2. 100 in.	5. 9 ft.	8. 15 yd.	11. 960 rd.
3. 115 in.	6. 25 ft.	9. 35 yd.	12. 1280 rd.

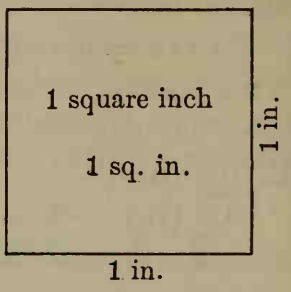
Reduce to feet, or to feet and inches:

13. 150 in.	15. 360 in.	17. 600 in.	19. 960 in.
14. 165 in.	16. 400 in.	18. 720 in.	20. 972 in.

Reduce to yards, or to yards and one lower unit:

21. 60 ft.	23. 500 ft.	25. 72 in.	27. 50 in.
22. 65 ft.	24. 750 ft.	26. 84 in.	28. 6 ft. 8 in.

169. A Square. A flat surface that has four equal sides and four equal angles is called a *square.*



If each side is 1 in. long, the square is 1 square inch; if each side is 1 ft. long, the square is 1 square foot; if each side is 1 yd. long, the square is 1 square yard; and so on.

170. Area. The number of square units in a surface is called its *area.*

For example, if this figure represents a surface 4 yd. long and 2 yd. wide, the area is $2 \times 4 \times 1$ square yard, or 8 square yards.

171. Square Measure. The common table of square measure is as follows:

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

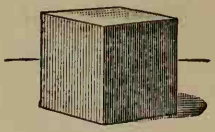
EXERCISE 87

Reduce to the next lower units:

1. 13 sq. ft.	4. 43 sq. yd.	7. 6 A.	10. 5 sq. mi.
2. 28 sq. ft.	5. 62 sq. yd.	8. 15 A.	11. 25 sq. mi.
3. 60 sq. ft.	6. 124 sq. rd.	9. 35 A.	12. 150 sq. mi.

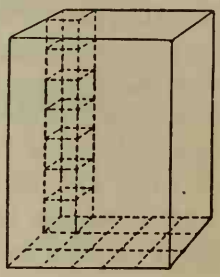
13. Reduce 75 sq. ft. 36 sq. in. to square inches.
14. Reduce 2 sq. yd. 58 sq. in. to square inches.
15. Reduce 52 sq. yd. 8 sq. ft. to square feet.
16. Reduce 67,592 sq. in. to higher units.
17. Reduce 47,916 sq. ft. to higher units.
18. Reduce 22,471 sq. yd. to higher units.

172. A Cube. A solid bounded by six equal squares is called a *cube.*



If each edge is 1 in., the cube is 1 cubic inch; if each edge is 1 ft., the cube is 1 cubic foot; and so on.

173. Volume. The number of cubic units a body contains is called its *volume.*



For example, if this figure represents a solid 5 ft. long, 3 ft. wide, and 7 ft. high, each column of cubes as here shown contains 7×1 cubic foot. But 3×5 such columns may be placed on the bottom of the solid, so the volume is $3 \times 5 \times 7 \times 1$ cubic foot, or 105 cubic feet.

174. Cubic Measure. The common table of cubic measure is as follows:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

A cubic yard of earth is considered a load.

EXERCISE 88

Reduce to the next lower units:

1. 2 cu. ft.	**4.** 125 cu. ft.	**7.** 25 cu. yd.	**10.** 25 cd.
2. 25 cu. ft.	**5.** $33\frac{1}{3}$ cu. ft.	**8.** 36 cu. yd.	**11.** 125 cd.
3. 75 cu. ft.	**6.** $66\frac{2}{3}$ cu. ft.	**9.** 125 cu. yd.	**12.** 500 cd.

Reduce to higher units:

13. 3456 cu. in.	**15.** 2457 cu. ft.	**17.** 2000 cu. in.
14. 8640 cu. in.	**16.** 4900 cu. ft.	**18.** 4260 cu. in.

19. Reduce 25 cu. ft. 65 cu. in. to cubic inches.

20. Reduce 15 cu. yd. $10\frac{1}{2}$ cu. ft. to cubic inches.

21. Reduce 1290 cu. ft. to cords; to cubic yards.

175. Avoirdupois Weight. The common weight used in the United States is called *avoirdupois weight.*

The table is as follows:

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

100 lb. is sometimes called a *hundredweight* (cwt.). The ton of 2000 lb. is sometimes called the *short ton*, there being an old ton of 2240 lb., known as the *long ton*, which is used in the customhouse and in some wholesale transactions in mining products. There is also the *long hundredweight* of 112 lb., but it is rarely used except in English trade. A ton of loose hay fills about $8 \times 8 \times 8$ cu. ft., or 512 cu. ft., of space, but this varies greatly with the condition of the hay. A keg of nails, a cental of corn, or a quintal of fish weighs 100 lb. A barrel of flour weighs 196 lb., a barrel of salt 280 lb., and a barrel of pork or of beef 200 lb. A cask of lime weighs 240 lb.

176. Troy and Apothecaries' Weight. Goldsmiths still use a system known as *troy weight*, and in compounding medicines apothecaries use a table of their own. These are not of enough importance to be generally taught, but are inserted here for reference.

The table of troy weight is as follows:

24 grains (gr.) = 1 pennyweight (pwt. or dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

The avoirdupois pound contains 7000 gr., the troy pound 5760 gr.

A *carat* weight, used in weighing diamonds, is commonly taken in the United States as 3.2 grains. The word *carat*, meaning $\frac{1}{24}$, is used also in speaking of the purity of gold. Hence 18 carats fine means $\frac{18}{24}$ pure gold and $\frac{6}{24}$ alloy.

The table of apothecaries' weight is as follows:

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb.)

Physicians often use the metric system, explained later.

EXERCISE 89

1. In this load of hay there are 2150 lb. Express this as tons and a common fraction.

2. If there were 1850 lb. in the load, what decimal of a ton would there be?

3. How many grains are there in 56 troy pounds?

4. The avoirdupois pound is how many times as heavy as the troy pound?

Reduce to the next lower units:

5. 16 lb. **7.** $2\frac{1}{2}$ lb. **9.** 16 T. **11.** $28\frac{1}{2}$ T.

6. 27 lb. **8.** $8\frac{1}{4}$ lb. **10.** 23 T. **12.** $16\frac{1}{4}$ T.

Reduce to higher units:

13. 176 oz. **14.** 200 oz. **15.** 32,000 oz. **16.** 10,000 lb.

Reduce to ounces:

17. $29\frac{3}{4}$ lb. **18.** $56\frac{5}{8}$ lb. **19.** $54\frac{3}{16}$ lb. **20.** 27 lb. 5 oz.

Reduce to pounds:

21. 27 T. **22.** 2.7 T. **23.** 144 oz. **24.** 3 T. 700 lb.

177. Liquid Measure. The common table of liquid measure is as follows:

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

A gallon contains 231 cubic inches.

These measures also are in common use:

$31\frac{1}{2}$ gallons = 1 barrel (bbl.)
63 gallons = 1 hogshead (hhd.)

Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads. Whenever barrels or hogsheads are used as *measures*, a barrel means $31\frac{1}{2}$ gallons, and a hogshead means 63 gallons. A barrel contains about $4\frac{1}{5}$ cu. ft.

Apothecaries and physicians use the following table of liquid measure, but it need not be learned:

60 minims (℥) = 1 dram (℥ lx. or f. ʒ)
8 fluid drams = 1 ounce (fl. drm. viij. or f. ℥)
16 ounces = 1 pint (fl. oz. xvj. or O)
8 pints = 1 gallon (cong.)

EXERCISE 90

Reduce to the next lower units:

1. 27 pt.	**3.** 69 qt.	**5.** 89 gal.	**7.** $64\frac{1}{4}$ gal.
2. 46 pt.	**4.** 87 qt.	**6.** 68 gal.	**8.** $62\frac{1}{2}$ qt.

Reduce to higher units:

9. 96 gi. **11.** 35 gi. **13.** 300 qt. **15.** $92\frac{1}{2}$ qt.

10. 96 pt. **12.** 85 pt. **14.** 475 pt. **16.** $75\frac{1}{2}$ pt.

Reduce to pints:

17. 396 gi. **18.** 892 gi. **19.** $126\frac{1}{2}$ qt. **20.** $79\frac{1}{2}$ gal.

Reduce to quarts:

21. 792 gi. **22.** 298 pt. **23.** $926\frac{3}{4}$ gal. **24.** 72 gal. 3 qt.

Reduce to the next lower units:

25. 64 lb. **27.** $9\frac{3}{4}$ lb. **29.** 96 T. **31.** $48\frac{3}{4}$ T.

26. $5\frac{1}{2}$ cu. ft. **28.** $7\frac{1}{2}$ sq. ft. **30.** $6\frac{1}{2}$ sq. rd. **32.** $2\frac{1}{2}$ A.

Reduce to higher units:

33. 528 oz. **35.** 375 oz. **37.** 35,000 oz. **39.** 12,500 lb.

34. 624 oz. **36.** 450 oz. **38.** 32,176 oz. **40.** 75,550 lb.

Reduce to ounces:

41. $\frac{3}{4}$ lb. **43.** $72\frac{3}{8}$ lb. **45.** $96\frac{7}{8}$ lb. **47.** 1 T. 200 lb.

42. $\frac{7}{32}$ lb. **44.** $29\frac{5}{32}$ lb. **46.** 29.75 lb. **48.** 9 lb. 13 oz.

Reduce to pounds:

49. $42\frac{3}{4}$ T. **51.** 3.6 T. **53.** 432 oz. **55.** 2 T. 800 lb.

50. $65\frac{4}{5}$ T. **52.** $3.3\frac{1}{3}$ T. **54.** 4832 oz. **56.** 5 T. 650 lb.

57. How many pounds in $26\frac{1}{2}$ long tons?

58. How many pounds in 1750 hundredweight?

59. How many more pounds in 86 long hundredweight than in 86 common hundredweight?

60. These children are standing by a cord of wood. It is 8 ft. long and the wood is cut in 4-foot lengths. How high is the pile?

61. How many cubic feet are there in $3\frac{3}{4}$ cords?

62. A pile of 4-foot wood is 4 rd. long and 6 ft. high. How many cords does it contain?

63. A pile of 4-foot wood is 75 ft. long and 7 ft. high. How many cords does it contain?

64. How much is a pile of 4-foot wood 60 ft. long and 6 ft. high worth at \$3.75 a cord?

65. How much is a pile of 4-foot wood 75 ft. long and 7 ft. high worth at \$4 a cord?

66. A pile of wood 8 ft. long and 4 ft. high is usually called a cord, the price varying with the length of the wood. How many cords in a pile 28 ft. long and 6 ft. high?

67. A pile of wood is 50 ft. long and 5 ft. high. How many cords does it contain?

68. How much is a pile of wood 30 ft. long and 5 ft. high worth at \$3.50 a cord?

178. Dry Measure. The common table of dry measure is as follows:

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel contains 2150.42 cu. in., or about $1\frac{1}{4}$ cu. ft. A dry quart contains 67.2 cu. in., while the liquid quart contains only 57.75 cu. in. In measuring grain, seeds, and small fruits the measure must be *even* full. In measuring apples, potatoes, and other large articles the measure must be *heaping* full.

The various states prescribe the weight of certain farm products per bushel. These weights vary and are given in the Appendix.

EXERCISE 91

Reduce to the next lower units:

1. 37 qt.	5. $16\frac{1}{2}$ qt.	9. 48 pk.	13. 47 bu.
2. 49 qt.	6. $37\frac{1}{2}$ qt.	10. 59 pk.	14. $56\frac{1}{2}$ bu.
3. 99 qt.	7. $87\frac{1}{2}$ qt.	11. $67\frac{1}{4}$ pk.	15. $92\frac{3}{4}$ bu.
4. 256 qt.	8. $248\frac{1}{2}$ qt.	12. $268\frac{3}{4}$ pk.	16. $427\frac{1}{4}$ bu.

Reduce to higher units:

17. 72 pt.	20. 64 qt.	23. 96 pk.	26. 156 pt.
18. 76 pt.	21. 60 qt.	24. 89 pk.	27. 156 qt.
19. 98 pt.	22. 98 qt.	25. 75 pk.	28. 156 pk.

Reduce to pints:

29. 72 qt.	**32.** 49 pk.	**35.** 2 bu.	**38.** 2 pk. 3 qt.
30. $85\frac{1}{2}$ qt.	**33.** $53\frac{1}{2}$ pk.	**36.** 35 bu.	**39.** 5 pk. 7 qt.
31. $68\frac{1}{2}$ qt.	**34.** $75\frac{3}{4}$ pk.	**37.** $28\frac{1}{2}$ bu.	**40.** 2 bu. 1 pk.

Reduce to quarts:

41. 38 pk.	**43.** $59\frac{3}{4}$ pk.	**45.** 75 bu.	**47.** 2 bu. 3 pk.
42. $48\frac{1}{2}$ pk.	**44.** $78\frac{1}{8}$ pk.	**46.** $87\frac{1}{2}$ bu.	**48.** 9 bu. 1 pk.

Reduce to pecks:

49. 72 bu.	**50.** $89\frac{1}{2}$ bu.	**51.** 144 qt.	**52.** 576 pt.

Reduce to bushels:

53. 336 pk.	**55.** 772 pk.	**57.** 352 qt.	**59.** 1984 pt.
54. 516 pk.	**56.** 956 pk.	**58.** 1952 qt.	**60.** 6336 pt.

61. A bushel contains 2150.42 cu. in. How many bushels of wheat can be put in a bin containing 645,126 cu. in.?

62. Allowing $1\frac{1}{4}$ cu. ft. to a bushel, how many bushels of wheat can be put in a bin containing 900 cu. ft.?

63. If a bushel of wheat weighs 60 lb., what will be the weight of the wheat in Ex. 62?

64. A dry quart contains 67.2 cu. in. How many quarts in a measure containing 336 cu. in.?

65. If a bushel of oats weighs 32 lb., what will be the weight of the oats required to fill a bin that contains 875 cu. ft., allowing $1\frac{1}{4}$ cu. ft. to a bushel?

66. There are 231 cu. in. in a gallon and 537.6 cu. in. in a peck. From these measures verify the number of cubic inches in a liquid quart and in a dry quart as given in § 178 below the table.

179. Surveyors' Measure. Surveyors use a chain 4 rods, or 66 feet, long. This chain has 100 links, each 7.92 in. long. They use a table as follows:

Long Measure	*Square Measure*
7.92 in. = 1 link (l.)	**16 sq. rd. = 1 sq. ch.**
25 links = 1 rod (rd.)	**10 sq. ch. = 1 A.**
4 rods, or 100 links = 1 chain (ch.)	**640 A. = 1 sq. mi.**
80 chains = 1 mile (mi.)	**1 sq. mi. = 1 section (sec.)**
	36 sec. = 1 township (T.)

EXERCISE 92

Reduce to the next lower units:

1. 300 l.	**6.** 27 mi.	**11.** 29 sq. ch.	**16.** 36 sq. mi.
2. 275 l.	**7.** $16\frac{1}{2}$ mi.	**12.** $3\frac{3}{4}$ sq. ch.	**17.** 75 sq. mi.
3. 210 rd.	**8.** $23\frac{3}{4}$ mi.	**13.** $5\frac{7}{8}$ sq. ch.	**18.** 125 sq. mi.
4. 425 ch.	**9.** $42\frac{1}{8}$ mi.	**14.** 92 A.	**19.** 630 sq. mi.
5. 575 ch.	**10.** $16\frac{7}{8}$ mi.	**15.** 750 A.	**20.** 960 sq. mi.

Reduce to chains:

21. 700 l.	**23.** 976 rd.	**25.** 664 rd.	**27.** 26 mi.
22. 700 rd.	**24.** 824 rd.	**26.** 792 in.	**28.** 37 mi.

Reduce to acres and decimals:

29. 70 sq. ch.	**31.** 96 sq. ch.	**33.** 3 A. 4 sq. ch.
30. 320 sq. rd.	**32.** $1\frac{3}{4}$ sq. mi.	**34.** 7 A. 8 sq. ch.

35. How many acres in a field that contains 975 sq. ch.?

36. How many acres in a field that contains 720 sq. rd.?

37. If a square is 1 mi. on a side, how many acres does it contain? If $\frac{1}{2}$ mi. on a side, how many acres does it contain?

180. Time Measure. The table of time is as follows:

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
366 days = 1 leap year
100 years = 1 century

There are 12 months (mo.) or about 52 weeks in a year. The names of the months, called calendar months, are:

	da.		da.
1. January (Jan.)	31	7. July	31
2. February (Feb.)	28 or 29	8. August (Aug.)	31
3. March (Mar.)	31	9. September (Sept.)	30
4. April (Apr.)	30	10. October (Oct.)	31
5. May	31	11. November (Nov.)	30
6. June	30	12. December (Dec.)	31

The number of days in each month may be easily remembered by committing to memory the following lines:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
Till leap year gives it twenty-nine."

We usually write 15 minutes after 10 in the forenoon thus: 10 hr. 15 min. A.M., 10.15 A.M., or 10 15 A.M. We usually write 20 minutes before 3 in the afternoon thus: 2 hr. 40 min. P.M., 2.40 P.M., or 2 40 P.M.

A *year* is the time in which the earth performs one revolution in its orbit around the sun, and consists of 365 da. 5 hr. 48 min. 46 sec.; that is, nearly $365\frac{1}{4}$ days. As there are 365 days in a common year, a common year lacks nearly $\frac{1}{4}$ of a day of being a solar year, and this defect is made up by reckoning for some years (leap years) 366 days.

The leap years are the years whose dates are exactly divisible by 4; except in the case of complete hundreds, and these are leap years if exactly divisible by 400. Thus, 1600 and 1904 were leap years; 1800, 1900, and 1909 were not.

EXERCISE 93

Reduce to the next lower units:

1. 360 min.	4. 16 da.	7. 52 wk.	10. 8 yr.
2. 18 hr.	5. 36 da.	8. 26 wk.	11. 16 yr.
3. 36 hr.	6. $48\frac{1}{2}$ da.	9. $46\frac{3}{7}$ wk.	12. 32 yr.

Reduce to higher units:

13. 360 sec.	15. 504 hr.	17. 132 mo.	19. 25 yr.
14. 720 min.	16. 168 hr.	18. $182\frac{5}{8}$ da.	20. 365 da.

21. How many days from March 5 to June 5?

22. How many days from June 10 to November 10?

23. Allowing $365\frac{1}{4}$ da. to the year, how many days in half a century?

24. Allowing $365\frac{1}{4}$ days to the year, how many years in 182,625 da.?

25. Name the leap years in the following: 1925, 1926, 1927, 1928, 1929, 1930, 1940, 1950, 1960, 1972, 1982, 2000.

26. January 1, 1910, being Saturday, on what day of the week does January 1, 1911 fall?

27. Reduce 15 hr. 32 min. 17 sec. to seconds.

28. Reduce 2 da. 7 hr. 29 min. to minutes.

29. Reduce 8 wk. 5 da. 2 hr. 3 min. to minutes.

30. How many hours will the month of February, 1948, contain? How many minutes?

31. A lunar month contains 2,551,443 seconds. Express a lunar month in units of higher denominations.

32. A man received \$2.75 a day every week day during March, in a year when that month had 5 Sundays. How much did he receive during the month?

181. Circle. A plane figure bounded by a curved line all points of which are equally distant from a point within is called a *circle.*

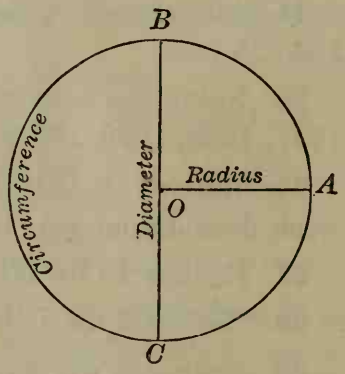
182. Circumference. The bounding line of a circle is called the *circumference.*

183. Center. The point within a circle from which all points in the circumference are equally distant is called the *center* of the circle.

184. Arc. Any portion of a circumference is called an *arc.*

185. Diameter and Radius. A straight line drawn through the center of a circle, terminated at both ends by the circumference, is called a *diameter.* A straight line drawn from the center to any point in the circumference is called a *radius.* A radius is equal to half a diameter.

186. Measuring Arcs. The circumference of every circle is divided into 360 equal parts called *degrees.*



Corresponding to every arc, like AB in the figure, is an angle at the center, AOB. Therefore to every degree on the circumference (*arc degree*) there is a degree at the center (*angle degree*), and an arc has as many degrees as its angle.

187. Right Angle and Perpendicular. The angle at the center of a circle corresponding to one fourth of a circumference is called a *right angle*, and the two lines that form a right angle are said to be *perpendicular* to each other.

Thus angle AOB is a right angle, and OB is perpendicular to OA. A right angle, therefore, contains $\frac{1}{4}$ of 360 degrees, or 90 degrees.

An angle less than a right angle is called an *acute angle*, and one greater than a right angle is called an *obtuse angle.*

188. Angle Measure and Arc Measure. The following are the tables of circular measure:

Angle Measure

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 4 right angles

Arc Measure

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference

That is, angle measures and arc measures are practically the same.

A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

189. Miscellaneous Measures. The following tables are used:

Numbers	*Paper*
12 units = 1 dozen (doz.)	**24 sheets = 1 quire**
12 dozen = 1 gross (gr.)	**20 quires = 1 ream**
12 gross = 1 great gross	**2 reams = 1 bundle**
20 units = 1 score	**5 bundles = 1 bale**

The great gross and score are becoming obsolete. For convenience in counting, 500 sheets are now commonly called a ream.

EXERCISE 94

Express as seconds:

1. 17′. 3. 48′. 5. 3°. 7. 3° 5′. 9. 8° 5′ 8″.
2. 32′ 4. 56′. 6. 27° 8. 4° 8′. 10. 9° 6′ 2″.

Express as minutes:

11. 480″. 12. 900″. 13. 27°. 14. 9° 9′. 15. 27° 56′.

Express as degrees:

16. 4800″. 17. 1800″. 18. 2700′. 19. 108,000′.

190. Addition of Compound Numbers. The general principles of adding compound numbers are the same as those of adding integers.

The only difference arises from the fact that in integers ten units of each order always make one unit of the next higher, while in compound numbers the number of units of a lower denomination required to make one unit of the next higher varies with the denomination we are using.

Required to add 2 hr. 38 min. 42 sec., 3 hr. 25 min. 36 sec., and 5 hr. 17 min. 58 sec.

Arranging as in ordinary addition, so that units of the same denomination stand in the same column, we have the work as here shown.

hr.	min.	sec.
2	38	42
3	25	36
5	17	58
11	22	16

The sum of the seconds is 136 sec., or 2 min. 16 sec. We write the 16 sec. under the seconds, and add the 2 min. to the minutes. The sum of the minutes is 82 min., or 1 hr. 22 min. Writing the 22 min., we add the 1 hr. to the hours. The sum of the hours is 11 hr. Hence the result is 11 hr. 22 min. 16 sec.

In practical life to-day we rarely have to add numbers of more than two denominations, and hence the examples given in this book are in general of not more than two denominations.

EXERCISE 95

Add:

1.

ft.	in.
3	9
7	10
12	8

2.

ft.	in.
29	11
16	8
37	6

3.

sq. ft.	sq. in.
96	36
72	87
81	92

4.

sq. ft.	sq. in.
39	47
46	83
92	54

5.

cu. ft.	cu. in.
87	921
94	436
76	997

6.

cu. ft.	cu. in.
37	692
49	875
89	981

7.

lb.	oz.
147	7
962	12
321	8

8.

lb.	oz.
482	15
621	12
537	8

9.

gal.	qt.
29	3
37	2
16	1
19	3
36	2

10.

gal.	qt.
36	3
48	1
29	2
43	2
51	3

11.

qt.	pt.
17	1
36	1
49	1
27	
35	1

12.

gal.	pt.
47	5
36	4
42	6
71	1
43	5

13.

bu.	pk.
29	3
36	2
48	3
73	1
49	2

14.

pk.	qt.
19	3
17	6
23	4
19	7
13	5

15.

ft.	in.
17	8
26	7
32	5
41	7
20	9

16.

ft.	in.
47	8
69	7
33	9
83	6
50	

17.

wk.	da.
12	4
32	5
16	6
10	5
13	3

18.

hr.	min.	sec.
2	17	36
3	39	48
5	17	50
3	56	49
5	48	35

19.

°	′	″
5	37	49
9	26	
2	52	46
	48	32
6	15	9

20.

hr.	min.	sec.
17	23	32
23	15	48
4	7	15
16	19	49
43	6	33

21. Add 16° 27′ 32″, 37° 48′ 16″, 48° 29′ 36″, 28° 13′ 23″.

22. Add 2 A. 40 sq. rd., 3 A. 80 sq. rd., 4 A. 30 sq. rd., 10 sq. rd.

23. Add 5 sq. rd. 6 sq. yd., 7 sq. rd. $20\frac{1}{4}$ sq. yd., 6 sq. rd. 4 sq. yd.

24. Add 9 cu. yd. 18 cu. ft., 7 cu. yd. 12 cu. ft., 24 cu. ft. 920 cu. in., 808 cu. in.

25. Add $2\frac{1}{2}$ lb., 4 lb. 4 oz., $3\frac{3}{4}$ lb., 5 lb. 8 oz., 6 lb. 9 oz., $5\frac{1}{4}$ lb., $2\frac{1}{2}$ lb., 8 oz.

191. Subtraction of Compound Numbers. The general principles of subtracting compound numbers are the same as those of subtracting integers, except as explained in addition.

For example, from 22 hr. 16 min. 27 sec. required to subtract 14 hr. 25 min. 40 sec.

hr.	min.	sec.
22	16	27
14	25	40
7	50	47

Since we cannot subtract 40 sec. from 27 sec., we take 1 min. (60 sec.) of the 16 min. Then 60 sec. + 27 sec. = 87 sec., and 87 sec. − 40 sec. = 47 sec. Since we cannot subtract 25 min. from 15 min., we take 1 hr. (60 min.) of the 22 hr. Then 60 min. + 15 min. = 75 min., and 75 min. − 25 min. = 50 min. Then 21 hr. − 14 hr. = 7 hr. The difference is 7 hr. 50 min. 47 sec.

EXERCISE 96

Subtract:

1.

ft.	in.
29	7
16	10

2.

ft.	in.
82	$5\frac{1}{2}$
64	7.25

3.

sq. ft.	sq. in.
78	92
29	106

4.

sq. ft.	sq. in.
69	75.25
49	$26\frac{1}{2}$

5.

yd.	ft.
12	2
11	1

6.

bu.	qt.
28	24
19	17

7.

A.	sq. rd.
58	82
42	120

8.

hr.	min.
19	40
12	25

9.

cu. ft.	cu. in.
126	750
78	1025

10.

cu. ft.	cu. in.
250	67
175	998

11.

lb.	oz.
251	4
176	12

12.

lb.	oz.
173	
69	10

13.

gal.	qt.
42	2
29	3

14.

qt.	pt.
63	
28	1

15.

bu.	pk.
42	1
26	2

16.

min.	sec.
106	27
49	49

192. Multiplication of Compound Numbers. Required to multiply 3 ft. 9 in. by 5.

Since 5×9 in. $= 45$ in., or 3 ft. 9 in., we write 9 in. under the inches, and reserve the 3 ft. to add to the product of 5×3 ft. Then 5×3 ft. $= 15$ ft., and 15 ft. $+ 3$ ft. $= 18$ ft. Therefore the product is 18 ft. 9 in.

ft.	in.
3	9
	5
18	9

EXERCISE 97

Multiply:

1. 37 ft. 6 in. by 7.
2. 49 ft. 11 in. by 15.
3. 64 ft. 10 in. by $16\frac{1}{2}$.
4. 35 qt. 1 pt. by 17.
5. 62 gal. 3 qt. by 25.
6. 48 bu. 2 pk. by 62.
7. 27 sq. ft. 60 sq. in. by 7.
8. 6 cu. ft. 700 cu. in. by 9.
9. 27 cu. ft. 200 cu. in. by 8.
10. 28 lb. 4 oz. by 35.
11. 48 min. 25 sec. by 16.
12. 55 min. 30 sec. by 20.

13. One side of a square is 7 ft. 9 in. What is the perimeter?

14. If a barrel contains 31 gal. 2 qt., how much water will 15 barrels hold?

15. If one can of fruit weighs 1 lb. 12 oz., what will a dozen such cans weigh?

16. If a school has 6 recitations a day, each of 45 min., how many hours and minutes of recitations a day?

17. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a tank that contains 48 cu. ft.?

18. A year being 365 da. 5 hr. 48 min. 46 sec., how many days, hours, minutes, and seconds in 6 years?

19. If the height of each story above the first or ground story in a city office building is 11 ft. 9 in., and that of the first story is 15 ft. 8 in., and there are nine stories in all, what is the height of the building?

193. Division of a Compound Number by an Abstract Number. Required to divide 7 hr. 5 min. 24 sec. by 2.

	hr.	min.	sec.
2)	7	5	24
	3	32	42

Arranging as in ordinary division, we have 7 hr. ÷ 2 = 3 hr., with remainder 1 hr., or 60 min. Then 60 min. + 5 min. = 65 min., and 65 min. ÷ 2 = 32 min., with remainder 1 min., or 60 sec. Then 60 sec. + 24 sec. = 84 sec., and 84 sec. ÷ 2 = 42 sec. Therefore the quotient is 3 hr. 32 min. 42 sec.

In most cases in practical business the dividend has only two denominations. Indeed, it is so much more common to write $2\frac{1}{2}$ bu., $7\frac{1}{2}$ lb., 28.25 A., instead of 2 bu. 2 pk., 7 lb. 8 oz., 28 A. 40 sq. rd., that compound numbers are not nearly as important as formerly.

194. Division of One Compound Number by Another. Required to divide 41 ft. 3 in. by 2 ft. 9 in.

Here it is advisable to reduce both numbers to the same denomination, and it makes no difference in principle whether we reduce to feet or to inches.

Reducing to inches, we have 41 ft. 3 in. = 495 in. and 2 ft. 9 in. = 33 in. Then 495 in. ÷ 33 in. = 15.

Or, reducing to feet, we have

$$41\tfrac{1}{4} \text{ ft.} \div 2\tfrac{3}{4} \text{ ft.} = \frac{\cancel{4}}{\cancel{11}} \text{ of } \frac{\overset{15}{\cancel{165}}}{\cancel{4}} = 15.$$

We aim to choose the denomination that makes the work the shorter.

EXERCISE 98

Divide:

1. 9 ft. 8 in. by 4.
2. 175 ft. 4 in. by 8.
3. 34 lb. 6 oz. by 11.
4. 29 yd. 18 in. by 6.
5. 47 mi. 288 ft. by 12.
6. 4 hr. 3 min. 30 sec. by 10.
7. 25 ft. 1 in. by 3 ft. 7 in.
8. 60 ft. 9 in. by 6 ft. 9 in.
9. 8 hr. 3 min. by 2 hr. 45 sec.
10. 59 yd. 21 in. by 5 yd. 15 in.
11. 240 lb. 12 oz. by 26 lb. 12 oz.
12. 21 mi. 1620 ft. by 1 mi. 345 ft.

13. How many planks, each 9 ft. 6 in. long, will it take to reach 76 ft.?

14. How many cans, each containing $\frac{1}{8}$ qt., can be filled from 7 gal. of sirup?

15. If a train travels 20 mi. in 24 min. 20 sec., how long does it take to go a mile?

16. How many rods are there in 44 yd.? How many yards are there in 44 rd.?

17. From 9 gal. of sirup 40 cans are filled. How many pints does each can hold?

18. If a train moves 73 ft. 4 in. in a second, how many seconds will it take to go a mile?

19. How many lots, each containing 2 A. 20 sq. rd., can be cut from a lot containing 17 A.?

20. If a train is traveling at the rate of a mile in 72 sec., how many miles will it travel in 42 min.?

21. If a man walks at the rate of a mile in 19 min., how many miles can he walk in 1 hr. 54 min.?

22. If a man can build 30 ft. 8 in. of fence a day, how many days will it take him to build 214 ft. 8 in.?

23. A steamship changed its longitude 52° 30′ in 7 days. How many degrees of longitude did it sail per day?

24. If each tile has a surface area of 1 sq. ft. 12 sq. in., how many tiles will be required to cover a space containing 156 sq. ft.?

25. A real estate dealer has $3\frac{1}{2}$ A. of land that he purposes to cut into five equal lots. How many square rods will there be in each lot?

26. A man is walking at the rate of a mile in 21 min., and a train is going at the rate of a mile in 72 sec. The train is traveling how many times as fast as the man?

EXERCISE 99

Review Problems

1. A schoolroom is 32 ft. 8 in. long. Express the length in inches.

2. A room is 282 in. wide. Express the width in feet and inches; in feet and a fraction.

3. How much will a strip of carpet cost at 50¢ a yard, the strip being 16 ft. 6 in. long?

4. A wheel is 11 ft. in circumference. How many times will it turn in going $2\frac{1}{2}$ mi.?

5. If a lot is 197 ft. 9 in. long and $82\frac{1}{4}$ ft. wide, the length is how many rods greater than the width?

6. If a man is walking 88 yd. a minute, he is walking at the rate of how many miles per hour?

7. If a cubic inch of cast iron weighs $4\frac{5}{36}$ oz., how many pounds does a cubic foot weigh?

8. If a cubic foot of ice weighs 57 lb. 8 oz., how many cubic feet of ice will it take to weigh a ton?

9. The radius of the earth at the equator is estimated to be 20,926,202 ft. Express this in miles and feet.

10. If a pile of wood contains 2760 cu. ft., how many cords does it contain?

11. Express 2 ft. 6 in. as inches; as feet and a decimal; as a fraction of a yard; as a fraction of a rod.

12. A horse is 14 hands high. Express this in inches; in feet and inches; in feet and a common fraction; in feet and a decimal fraction.

13. A cubic foot of water weighs 1000 oz. How many pounds is this? Express the result in pounds and ounces, and in pounds and a common fraction of a pound.

14. How many pounds does 5 cu. yd. 18 cu. ft. of earth weigh, if a cubic yard weighs 1 T. 825 lb. ?

15. If the average weight of a certain kind of water pipe is 15 lb. 8 oz. per foot, how many feet of the pipe will it take to weigh a ton ?

16. If the water in a certain harbor is 14 fathoms deep, what is the depth in feet ? If the depth is 165 ft., what is the depth in fathoms ?

17. If a man takes 100 steps of 0.8 yd. each in a minute, in how many hours and minutes will he walk a distance of 16 miles ?

18. When sound travels 1116 ft. a second, how far distant is the lightning flash when a man hears the thunder 9 seconds after he sees the flash ?

19. If a cubic foot of ice weighs 57 lb. 8 oz., what will be the weight in tons of the mass of ice 1 ft. thick that covers a pond whose surface is 100,000 sq. yd. ?

20. A train leaves Detroit at 5 min. past 8, and reaches Toledo, 59.6 mi. distant, at 5 min. before 10. What is the average time per mile ? Answer to two decimal places.

21. A train leaves Chicago at 8.45 A.M., and reaches Detroit, 284.74 mi. distant, at 3.30 P.M. What is the average time per mile ? Answer to two decimal places.

22. On a trip of three miles an automobile required 2 min. 34 sec. for the first mile, 2 min. 17 sec. for the second, and 1 min. 56 sec. for the third. How long did it take for the three miles ?

23. When a man was three miles from his railroad station he had 20 min. 30 sec. before his train was due to leave. He then drove one mile in 6 min. 48 sec., and a second mile in 6 min. 54 sec. What is the longest time in which he can drive the last mile and still catch his train ?

EXERCISE 100

Review Questions

1. What are denominate numbers? compound numbers?

2. What is meant by reduction descending? Illustrate.

3. What is meant by reduction ascending? Illustrate.

4. Write the common table of length. Make up and solve an example in addition of three compound numbers.

5. Write the common table of square measure. Make up and solve an example in subtraction in this table.

6. Write the common table of cubic measure. Make up and solve an example in multiplication in this table.

7. Write the common table of avoirdupois weight. Make up and solve an example in division in this table.

8. Write the common table of liquid measure. Make up and solve an example in division in this table.

9. Write the common table of dry measure. Make up an example involving this table, and solve it.

10. Why is it necessary to have some years of 366 days, and how often do they occur? What is the exception to this rule?

11. What is a circle? a circumference? an arc?

12. Distinguish between a radius and a diameter.

13. How are arcs measured? How are angles measured? Why does the same table answer for both?

14. How does the addition of compound numbers differ from that of integers? Illustrate.

15. How does the subtraction of compound numbers differ from that of integers? Illustrate by the case of $42 - 39$, and 4 ft. 2 in. $-$ 3 ft. 9 in.

16. How does the multiplication of a compound number differ from that of an integer? Illustrate.

CHAPTER XI

HOW TO SOLVE PROBLEMS

195. What we have Learned. We have now learned the operations of addition, subtraction, multiplication, and division with integers, common fractions, decimals, and compound numbers. We have solved also a great many problems involving these different operations and classes of numbers. We are now ready to consider more fully the methods of solving problems.

196. Three Necessary Things. In solving a problem there is one thing that the teacher will ask you to do, and two things that may be required. These three are: (1) to work rapidly and accurately; that is, to take the shortest road to the answer, and to be certain the answer is correct; (2) to put neatly on paper not merely the operation but a brief explanation; (3) to give a brief analysis or oral explanation.

For example, if 5 yd. of cloth cost \$2.10, how much will 12 yd. cost?

(1) *The number work:*

$$\begin{array}{r} .42 \\ 12 \times \$2.1\not{0} \\ \hline \not{5} \end{array} \qquad \begin{array}{r} \$.42 \\ 12 \\ \hline 84 \\ 42 \\ \hline \$5.04 \end{array}$$

(2) *The written explanation:*

5 yd. costs \$2.10.
1 yd. costs $\frac{1}{5}$ of \$2.10.
12 yd. costs $12 \times \frac{1}{5}$ of \$2.10, or \$5.04.

(3) *The oral analysis:*

Since 5 yd. cost \$2.10, 1 yd. will cost $\frac{1}{5}$ of \$2.10, and 12 yd. will cost $12 \times \frac{1}{5}$ of \$2.10, or \$5.04.

Teachers will not need to call for all this work with every example. Sometimes it will be necessary to emphasize (2) and sometimes (3), but the important thing is that (1) should be quickly and neatly, and, above all, *accurately* done.

197. Estimating the Answer. One of the best ways to avoid absurd answers is to *estimate the result* in advance. Write down this estimate and compare it with the answer, and if there is a great difference look over your work again.

For example, if 5 yd. cost \$2.10 we know 12 yd. will cost nearly $2\frac{1}{2}$ times as much, or somewhere near \$5. When we solve, if we find such a result as \$50.40, we see at once that there is a mistake, probably in the position of the decimal point. The correct result is \$5.04.

Always estimate the answer in advance, write down the estimate, and compare it with the result. If there is a great difference the work is probably wrong.

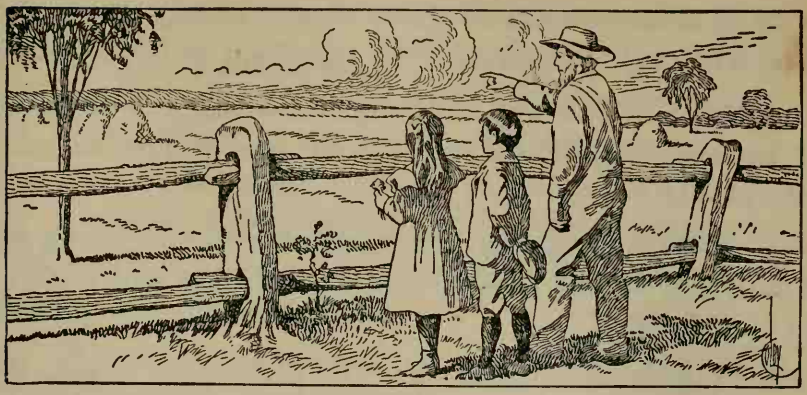
198. Accuracy. In any single operation the work is *right* or it is *wrong*. A business man will not excuse a bookkeeper who writes \$9250.75 instead of \$90,250.75. Only a zero is missing, but it means a difference of over \$80,000.

A pupil may be excused if he does not understand the wording or the meaning of a problem, but for inaccurate numerical work he must be judged as he will be judged in business, — his work is right or else it is wrong. Accuracy is far more important than speed.

Always check the number work in some way. The first consideration in arithmetic is accuracy.

One of the simplest checks is estimating the answer in advance, or seeing if it is reasonable after the work is performed.

199. Businesslike Work. Work as business men work, putting the additions and subtractions in columns except when a written analysis is required.



EXERCISE 101

Problems of the Farm

1. This farmer is pointing to a field that is 80 rd. long and 40 rd. wide. How many acres does it contain?

2. He explains that a cubic foot of damp clay loam contains on an average 18 lb. of water. What is the weight of the water to the depth of a foot in such loam in this field?

3. He thinks of replacing the fence around this field by a 4-strand barbed wire fence. How many pounds of barbed wire will he need, allowing 1 lb. of barbed wire to the rod?

4. He has an artesian well producing 3.265 cu. ft. of water per minute. If a cubic foot of water weighs 62.5 lb., what is the weight of the water produced in an hour?

5. In his barn he allows 350 cu. ft. of solidly stacked hay to the ton. How many tons in a mow containing 4375 cu. ft.?

6. If his corn fodder loses $\frac{3}{5}$ of its weight in curing, how much will 28 T. of green fodder weigh after it is cured?

7. Taking the annual rainfall of Indiana as 41.5 in., what will be the weight of water that will fall on 1 sq. ft. of land per year? on 1 sq. rd.? on 1 A.?

8. If a farmer has 64 A. of wheat yielding $24\frac{1}{4}$ bu. per acre, what will it cost him for threshing at 6¢ a bushel?

9. What is the cost of $15\frac{1}{2}$ bu. of wheat at 80¢, 17 bu. of potatoes at 28¢, 13 bu. of corn at 45¢, and 21 bu. of oats at 41¢?

10. If a man and team can seed 8 A. a day, how long will it take him to seed a field of 96 A., and what will it cost at $4.75 a day?

11. Supposing they are able to plow 2 A. a day, how much will it cost to plow a 76-A. field at $4.75 a day?

12. If three fourths of the weight of potatoes is water, and if a bushel of potatoes weighs 60 lb., what is the weight of water in 275 bu. of potatoes?

13. If two twenty-fifths of the weight of barley is water, and if a bushel of barley weighs 48 lb., what is the weight of water in 275 bu. of barley?

14. When wheat is worth 92¢ a bushel, what is the value of the wheat grown on 75 A., if the average yield per acre is 22 bu.?

15. If wheat weighs 60 lb. per bushel, and the average yield is $24\frac{1}{2}$ bu. per acre, what will be the weight of the wheat grown on a field of $42\frac{1}{2}$ A.?

16. An experiment has shown that it requires a pull of 92 lb. to draw a ton over a good gravel road, and $2.3\frac{1}{2}$ times as much to draw the same load over a common dirt road. What pull is required for the dirt road?

17. It is estimated that an 8-ft. pine pillar 4 in. square at the base will support 12.1 T. of the weight of a barn floor. At this rate, how much will a pillar 4 in. thick and 8 in. wide support? a pillar 4 in. thick and 10 in. wide? a pillar 4 in. thick and 12 in. wide?

18. If a bushel of wheat weighs 60 lb., how many pounds of wheat will be produced by a field of 80 A., where the average yield is $23\frac{1}{2}$ bu. to the acre? How many tons?

19. If the wheat in Ex. 18 is worth 82¢ a bushel, what is the value of the crop?

20. If it costs 52¢ an acre to cut the wheat mentioned in Ex. 18, and 6¢ a bushel to thresh it, what will be the cost of cutting and threshing?

21. If during a rainstorm 0.25 in. of rain fell on the field mentioned in Ex. 18, how many cubic inches of water fell on 1 sq. ft.? on 1 sq. rd.? on 1 A.? on the whole field?

22. Reduce each answer in Ex. 21 to cubic feet.

23. A cubic foot of water weighing 62 lb. 8 oz., find the weight of the water that fell on the whole field in Exs. 21, 22.

24. Using the last result of Ex. 22, find the weight of the water that falls on the field in a year, if the rainfall is 36 in.

25. If one twentieth part of wheat, by weight, is water, how many tons of water in the wheat produced by the field mentioned in Ex. 18?

26. If growing plants use one eighth of the water that falls on a field, what is the weight of water used by the wheat of Ex. 18 in a year, as found from the result of Ex. 24?

27. If in every bushel of wheat there are 20 oz. of nitrogen, what is the weight of the nitrogen in the wheat grown on the field in Ex. 18?

28. Suppose the soil in Ex. 18 to be cultivated to the depth of 4 in., what part of a cubic foot is cultivated to every square foot of area? How many cubic feet to every square rod? to 1 A.? to the entire 80 A.? What does the cultivated soil all weigh, at 75 lb. to the cubic foot?

200. Goods sold by the Hundred, the Thousand, or the Ton. It is the custom to sell certain kinds of goods by the hundred, certain kinds by the thousand, and certain kinds by the ton. In such cases it is frequently necessary to find the cost of fractional parts of these amounts.

In quoting prices, C is often used for hundred and M for thousand. At \$5.75 per C, means at \$5.75 per hundred; and at \$10 per M, means at \$10 per thousand.

The following examples will illustrate.

(1) Find the cost of 2675 laths at 44¢ per C.

\$0.44
$26\frac{3}{4}$
33
264
88
\$11.77

$2675 = 26\frac{3}{4}$ C.

Since 100 laths cost \$0.44, $26\frac{3}{4}$ C will cost $26\frac{3}{4} \times$ \$0.44, or \$11.77.

If the problem had asked the cost of 2638 laths, it would have been easier to multiply 26.38 by .44 than .44 by 26.38, and the result would have been the same (§ 42).

(2) Find the cost of 3924 feet of boards at \$17.50 per M.

\$3.924
$17\frac{1}{2}$
1962
27468
3924
\$68.670

$3924 = 3.924$ M.

Since 1000 ft. of boards cost \$17.50, 3.924 M ft. will cost $3.924 \times$ \$17.50, or \$68.67.

Here it is more convenient in practice to use $17\frac{1}{2}$ as the multiplier, changing 3.924 to \$3.924 (§ 42). In explaining the work, however, use 3.924 as the multiplier as above.

(3) Find the cost of 8712 pounds of anthracite coal at \$6.50 per ton.

\$8.712
$3\frac{1}{4}$
2178
26136
\$28.314

Since 2000 lb. of anthracite coal cost \$6.50, therefore 1000 lb. will cost $\frac{1}{2}$ of \$6.50, or \$3.25.

$8712 = 8.712$ M.

Since 1000 lb. of coal cost \$3.25, 8.712 M lb. will cost $8.712 \times \$3.25 = \$28.31$.

Here again it is more convenient to interchange multiplicand and multiplier, using $3\frac{1}{4}$ instead of 3.25 for the latter, and changing 8.712 to \$8.712.

EXERCISE 102

1. What will 9275 laths cost at 45¢ per C?

2. What will 27,625 laths cost at 43¢ per C?

3. What will 48,250 bricks cost at $8.10 per M?

4. What will 4925 paving stones cost at $9.22 per C?

5. What will 2735 ft. of boards cost at $18.75 per M?

6. What will 3690 ft. of boards cost at $22.50 per M?

7. What will 16,250 ft. of boards cost at $24.75 per M?

8. What will 7 T. 935 lb. of coal cost at $6.25 per ton?

9. At 25¢ per C, what is the freight on 32,500 lb. of flour?

10. If 7275 lb. of plaster cost $29.10, what is the price per ton?

11. What will 8275 tiles for the roof of a building cost at $9.25 per M?

12. What is the cost of coal per ton when 27,770 lb. cost $83.31?

13. At $19.50 per M, how many feet of boards can be bought for $146.25?

14. If a man buys 17,250 lb. of coal at $5.90 per ton, how much does it cost him?

15. If a dealer pays $3.10 per M for shingles and sells them for $3.80, how much does he gain on 29 M?

16. A coal dealer retails coal at $6.50 per ton. If it costs $3.25 at the mine, $1.70 for freight, and 80¢ for handling, what is his profit on 250 T. 750 lb.?

17. A lumber dealer buys 59,500 ft. of lumber at $16 per M. He sells 48,000 ft. at $19.50 per M, and the balance at $19 per M. How much is his whole profit?

201. Advantage of Cancellation. There is often an advantage in stating a problem in a single step, endeavoring then to simplify the work by cancellation.

For example, a man in rapid walking takes 120 steps a minute, averaging 26.4 in. each. How long will it take him to walk a mile?

Since he walks 26.4 in. in taking 1 step and takes 120 steps a minute, in 1 min. he will walk 120×26.4 in. Since in 1 mi. there are 5280 ft., or 5280×12 in., it will take him as many minutes to walk a mile as 5280×12 in. divided by 120×26.4 in. Therefore we have

$$\frac{5280 \times 12 \text{ in.}}{120 \times 26.4 \text{ in.}} = \frac{\overset{20}{\cancel{5280}} \times \cancel{12} \times \cancel{10}}{\underset{\cancel{12}}{\cancel{120}} \times \underset{\cancel{33}}{\cancel{264}}} = 20.$$

It will therefore take the man 20 minutes to walk 1 mile.

EXERCISE 103

Problems in Transportation

1. A manufacturer ships 2500 lb. of brass bedsteads from New York to Chicago. Such freight takes the first-class rate, 75¢ per hundred pounds. The manufacturer agrees to pay two thirds of the freight. How much does he pay?

2. He ships 1800 lb. of iron bedsteads, this kind of freight taking the second-class rate, 65¢ per hundred pounds. He agrees to pay $\frac{1}{3}$ of this freight. How much does he pay?

3. He ships 650 lb. of furniture casters in boxes, this kind of freight taking the third-class rate, 50¢ per hundred pounds. If he pays three fourths of this freight, how much does he pay?

4. He ships 900 lb. of crated bamboo bedsteads, this kind of freight taking double the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

5. He ships 1200 lb. of crated bamboo cabinets, the rate being three times the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

6. A dealer ships some bundles of green hides weighing 1900 lb., these taking the fourth-class rate, which is $\frac{7}{13}$ of the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

7. A dealer ships 2200 lb. of dry hides, the rate being $1\frac{1}{2}$ times the first-class rate. If the first-class rate is 90¢ per hundred pounds, how much is the freight?

8. Car loads of green hides take the fifth-class rate. If the first-class rate is 80¢ per hundred pounds, and the fifth-class two fifths as much, and a dealer ships a car load of 36,000 lb., how much is the freight?

9. Wheat in car-load lots takes the sixth-class rate. If the first-class rate is 90¢ per hundred pounds, and the sixth-class one third as much, and a dealer ships a car-load lot of 40,000 lb., how much is the freight?

10. If a dealer sends by express 250 lb. of goods, the rate being $3.50 per hundred pounds, and he pays half of the transportation charges, how much does he pay?

11. A dealer sent an express package weighing 150 lb. and agreed to pay $\frac{2}{5}$ of the transportation charges. If the rate was $2.50 per hundred pounds, how much did he pay?

12. A canal boat carries 211.75 T. of freight, the rate being 20¢ per hundredweight. If $\frac{3}{7}$ of the freight is owned by one man, $\frac{2}{7}$ by another, and the rest by a third, what is the charge to be paid by each?

13. If a drayman in a city averages 3 loads of freight a day, and receives on an average $2.10 a load, how much will he earn in a month of 26 working days?

202. Problems in Profit and Loss. In business the merchant's chief problem relates to the profit on his goods.

For example, a merchant bought 800 yd. of linen lawn at $67\frac{1}{2}¢$ a yard, and sold 725 yd. at $80¢$ a yard, and the rest at a bargain sale at $65¢$ a yard. Find his profit.

$\$0.67\frac{1}{2}$	$\$0.65$	$\$725$
800	75	.80
400	325	$\$580.00$
536	455	48.75
$\$540$	$\$48.75$	$\$628.75$
		540.
		$\$88.75$

The written analysis is as follows:

1. $800 \times \$0.67\frac{1}{2} = \$540$, the cost.
2. 800 yd. $-$ 725 yd. $= 75$ yd., sold at bargain sale.
3. $75 \times \$0.65 = \$48.75$, received at bargain sale.
4. $725 \times \$0.80 = \$580$, received at regular sale.
5. $\$580 + \$48.75 = \$628.75$, total receipts.
6. $\$628.75 - \$540 = \$88.75$, profit.

EXERCISE 104

Problems in Profit and Loss

1. A merchant buys 12 gross of cards of hooks and eyes at $5.04 per gross. He sells them at $5¢$ a card. Required his profit.

2. A merchant buys 480 yd. of torchon lace at $7\frac{1}{2}¢$ a yard. He sells 420 yd. at $10¢$ a yard, and the rest at a bargain sale at $5¢$ a yard. Required his profit.

3. A manufacturer delivers to a wholesale dealer 22,320 yd. of linen crash at $6\frac{1}{8}¢$ a yard. The wholesale dealer disposes of it at a profit of $1¢$ a yard to a retail dealer, who sells it for $10¢$ a yard. The retail dealer pays $6.25 freight on the goods. Required his profit.

4. A dealer buys 75 sewing machines at $40 each. He pays $56.25 freight on the machines and sells them for $45 apiece. How much is his profit?

5. A merchant buys 840 yd. of muslin at 7¢ a yard, and sells it at the rate of a dozen yards for a dollar. How much is his profit on the lot?

6. A merchant buys 12 gross of spools of silk at $5.76 a gross. He sells the spools at 60¢ a dozen. How much is his profit?

7. A dealer buys 275 lb. of twine at 28¢ a pound. He sells it at 35¢ a pound. How much is his profit per pound? on the whole lot?

8. If a dealer buys 960 yd. of scrim for $300 and sells three fourths of it at 35¢ a yard and the rest at 30¢ a yard, how much is his profit?

9. A dealer buys 250 reams of paper at $1.22½ a ream. He sells 7 reams, which were damaged, at 75¢ a ream, and the rest at $1.35 a ream. How much is his profit?

10. A dealer buys 640 yd. of damask linen at 87½¢ a yard. At what price per yard must he sell it to make a profit of $80 on the whole amount?

11. If a dealer buys 1600 yd. of Victoria lawn for $342, for how much per yard must he sell it to make a profit of 3⅝¢ a yard? What will his whole profit be?

12. A dry goods dealer buys 720 yd. of percaline at 19⅝¢ a yard. He sells 500 yd. at 25¢ a yard, 120 yd. at 23¢ a yard, and the rest at 15¢ a yard. How much is his profit?

13. A hardware dealer buys 3 doz. scissors at $3.12 a dozen, and a gross of shears at $5.40 a dozen. He sells the scissors at 35¢ a pair and the shears at 60¢ a pair. How much is his whole profit?

203. Unitary Analysis. A common form of analysis is illustrated in § 196, where we went from the cost of 5 yd. to the cost of 1 yd., and then to the cost of 12 yd.

An analysis that proceeds first to the unit and then to the required number of units is called *unitary analysis.*

In all such problems the words "at the same rate" are understood.

For example, if 50 bbl. of flour cost \$235, how many barrels can be bought for \$126.90?

If \$235 buys 50 bbl., \$1 will buy $\frac{50 \text{ bbl.}}{235}$, and \$126.90 will buy

$$126.90 \times \frac{\overset{10}{\cancel{50}} \text{ bbl.}}{\underset{47}{\cancel{235}}} = \frac{1269 \text{ bbl.}}{47} = 27 \text{ bbl.}$$

Sometimes the problem requires us to proceed to two or more units, as in some of the examples already given.

For example, if 16 horses eat 42 bu. of oats in 2 weeks, how many bushels will 32 horses eat in 3 weeks?

1. In 2 weeks 16 horses eat 42 bu.
2. In 1 week 16 horses eat $\frac{1}{2}$ of 42 bu.
3. In 1 week 1 horse eats $\frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.
4. In 3 weeks 1 horse eats $3 \times \frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.
5. In 3 weeks 32 horses eat $32 \times 3 \times \frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.

Canceling, we find the result to be 126 bu.

EXERCISE 105

1. If 100 laborers can do a piece of work in 12 days, how many will be needed to do it in 8 days?

2. How many men will it take to do a piece of work in 168 days that 108 men can do in 266 days?

3. If a stable has on hand enough oats to last 25 horses 105 days, how long will the oats last 15 horses?

4. How long will it take 16 men to do a piece of work that 18 men can do in 24 days?

5. If a clock gains 1 min. 15 sec. in 24 hours, how much will it gain in 42 hours at the same rate?

6. If 80 ft. of water pipe weigh 1240 lb., how much will 4 rd. of the pipe weigh?

7. If a candle $5\frac{3}{3}$ in. long burns 3 hr. 24 min., how many hours will a candle 6.3 in. long burn?

8. If a certain weight of sheet iron $\frac{3}{8}$ in. thick covers 15 sq. ft., how many square feet would the same weight of sheet iron $\frac{1}{4}$ in. thick cover?

9. If a wheel 12 ft. 1 in. in circumference makes 3432 revolutions in going a certain distance, how many revolutions will a wheel 11 ft. 11 in. in circumference make?

10. If a wheel 10 ft. 2 in. in circumference makes 3036 revolutions in going a certain distance, what is the circumference of a wheel that makes 3660 revolutions in going the same distance?

11. At the rate of $2\frac{3}{4}$ mi. an hour, I can walk a certain distance in 2 hr. 30 min. How long will it take me to walk the same distance at the rate of $3\frac{1}{8}$ mi. an hour?

12. At the rate of $2\frac{7}{8}$ mi. an hour, I can walk a certain distance in 2 hr. 20 min. What is my rate per hour if I walk the same distance in 2 hr. 33 min. 20 sec.?

13. Using 24 electric lights, a man's electricity bill is \$9 for 3 months. How much will it be for 4 months, if he uses half a dozen more lights, all lights running equally?

14. If 144 reams of paper are used in printing 7600 books, 84 sheets to a book, how many reams will be required to print 4900 books, containing 57 sheets of the same size?

15. If 27 sacks, each containing $3\frac{1}{3}$ bu., will hold a given quantity of grain, what must be the capacity of each sack that 18 sacks may hold the same quantity?

EXERCISE 106

Problems in Shipping our Food

1. A dealer ships 75 bu. of blueberries to market, using cases of 24 qt. each. How many cases does he need?

2. A gardener ships 1272 bunches of asparagus in boxes of 2 dozen bunches each. How many boxes does he need?

3. A market gardener ships 960 bu. of early potatoes in 3-peck boxes. How many boxes does he need?

4. A Michigan fruit grower ships 150 baskets of peaches to Chicago, each basket containing $\frac{1}{5}$ bu. If the dealers sell these by the quart, how many quarts will they sell?

5. A man ships 150 boxes of Damson plums to a city dealer. The boxes hold $\frac{1}{3}$ bu. each. The plums are sold in the city by the quart. How many quarts are there?

6. A dealer shipped 1250 lambs to Chicago. The average weight was 81 lb., and he paid \$4.75 per 100 lb. How much did he pay for the lot?

7. A Florida grower sends to the northern market 250 crates of strawberries, each containing 32 quart boxes. The berries retail at 40¢ a box. What is the total retail price?

8. A sheep raiser shipped a lot of lambs to a packing house, receiving \$342 for the lot. The total weight was 7200 lb. At this rate, how much should he receive for another shipment of 3200 lb.?

9. A grape grower ships a ton of grapes to market. Of these $\frac{9}{10}$ were put into 9-pound baskets, and the rest into 10-pound baskets. How many baskets did he ship?

10. A gardener shipped 260 lb. of horse-radish to a city market, where it was sold for \$9.85. If the transportation charges were 75¢, what was the net price per pound?

EXERCISE 107

Problems in Manufacturing

1. If the output of rope in a single year in this country was 200,824,974 lb., how many tons were manufactured?

2. Our silk mills consumed 11,572,783 lb. of raw silk in a single year. How many tons were consumed?

3. If the United States manufactured 35,553,731 gross of buttons in a certain year, how many buttons were manufactured?

4. If there were manufactured in the same year in this country 136,887,782 gross of pins, how many pins were manufactured?

5. A tannery paid $390 for tanning materials, which was 0.13 of the total cost of the materials it used. What did it pay for all the materials?

6. If our cotton mills use 3,743,089 bales of cotton in one year, weighing 1,873,074,716 lb., what is the average weight per bale? Answer to whole pounds only.

7. If the capital of a certain establishment is $75,500, and its annual business is 3.2 times its capital, what is its annual business?

8. If we have 1400 establishments engaged in manufacturing boots and shoes, and all but one twentieth are in the New England, North Central, and Middle Atlantic States, how many are in the other states?

9. A manufacturing establishment makes a certain article for which it pays 65¢ for material, 25¢ for labor, 24¢ for expenses and advertising, and 8¢ for putting the article on the market. It sells this article for $1.30. How much is its profit on 1728 such articles?

EXERCISE 108

Problems in Home Purchases

1. If 16 lb. of a certain kind of biscuit can be bought for $1.44, how much will 10 lb. cost at the same rate?

2. A lady can buy pastry flour in 10-pound bags at 44¢, or in 24½-pound bags at 98¢. How much does she save a pound by buying the flour in the larger bags?

3. A dealer sells Ceylon tea at 68¢ a pound and 5-pound caddies at $3.14. If two families want 2½ lb. each, how much will each save if they buy a caddy and divide it?

4. A dealer sells coffee in half-pound bags at 17¢, and 5-pound cans at $1.64. If a woman wishes 5 lb., how much does she save by purchasing it by the can?

5. A woman can buy Dutch cocoa in ¼-pound boxes at 22¢, or in 5-pound cans at $2.78. If she wishes 5 lb., how much does she save by purchasing it by the can?

6. A woman can buy maple sirup at 44¢ a quart, or in gallon cans at $1.38. If she wishes 4 qt., how much does she save by purchasing it by the can?

7. A dealer sells large glass jars of comb honey at $5.06 a dozen or 44¢ each. How much more will he receive by selling a gross by the jar than by the dozen?

8. A lady buys a pound each of mustard seed, allspice, and cayenne pepper, paying 60¢ for the three. The mustard seed costs 12¢ a pound, and the allspice costs the same as the pepper. How much does the allspice cost?

9. A woman bought a can each of cinnamon, nutmeg, and pepper, paying 39¢ for the three. The pepper cost 9¢, and the nutmeg cost twice as much as the cinnamon. How much did the cinnamon cost?

204. Longitude and Time. One class of problems that arises when we think of the change in time as we travel or telegraph east or west relates to longitude and time.

205. Meridian. The shortest line on the earth's surface passing from one pole to the other is called a *meridian.*

206. Longitude. The distance in circular measure east or west of a certain meridian agreed upon is called *longitude.*

207. Prime Meridian. A meridian agreed upon as the one from which longitude is reckoned is called a *prime meridian.*

The prime meridian now used by most of the maritime nations of the world is that which passes through the National Observatory at Greenwich, England.

A place 75° 30′ 20″ west of this meridian is said to be in 75° 30′ 20″ west longitude. A place 15° 25′ east of this meridian is said to be in 15° 25′ east longitude.

Since 180° is halfway around the earth from the prime meridian there can be no more than 180° E. (that is, east longitude) or 180° W. (that is, west longitude).

208. Difference in Longitude. When two places are both east or both west of the prime meridian, the difference in longitude is evidently found by subtracting one longitude from the other.

When one place is east and the other is west of the prime meridian, the difference in longitude is evidently found by adding the two longitudes.

This is clearly seen by looking at a map or globe of the world.

Since the earth is spherical, there are always two differences in longitude between any two places, and we may go from one longitude to the other by traveling either east or west. If, therefore, the difference is more than 180°, we may subtract this from 360° to get the smaller difference. Thus the difference between 100° E. and 100° W. is 200° or 160°, depending on which way we think of going from one longitude to the other. It is usually better to take the smaller number, although occasionally it is more convenient to use the larger.

209. Relation of Longitude to Time. Since the earth makes one complete revolution upon its axis in 24 hr., a point on the surface passes through 360° in 24 hr. That is,

360° corresponds to 24 hr.
1° corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.
1′ corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.
1″ corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

Hence we have the following correspondence:

Longitude to Time		*Time to Longitude*	
360° . . .	24 hr.	24 hr. . . .	360°
1° . . .	$\frac{1}{15}$ hr. = 4 min.	1 hr. . . .	15°
1′ . . .	$\frac{1}{15}$ min. = 4 sec.	1 min. . . .	$\frac{1}{4}° = 15'$
1″ . . .	$\frac{1}{15}$ sec.	1 sec. . . .	$\frac{1}{4}' = 15''$

210. Finding Difference in Time. Given the difference in longitude, the difference in time may be found.

For example, two ships at sea are 25° 24′ 15″ of longitude apart. What is the difference in time?

Looking at the above table, we find the difference in time corresponding to 1°, 1′, and 1″, and then we obtain the following:

25×4 min. = 100 min. = 1 hr. 40 min.
24×4 sec. = 96 sec. = 1 min. 36 sec.
$15 \times \frac{1}{15}$ sec. = 1 sec.

Therefore the difference in time is 1 hr. 41 min. 37 sec.

Therefore, *given the difference in longitude to find the difference in time, multiply 4 minutes of time by the number of degrees, 4 seconds of time by the number of minutes of longitude, and* $\frac{1}{15}$ *of a second of time by the number of seconds of longitude, and add the products.*

211. Later to the East, Earlier to the West. Because the earth turns from the west to the east, the sun appears to go from east to west. It is therefore 1 hr. later 15° east of here, and 1 hr. earlier 15° west.

EXERCISE 109

Find the difference in time between two places, given the following differences in longitude:

1. 15° 15′.
2. 15′ 15″.
3. 30° 15′.
4. 45° 30′.
5. 60° 30″.
6. 25° 15′.
7. 32° 30′.
8. 48° 45′.
9. 127° 15″.
10. 168° 30″.
11. 5° 10′ 15″.
12. 7° 25′ 30″.
13. 28° 42′ 30″.
14. 62° 27′ 45″.
15. 128° 6′ 7½″.

16. Two ships are 32° 40′ 30″ of longitude apart. What is the difference in time?

17. Two observatories are 42° 27′ 45″ of longitude apart. What is the difference in time?

18. A ship in 15° W. sends a wireless telegram at noon. If the telegram reaches Greenwich immediately, at what time does it arrive at Greenwich?

19. A ship in 58° 17′ 20″ W. receives at noon a wireless telegram from one in 43° 2′ 5″ W. When was it sent?

20. Italy uses the time of 15° E., and Indiana that of 90° W. When it is noon in Italy, what time is it in Indiana?

21. Constantinople uses for railway purposes the time of 30° E., and New England uses that of 75° W. When it is 1 P.M. in Constantinople, what time is it in New England?

22. A ship at sea carries a chronometer that shows Greenwich time. A ship is in longitude 37° 42′ 30″ W. at noon. What is the time by the Greenwich chronometer?

23. A traveler sailing from New York has his watch set by 75th meridian time. Two days later he finds that the ship is in longitude 55° W. Is his watch faster or slower than ship time, and how much?

212. Finding Difference in Longitude. If the difference in time between two ships is 2 hr. 17 min. 22 sec., what is the difference in longitude?

Consulting the table (§ 209), we find the difference in longitude corresponding to 1 hr., 1 min., and 1 sec., and then obtain:

$$2 \times 15^\circ = 30^\circ$$
$$17 \times 15' = 255' = 4^\circ 15'$$
$$22 \times 15'' = 330'' = 5' 30''$$

Therefore the difference in longitude is 34° 20′ 30″.

Given the difference in time to find the difference in longitude, multiply 15°, 15′, 15″ respectively by the number of hours, minutes, and seconds of time, and add the products.

EXERCISE 110

Find the difference in longitude, given the following differences in time:

1. 2 hr. 3 min.
2. 4 hr. 7 min.
3. 6 hr. 9 min.
4. 10 hr. 4 sec.
5. 11 hr. 8 sec.
6. 4 hr. 20 min.
7. 5 hr. 25 min.
8. 7 hr. 6 min. 8 sec.
9. 9 hr. 35 min. 40 sec.
10. 11 hr. 58 min. 48 sec.

11. When it is noon by the Greenwich chronometer it is 2 P.M. where a ship is. What is the ship's longitude?

12. When it is 3 P.M. by the Greenwich chronometer it is noon where a ship is. What is the ship's longitude?

13. The captain of another ship finds that when it is noon where the ship is it is 2 P.M. by the Greenwich chronometer. What is the longitude of the ship?

14. An ocean traveler whose watch is set by Greenwich time finds that when it is noon by ship time it is 1.45 P.M. by his watch. What is his longitude?

213. Standard Time. For practical purposes most of the civilized world is now divided into sections, and all places in the same section use the time of a certain meridian in that section. Generally this meridian is 15°, 30°, 45°, or other multiple of 15° east or west of Greenwich.

Thus England uses the time of 0° (Greenwich meridian); Germany, Switzerland, Italy, and central Europe in general use that of 15° E.; Japan uses that of 135° E.; and Australia, on account of its size, is divided into different sections.

A system in which all places in a section of territory near some chosen meridian use the time of that meridian is called a system of *standard time.*

The following is the standard-time map of the United States:

The irregular boundaries of the sections are caused by the desire to have the division lines pass through important railway centers.

When it is noon by Eastern Time (75° W.) it is 11 A.M. by Central Time (90° W.), 10 A.M. by Mountain Time (105° W.), 9 A.M. by Pacific Time (120° W.), and 5 P.M. by Greenwich (0°) time.

In Exercise 111 every example refers to standard time.

EXERCISE 111

1. When it is noon in Wisconsin, what time is it in Wyoming? in California? in New York? in Missouri?

2. When it is 11 A.M. in Minnesota, what time is it in Colorado? in eastern Nebraska? in Vermont? in England?

3. When it is 1 P.M. in Montana, what time is it in Iowa? in Indiana? in Seattle? in Boston? in London?

4. When it is 2.30 P.M. in eastern Utah, what time is it in southeastern Idaho? in Arkansas? in New Hampshire?

5. When it is 9.45 A.M. in San Francisco, what time is it in eastern Oklahoma? in Louisiana? in eastern Florida? in Alabama? in Maine?

6. When it is 11.45 P.M. in Los Angeles, what time is it in western Tennessee and Kentucky? in Mississippi? in eastern Virginia? in Connecticut? in England?

7. At 9 P.M. a telegram is sent from London to San Francisco. Allowing an hour for transmission and delivery, at what time will it reach its destination?

8. At 2 A.M. on January 1 a telegram is sent from Constantinople (time longitude 30° E.) to Portland, Oregon. Allowing an hour for transmission, on what day and at what hour will it reach its destination?

9. If an event happens in Japan (135° E.) at 10 P.M. and the news is telegraphed without delay, at what time will it reach Italy? England? New York? San Francisco?

10. The longitude of San Francisco is 122° 25′ $40\frac{4}{5}$″ W. What is the difference between local (the real meridian) time and standard time? Which is the faster?

11. The longitude of New York is 73° 58′ $25\frac{1}{2}$″ W. What is the difference between local time and standard time? Which is the faster?

EXERCISE 112

Problems without Numbers

1. If you know the cost of a certain number of things and wish to know the cost of one, how do you proceed?

2. If you know the cost of one thing and wish to find the cost of a certain number of things, how do you proceed?

3. If you know the cost of a fraction of anything and wish to know the cost of the whole, how do you proceed?

4. If you know the cost of a certain number of things and wish to know the cost of a certain other number of the same things, how do you proceed?

5. If you know the cost of a gallon of anything and wish to know the cost of a pint, how do you proceed?

6. If you know the price of cloth per yard, how do you find the cost of a certain number of inches?

7. If you know the price of cloth per yard, how do you find the cost of a certain number of yards plus a certain number of inches?

8. If you know the circumference of a wheel, how do you find how many times the wheel turns in going a mile?

9. If you know how many times a wheel turns in going a mile, how do you find the circumference?

10. If you know the freight rate per hundred pounds, how do you find the freight charge on a ton?

11. If you know the freight rate per hundred pounds, how do you find the freight charge on a certain number of tons plus a certain number of pounds?

12. If you have the product of several numbers to be divided by the product of several others, how may you perform the operations with least labor?

13. If you know what a merchant pays for a certain article, and his selling price, how do you find his profit on a certain number of those articles?

14. How may a merchant sell part of a lot of goods below cost and still make a profit on the lot? Illustrate.

15. If a certain number of men can do a piece of work in a week, how long will it take one man to do the same work?

16. If a certain number of men can do a piece of work in a week, working full time each day, how long will it take half as many men working half time each day?

17. If you know the cost of a certain number of feet of water pipe, how do you find the cost of a certain number of rods of the pipe?

18. A man buys a certain number of pecks of chestnuts. How do you find how many quarts he buys?

19. A man buys a certain number of pecks of chestnuts. How do you find how many bushels he buys?

20. If you know what fraction of the weight of green corn is water, and know the weight of the green corn, how do you find what the weight will be after three fourths of the water is dried out?

21. If you know how much water flows through a pipe in a minute, how do you find how much water will flow through in a whole day?

22. If you know how much water flows through a pipe in a day, how do you find how much water flows through in a second?

23. If you know the difference in longitude between two places, how do you find the difference in time?

24. If you know the difference in time between two places, how do you find the difference in longitude?

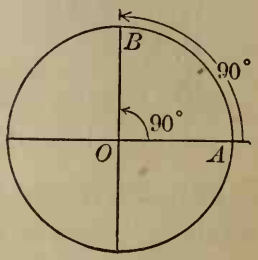
CHAPTER XII

PRACTICAL MEASUREMENTS

214. Terms Used. Most of the names used in measuring are already familiar to the class. Some of the more important are repeated here, and a few new names are added.

215. Angles. When two straight lines meet they are said to form an *angle.*

An angle of 90° is called a *right angle.*



In the figure the angle *AOB* is a right angle.

When two lines form a right angle they are said to be *perpendicular* to each other (§ 187).

An angle smaller than a right angle is called an *acute angle.*

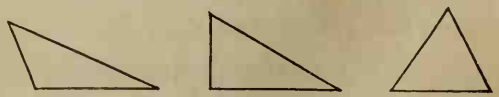
Acute angle

An angle greater than a right angle is called an *obtuse angle.*

Obtuse angle

216. Triangles. A plane figure bounded by three straight lines is called a *triangle.*

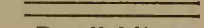
A triangle that contains an obtuse angle is called an *obtuse triangle;* a triangle that contains a right angle is called a *right triangle;* a triangle that contains only acute angles is called an *acute triangle.*


Obtuse Right Acute

The side on which a triangle rests is called the *base,* and the opposite corner is called the *vertex.*

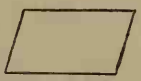
The perpendicular distance from the vertex to the line of the base is called the *altitude* or *height* of the triangle.

217. Parallel Lines. Lines that are everywhere equally distant from each other are called *parallel lines.*

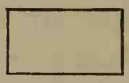

Parallel lines

218. Parallelogram. A plane figure bounded by two pairs of parallel lines is called a *parallelogram.*

The *base* of a parallelogram is the side on which it rests. The *altitude* is the perpendicular distance from the opposite side to the base. The opposite parallel sides of a parallelogram are equal.


Parallelogram

219. Rectangle. A parallelogram whose angles are right angles is called a *rectangle.*


Rectangle

220. Square. A rectangle whose sides are equal is called a *square* (§ 169).

Square

221. Polygon. A plane figure bounded by straight lines is called a *polygon.*

Triangles and parallelograms are special kinds of polygons.

222. Perimeter. The distance around a polygon is called the *perimeter.*

EXERCISE 113

1. What is the cost of fencing a square lot 8 rd. on a side, if the fence costs 30¢ a foot?

2. What is the perimeter of a triangle that is 26 yd. 27 in. on each side?

3. What is the perimeter of a parallelogram two of whose sides are respectively 17 ft. 11 in. and 9 ft. 6 in.?

4. What is the perimeter of a triangle whose sides are 27 ft. 8 in., 24 ft. 11 in., and 29 ft. 7 in.?

5. What is the cost of fencing a rectangular field 16 rd. long and 10 rd. wide, if the fence costs $62\frac{1}{2}$¢ a yard?

6. How many feet of molding will be needed for the four walls of a room 16 ft. 4 in. long and 14 ft. 8 in. wide?

223. Measurement of a Rectangle. If this figure represents a rectangle 4 in. long by 2 in. wide, there are evidently 4 sq. in. in the lower row of squares, and since there are 2 rows, there are 2×4 sq. in., or 8 sq. in., in the rectangle.

If we know the area and one dimension, we may find the other dimension; for, since 8 sq. in. $= 2 \times 4$ sq. in., then 8 sq. in. $\div$ 4 sq. in. $= 2$, the number of units in the other dimension.

Express the length and breadth of a rectangle in the same linear unit; the product of these two numbers will express the area in square units of the same name as the linear unit.

The number of square units in a rectangle divided by the number of linear units in one dimension gives the number of linear units in the other dimension.

This means that the numbers are to be considered as abstract. The complete solutions with concrete numbers are given above.

The expression $4'' \times 7''$ means and is read 4 inches *by* 7 inches.

EXERCISE 114

Find the areas of rectangles with these bases and altitudes:

1. 127 ft., 46 ft.
2. 152 in., 35 in.
3. 225 yd., 82 yd.
4. 240 in., 11 ft.
5. 6 ft. 4 in., 3 ft.
6. 7 ft. 2 in., 5 in.
7. 3 ft. 6 in., 1 ft. 4 in.
8. 9 yd. 2 ft., 3 ft. 6 in.

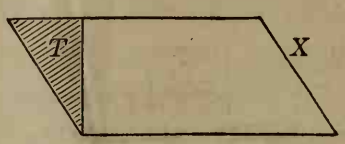
9. How many square rods in a rectangular field 32 rd. long and 16 rd. wide?

10. How many acres in a rectangular field 126 rd. long and 38 rd. wide?

11. A rectangular city lot is 129½ ft. long and 25 ft. 9 in. wide. Find the number of square feet in the lot.

12. A rectangular field containing 4.05 A. is 36 rd. long. How wide is it?

224. Measurement of a Parallelogram. If we cut off the triangle T and place it at X, the parallelogram will be changed to a rectangle of the same base and the same altitude.



Therefore, *the area of a parallelogram is equal to the product of the numbers expressing the base and altitude.*

The base and the altitude must be expressed in the same units.

Given the area and the altitude we may find the base, as in the case of a rectangle, and similarly for the area and base to find the altitude.

For example, if the base of a parallelogram is 3 ft. 8 in. and the altitude 1 yd., what is the area?

3 ft. 8 in. $= 3\frac{2}{3}$ ft.; 1 yd. $= 3$ ft.

$3 \times 3\frac{2}{3}$ sq. ft. $= 11$ sq. ft.

If the area of a parallelogram is 15 sq. ft. and the base is 5 ft., required the altitude.

15 sq. ft. $\div$ 5 sq. ft. $= 3$, the number of feet of altitude (§ 223).

EXERCISE 115

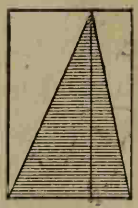
Find the areas of parallelograms with the following bases and altitudes:

1. 157 ft., 96 ft.
2. $32\frac{1}{2}$ ft., $15\frac{1}{2}$ ft.
3. 33 ft., 1 rd.
4. 27 ft., 7 yd.
5. $35\frac{1}{2}$ ft., 125 in.
6. 148 rd., 100 rd.
7. 27 ft. 6 in., 3 ft. 4 in.
8. 29 ft. 4 in., 4 ft. 8 in.
9. 36 yd. 27 in., 3 yd. 18 in.
10. 62 yd. 32 in., 17 yd. 20 in.

Find the altitudes of parallelograms with the following areas and bases:

11. 972 sq. ft., 36 ft.
12. 1170 sq. ft., 45 ft.
13. 1728 sq. ft., 18 yd.
14. 40 A., 100 rd.

225. Measurement of a Triangle. We see that this triangle may be cut into two parts, each of which is half of some rectangle, and that therefore the whole triangle is half of a rectangle of the same base and altitude.



Whatever the shape of the triangle, it may, in a similar way, be shown to be half of a parallelogram of the same base and altitude, and therefore (§ 224) equal to half of a rectangle of the same base and altitude.

Therefore, *the area of a triangle equals half the product of the numbers representing the base and altitude.*

EXERCISE 116

Find the areas of triangles with the following bases and altitudes:

1. 47 ft., 28 ft.
2. 75 ft., 25 ft.
3. $82\frac{1}{2}$ ft., 35 ft.
4. 160 rd., 25 rd.
5. 6 ft. 9 in., 27 in.
6. 3 yd. 18 in., 2 ft.
7. 7 ft. 2 in., 3 ft. 1 in.
8. 8 ft. 6 in., 4 ft. 8 in.
9. 12 ft. 9 in., 7 ft. 6 in.
10. 4 yd. 27 in., 2 ft. 4 in.

Find the altitudes of triangles with the following areas and bases:

11. 92 sq. ft., 23 ft.
12. 480 sq. ft., 48 ft.
13. 594 sq. ft., 54 ft.
14. 1053 sq. in., 39 in.

15. At $5.50 a square foot, what is the value of a triangular city lot with base 82 ft. and altitude 45 ft. 6 in.?

16. What is the area of a triangle with sides all equal, whose perimeter is 18 in. and altitude 5.19 in.?

17. A triangular gable to a house has a base 30 ft. and an altitude 10 ft. 6 in. Find the area of the gable.

226. A Trapezoid. A four-sided plane figure with two parallel sides is called a *trapezoid.*

Trapezoid

227. Area of a Trapezoid. In this figure two equal trapezoids have been placed together, one turned over so that the two make a parallelogram which equals a rectangle whose base is the sum of the two parallel sides.

Therefore, *the area of a trapezoid equals half that of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides.*

For example, the area of a trapezoid with altitude 6 in. and parallel sides 10 in. and 15 in., is $\frac{1}{2}$ of $6 \times (15 + 10)$ sq. in., or 75 sq. in.

EXERCISE 117

Find the areas of trapezoids whose altitudes are first given, followed by the two parallel sides:

1. 5″, 8″ and 10″.
2. 7″, 6″ and 12″.
3. 6″, 7″ and 11″.
4. 8″, 9″ and 12″.
5. 7″, 8″ and 14″.
6. 9″, 7″ and 15″.
7. 12″, 18″ and 24″.
8. 13″, 17″ and 21″.
9. 18″, 24″ and 30″.
10. 24″, 36″ and 42″.
11. 27″, 39″ and 45″.
12. 43″, 42″ and 61″.

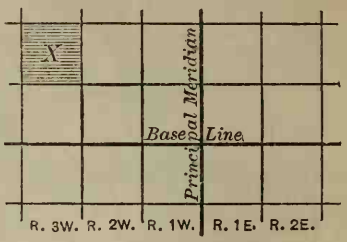
13. If the area of a trapezoid is 17,250 sq. ft., and the parallel sides are 250 ft. and 325 ft., what is the altitude?

14. If the area of a trapezoid is 22,000 sq. ft., the altitude 80 ft., and one of the parallel sides 225 ft., what is the other parallel side?

15. What is the area of a trapezoid one of whose parallel sides is 86 ft., the altitude being the same length and the other parallel side being twice as long?

228. Public Lands. In a large part of the United States the land was laid out by the government before it was sold to settlers.

Through a given tract of land a north-and-south line was chosen as the *principal meridian* for that tract, and an east-and-west line was chosen as the *base line.*



Lines were then run parallel to the principal meridian and other lines parallel to the base line, 6 mi. apart, thus cutting the land into *townships.*

Rows of townships running north and south are called *ranges.*

In the above map the portion marked *X* is numbered T. 2 N., R. 3 W., this meaning that it is the second township north of the base line, in the third range west of the principal meridian.

N

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	*Y*	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

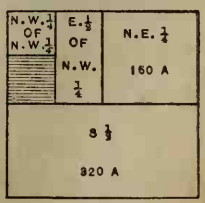
W *E*

S

Townships are divided into squares, numbered as shown in the second map, each being 1 mi. square. These divisions are called *sections.*

The second map represents township *X* of the first map.

Each section is then divided as shown in the third map, which represents the section *Y* (No. 21) of the second map.



The shaded portion of this map is described as follows: S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 21, T. 2 N., R. 3 W. This means the southwest quarter of the northwest quarter of section 21 of the second township north, in the third range west.

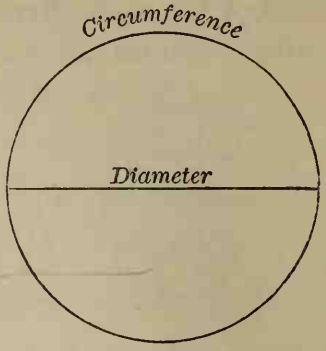
In those parts of the country where land is not laid out in this way, this subject may be omitted. Local conditions should influence the teaching of all such topics, and subjects of special importance in any community should be introduced.

EXERCISE 118

Write the following descriptions in full; draw a map locating the township with respect to the principal meridian and the base line, a second map locating the section in the township, and a third map locating the land in the section. Then find the area.

1. N.W. $\frac{1}{4}$, Sec. 3, T. 2 N., R. 2 E.
2. N.E. $\frac{1}{4}$, Sec. 17, T. 1 N., R. 3 W.
3. S.W. $\frac{1}{4}$, Sec. 36, T. 3 N., R. 2 E.
4. N. $\frac{1}{2}$, Sec. 25, T. 1 S., R. 1 W.
5. W. $\frac{1}{2}$, Sec. 26, T. 3 S., R. 1 E.
6. A farm is described as follows: S. $\frac{1}{2}$, Sec. 21, T. 4 S., R. 4 W. How much is it worth at $50 an acre?
7. A farm is described as follows: N.W. $\frac{1}{4}$, Sec. 23, T. 2 S., R. 3 W. How much is it worth at $60 an acre?
8. A stream runs through a farm cutting a field into two trapezoids. One of these trapezoids has for its parallel sides 40 rd. and 60 rd., and for its altitude 32 rd. How many acres are there in this portion of the farm?

229. Circumference of a Circle. If we measure around and across any circular object, we shall find that the circumference is about $3\frac{1}{7}$ times the diameter. It is shown in geometry that 3.1416 is more nearly correct than $3\frac{1}{7}$.



It is sufficient for most ordinary purposes to use $3\frac{1}{7}$ instead of 3.1416.

Thus, if the diameter of a wheel is 7 ft. the circumference is $3\frac{1}{7} \times 7$ ft., or 22 ft. If we use the multiplier 3.1416, the result is 3.1416×7 ft., or 21.9912 ft.

EXERCISE 119

Using $3\frac{1}{7}$, find the circumference, given the diameter:

1. 105 ft.	**3.** 25.9 ft.	**5.** 0.364 in.	**7.** 4970 in.
2. 18.9 ft.	**4.** 2.94 ft.	**6.** 4270 in.	**8.** 51.10 in.

Using $3\frac{1}{7}$, find the circumference, given the radius:

9. 63 in.	**11.** 98 in.	**13.** 61.6 in.	**15.** 85.4 ft.
10. 77 in.	**12.** 210 in.	**14.** 68.6 in.	**16.** 8.82 ft.

Using 3.1416, find the circumference, given the diameter:

17. 27 ft.	**19.** 32 yd.	**21.** 0.62 ft.	**23.** 2.25 ft.
18. 43 ft.	**20.** 4.7 yd.	**22.** 0.75 ft.	**24.** 1.27 in.

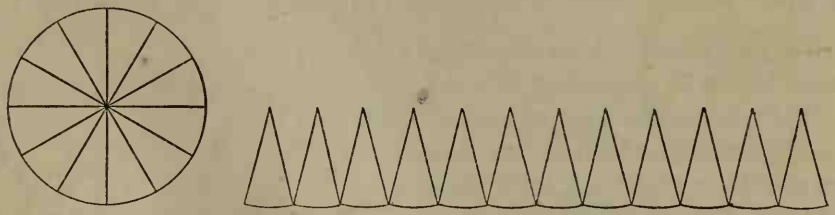
Using 3.1416, find the circumference, given the radius:

25. 27 in.	**27.** 61 in.	**29.** 1.9 yd.	**31.** 0.48 ft.
26. 38 in.	**28.** 4.8 ft.	**30.** 2.7 yd.	**32.** 0.62 ft.

Using 3.1416, find the diameter, given the circumference:

33. 62.832 ft.	**34.** 345.576 in.	**35.** 502.656 in.

230. Area of a Circle. We may cut a circle as here shown and separate it into a series of figures that are triangles except for their curved bases. If we should call these figures triangles their altitudes would be the radius of the



circle, and the sum of their bases would be the circumference. The area of each triangle (§ 225) equals half the product of the numbers representing the base and altitude. Therefore their total area equals half the product of the numbers representing the circumference and radius.

In geometry this is proved exactly, but this explanation is enough for our purpose.

Thus, if the radius is 2 in. and the circumference (§ 229) is 12.5664 in., the area is $\frac{1}{2}$ of 2×12.5664 sq. in., or 12.5664 sq. in.

The area of a circle equals half the product of the numbers representing the circumference and radius.

231. Area in Terms of Radius. We have found that

$$\text{area} = \tfrac{1}{2} \times \text{radius} \times \text{circumference} \quad (\S\,230)$$

and $$\text{circumference} = 3.1416 \times 2 \times \text{radius} \quad (\S\,229);$$

therefore $$\text{area} = \tfrac{1}{2} \times \text{radius} \times 3.1416 \times 2 \times \text{radius},$$

or, canceling, $$\text{area} = 3.1416 \times \text{square of radius}.$$

Therefore, *the area of a circle equals 3.1416 times the square of the radius.*

For example, if the radius is 2 in., the area is 3.1416×4 sq. in., or 12.5664 sq. in.

In Exercise 120 use 3.1416 instead of $3\frac{1}{7}$ (§ 229).

EXERCISE 120

Find the areas of circles with the following radii and circumferences:

1. 5 in., 31.416 in.
2. $4\frac{1}{2}$ in., 28.2744 in.
3. 4 in., 25.1328 in.
4. $3\frac{1}{2}$ in., 21.9912 in.
5. $5\frac{1}{2}$ in., 34.5576 in.
6. 10 in., 62.832 in.
7. 20 in., 125.664 in.
8. 30 in., 188.496 in.

Find the areas of circles with the following radii:

9. 1 in.
10. 4 in.
11. 3 in.
12. 5 in.
13. 10 in.
14. 20 in.
15. $\frac{1}{2}$ in.
16. $\frac{1}{3}$ in.
17. 0.1 in.
18. $\frac{1}{4}$ in.
19. $\frac{3}{4}$ in.
20. $\frac{2}{3}$ in.
21. $2\frac{1}{2}$ in.
22. $3\frac{1}{4}$ in.
23. 2.3 in.

24. What is the circumference of a water pipe that is 4 in. in diameter?

25. What is the area of a cross section of a water pipe that is 3 in. in diameter?

26. A water pipe is 4 in. in external diameter. If the iron is $\frac{1}{4}$ in. thick, what is the internal circumference?

27. A school globe is 15 in. in diameter. How long is the equator on the globe?

28. The equator on a school globe is 62.832 in. What is the radius of the globe?

29. What is the circumference of the largest circle that can be drawn in a square 10 ft. on a side?

30. What is the area inclosed between the circle and the sides of the square in Ex. 29?

31. What is the diameter of a circular plot of ground of which the circumference is 691.152 ft.? What is the area?

32. A pipe is 3.75 in. in external diameter. If the iron is $\frac{1}{4}$ in. thick, what is the area of an internal cross section?

232. Carpeting Rooms. Carpet is usually either 27 in. or 1 yd. wide, and in estimating the amount needed for a room, allowance must be made for matching the pattern.

In allowing for the waste in matching the pattern, it is not necessary to allow for loss on the first strip, since the other strips are cut to match that one. In the following examples, unless otherwise directed, the strips are to run lengthwise of the room.

For example, how much will it cost to carpet a room $13\frac{1}{2}' \times 16'$, the strips to run lengthwise, and the carpet being plain color, 27 in. wide, and worth 90¢ a yard?

$13\frac{1}{2}$ ft. $= 4\frac{1}{2}$ yd., the width,

and 16 ft. $= 5\frac{1}{3}$ yd., the length.

$4\frac{1}{2}$ yd. $\div \frac{3}{4}$ yd. $= 6$, the number of strips.

$6 \times 5\frac{1}{3}$ yd. $= 32$ yd.

$32 \times 90¢ = \$28.80$.

EXERCISE 121

Find the number of yards of plain carpet 27 in. wide that will be required for rooms of the following dimensions:

1. 24 ft., 15 ft.	4. 15 ft., 12 ft.	7. 25 ft., 18 ft.
2. 20 ft., 15 ft.	5. 17 ft., 12 ft.	8. 26 ft., 18 ft.
3. 22 ft., 18 ft.	6. 19 ft., 15 ft.	9. 32 ft., 21 ft.

10. What will it cost to carpet a room 19 ft. 8 in. long and 15 ft. wide with plain carpet 27 in. wide, at 75¢ a yard?

11. A floor 16 ft. wide is to be covered with 27-inch carpet, the strips running lengthwise. How many strips will be needed, and how much of the width of one strip will be lost by turning under?

12. What will it cost to cover a floor 18 ft. 6 in. long and 14 ft. 9 in. wide with carpet 1 yd. wide, worth $0.85 a yard, if the strips run lengthwise and $\frac{1}{8}$ yd. is allowed on every strip except the first for matching the pattern?

233. Papering Rooms. The standard width of wall paper is 18 in., and single rolls are 8 yd. long, double rolls being 16 yd. long.

Imported papers vary from this standard, both in width and in length of roll. The frieze or border is usually sold by the yard.

It is not easy to estimate exactly the amount of paper needed for a room, on account of the difficulty of matching patterns, particularly when there are doors and windows with spaces above. In general, however,

Measure the height of wall space to be covered, find how many strips can be cut from a roll, and allow two strips to every yard in the perimeter of the room.

A fraction of a roll is to be counted as a whole roll. By a roll is meant a single roll, unless the contrary is stated.

Paper hangers often use this rule for rooms of ordinary height, those in which the distance from the baseboard to the border does not exceed 8 ft.:

Find the number of feet in the perimeter of the room, leaving out the widths of the doors and the windows, and allow a single roll for every $3\frac{1}{2}$ feet.

If there were no loss in matching, and we allowed 8 ft. to each strip, 1 roll (24 ft.) would make 3 strips, and would cover $4\frac{1}{2}$ ft. in width. But on account of loss in matching and spaces over doors and windows, this is a fairly approximate basis for an estimate.

For example, how many double rolls of paper will be required for a room of ordinary height, $18' \times 16'$, with one door and three windows, each 4 ft. wide?

Perimeter of room	$= 2 \times (18' + 16')$	$= 68'$
Width of door and windows	$= 4 \times 4'$	$= 16'$
Deducting door and windows		$52'$

Since 1 single roll is allowed to every $3\frac{1}{2}$ ft., 1 double roll will be allowed to every 7 ft. Since $\frac{52}{7} = 7\frac{3}{7}$, therefore 8 double rolls are needed.

In Exs. 1–6 of the following exercise use this paper-hangers' rule.

EXERCISE 122

1. How many rolls of paper will be required for a room $15' \times 14'$, with 1 door and 2 windows, each $3'\ 6''$ wide?

2. How many rolls of paper will be required for a room $17'\ 6'' \times 14'\ 6''$, with 2 doors and 2 windows, each $3'\ 6''$ wide?

3. How many double rolls of paper will be required for a room $18'\ 8'' \times 16'\ 3''$, with 1 door and 3 windows, each $3'\ 6''$ wide?

4. How many double rolls of paper will be required for a room $20'\ 4'' \times 15'\ 8''$, with 2 doors and 3 windows, each $3'\ 8''$ wide?

5. How much will it cost for papering a room $15' \times 13'$, having 1 door and 1 window, each $3'\ 6''$ wide, the paper costing 35¢ a roll put on?

6. How much will it cost for papering a room $15'\ 6'' \times 14'\ 4''$, having 1 door and 2 windows, each $4'$ wide, the paper costing \$1.50 a double roll put on?

7. Find the cost of the border for each of the six rooms mentioned in the above examples, at 18¢ a yard, making no allowance for doors and windows.

8. Find the cost of plain paper for a room 16 ft. long by 15 ft. wide, using strips of 8 ft., at \$1.20 a double roll.

In this and the next example, compute the exact cost by the first method given in § 233.

9. Find the cost of plain paper for a room 24 ft. long by 16 ft. 6 in. wide, using strips of 8 ft., at \$1.30 a double roll.

10. Find the cost of plain paper for the ceiling of a hall 48 ft. long and 24 ft. wide, at \$1.40 a double roll.

11. Find the cost of plain paper for a ceiling 12 ft. long by 9 ft. wide, at \$0.65 a roll.

234. Plastering, Painting, and Kalsomining. The unit of plastering, painting, and kalsomining is usually the square yard. A common business rule for estimating such work is as follows:

From the total area of the wall or ceiling deduct half the area of the doors, windows, and other openings, and express the result to the nearest square yard.

It would not be fair to deduct the full area of the openings because some allowance should be made for the difficulty of plastering or painting around them.

The rule varies in different places, and an exact agreement in advance is necessary when work is to be done. In some places it is the custom to deduct even the full area of the openings, while in others a certain number of square feet is allowed.

Technical rules of this nature need not be memorized. They should be referred to when necessary. Teachers will find it helpful to use local rules in matters of this kind.

In giving dimensions, architects and contractors write first the length, then the width, and then the height.

At 21¢ a square yard, find the cost of plastering a room $18' \times 16' \times 10'$, if the room has a baseboard 10 in. high, two doors $7'\ 4'' \times 4'$, and three windows $6'\ 6'' \times 4'$.

Perimeter of room $= 2 \times (18' + 16') = 68'$

Height above baseboard $= 10' - 10'' = 9\frac{1}{6}'$

Total wall area $= 9\frac{1}{6} \times 68$ sq. ft. $= 623\frac{1}{3}$ sq. ft.

Area of ceiling $= 18 \times 16$ sq. ft. $= 288$ sq. ft.

Total area $= 911\frac{1}{3}$ sq. ft.

Height of doors above baseboard $= 7'\ 4'' - 10'' = 6'\ 6''$

Area of 2 doors $= 2 \times 4 \times 6\frac{1}{2}$ sq. ft. $= 52$ sq. ft.

Area of 3 windows $= 3 \times 4 \times 6\frac{1}{2}$ sq. ft. $= 78$ sq. ft.

Total area of openings $= 130$ sq. ft.

Half area of openings $= 65$ sq. ft.

Area to be paid for $= 846\frac{1}{3}$ sq. ft.

$= 94\frac{1}{27}$ sq. yd.

At 21¢ a square yard, 94 sq. yd. will cost $94 \times 21¢ = \$19.74$.

EXERCISE 123

Find the number of square yards to be covered in ceilings having the following dimensions:

1. 17′×15′.	**4.** 16′×14′.	**7.** 25′×20′.	**10.** 32′×29′.
2. 18′×16′.	**5.** 17′×16′.	**8.** 32′×25′.	**11.** 34′×26′.
3. 15′×13′.	**6.** 20′×19′.	**9.** 35′×28′.	**12.** 37′×25′.

If the cost of plastering is 22¢ per square yard, find the cost of plastering ceilings having the following dimensions:

13. 16′×15′.	**16.** 20′×18′.	**19.** 16′×16′.	**22.** 17′×14′.
14. 20′×15′.	**17.** 18′×15′.	**20.** 16′×13′.	**23.** 22′×20′.
15. 19′×15′.	**18.** 18′×14′.	**21.** 17′×13′.	**24.** 25′×22′.

Allowing $1\frac{1}{2}$¢ per square yard for labor and $\frac{1}{2}$¢ per square yard for material, find the cost of kalsomining ceilings having the following dimensions:

25. 13′×12′.	**28.** 16′×12′.	**31.** 20′×17′.	**34.** 35′×30′.
26. 14′×12′.	**29.** 19′×16′.	**32.** 25′×21′.	**35.** 40′×35′.
27. 15′×14′.	**30.** 21′×17′.	**33.** 30′×25′.	**36.** 50′×40′.

37. How much will it cost at $33\frac{1}{3}$¢ per square yard to plaster a room whose walls and ceiling have an area of 117 sq. yd., and have openings amounting to 9 sq. yd.?

38. A room is 18′ × 14′ × 9′, and has a baseboard 12 in. high, two doors 7′ × 3′ 6″, and two windows 6′ × 3′. How much will it cost to plaster the room at 30¢ per square yard?

39. A room is 22′ × 16′ × 10′, and has a baseboard 10 in. high, three doors 7′ 6″ × 4′, and four windows 6′ 6″ × 4′. How much will it cost to plaster the room at 25¢ per square yard?

235. Flooring. Flooring is measured by the 100 sq. ft., or by the 1000 sq. ft.

An area of 100 sq. ft. is called a *square*. The usual estimate of a contractor is by the square, although for small jobs a carpenter is generally employed by the day.

Carpenters usually allow $1\frac{1}{4}$ feet, board measure, for every square foot of flooring to be covered, there being some loss on account of the tongue-and-groove matching of the boards. If the boards are less than 3 in. in width, carpenters usually allow $1\frac{1}{3}$ feet for every square foot.

For example, how many feet of $2\frac{1}{4}$-inch flooring will be required for a room $15' \times 14'$?

The number of square feet is 15×14. Allowing $1\frac{1}{3}$ ft. of flooring to every square foot of floor, we have

$$15 \times 14 \times \tfrac{4}{3} \text{ ft.} = 280 \text{ ft.}$$

EXERCISE 124

Find the number of feet of $2\frac{1}{4}$-inch flooring material needed for rooms of the following dimensions:

1. $18' \times 14'$.
2. $14' \times 12'$.
3. $16' \times 12'$.
4. $20' \times 14'$.
5. $20' \times 16'$.
6. $22' \times 16'$.
7. $24' \times 16'$.
8. $20' \times 18'$.
9. $24' \times 18'$.

Find the number of feet of $2\frac{1}{2}$-inch flooring material needed for rooms of the following dimensions:

10. $16' \times 15'$.
11. $16' \times 13'$.
12. $20' \times 15'$.
13. $20' \times 17'$.
14. $22' \times 18'$.
15. $24' \times 20'$.

16. What is the cost of $2\frac{1}{2}$-inch birch flooring for a room $18' \times 15'$, at \$6.50 per 100 sq. ft. of floor surface?

17. What is the cost of laying a floor with $2\frac{1}{4}$-inch maple boards, the room being $24' \times 16'$, the labor costing \$10.75, and the lumber costing \$48 per M?

236. Lathing. Laths are put up in bundles of 100, each 4 ft. long, and a bundle is estimated to cover 6 sq. yd.

In estimating the number of bundles required, deduct the area of all openings.

For example, how many bundles of laths will be required for the ceiling of a room $17' \times 14'$?

$$17 \times 14 \text{ sq. ft.} = 238 \text{ sq. ft.} = 26\tfrac{4}{9} \text{ sq. yd.}$$

$$26\tfrac{4}{9} \text{ sq. yd.} \div 6 \text{ sq. yd.} = 4\tfrac{11}{27}.$$

Therefore 5 bundles will be required.

EXERCISE 125

1. How many bundles of laths will be required for the walls and ceiling of a room $16' \times 14'$, the ceiling being 8 ft. above the baseboard, allowing 74 sq. ft. for openings?

2. How many bundles of laths will be required for the walls and ceiling of a room $18' \times 16'$, the ceiling being 8 ft. above the baseboard, allowing 80 sq. ft. for openings?

3. If laths are worth 50¢ per bundle and it costs 20¢ per bundle to put them on, how much will it cost to cover 80 sq. yd. of surface?

4. At 72¢ a bundle put on, how much will it cost to lath the walls and ceiling of a room $20' \times 16'$, the ceiling being 8 ft. above the baseboard, allowing 77 sq. ft. for openings?

5. At 69¢ a bundle put on, how much will it cost to lath the walls and ceiling of a room $24' \times 16'$, the ceiling being 8 ft. 6 in. above the baseboard, allowing 65 sq. ft. for openings?

6. At 66¢ a bundle put on, how much will it cost to lath the walls and ceiling of a schoolroom $40' \times 30'$, the ceiling being 14 ft. above the baseboard, allowing 172 sq. ft. for openings?

237. Shingling. Shingles are put up in bunches of 250, each shingle being 16 in. long and averaging 4 in. wide, so that a shingle laid $4\frac{1}{2}$ in. to the weather will cover 18 sq. in., and 8 shingles will cover 1 sq. ft., and 1000 shingles will cover 125 sq. ft.

Many carpenters allow 1000 shingles to 120 sq. ft., thus providing for waste. Others allow 1000 shingles to 100 sq. ft.

EXERCISE 126

Allowing 1000 shingles to 120 sq. ft., find the number of bunches required to cover roofs of the following areas:

1. 1600 sq. ft.
2. 1750 sq. ft.
3. 1440 sq. ft.
4. 1860 sq. ft.
5. 1880 sq. ft.
6. 1820 sq. ft.

Allowing 1000 shingles to 110 sq. ft., find the number of bunches required to cover roofs of the following areas:

7. 1680 sq. ft.
8. 1620 sq. ft.
9. 2160 sq. ft.

10. At \$1 a bunch, how much will the shingles cost for a space of 2000 sq. ft., allowing 1000 shingles to 125 sq. ft.?

11. At \$1.10 a bunch, how much will the shingles cost to cover 1960 sq. ft., allowing 1000 shingles to 120 sq. ft.?

12. If 4 bunches of shingles cover 125 sq. ft., and cost \$1.20 a bunch, and if we allow $1\frac{1}{4}$ lb. of cut nails to the bunch, at 4¢ a pound, and pay a man 30¢ an hour, and he uses a bunch of shingles an hour, how much will it cost to cover the two sides of a roof each 40′ by 32′?

13. Allowing 4 bunches of shingles at \$1.15 a bunch to every 115 sq. ft., 7 lb. of nails at $3\frac{1}{2}$¢ a pound to each 1000 shingles, and 4 hr. of labor at 30¢ an hour to 1000 shingles, how much will it cost to shingle a barn 60 ft. long, each side of the roof being 25 ft. wide?

238. Paving. Paving and the laying of concrete walks and floors is estimated by the square foot or the square yard.

EXERCISE 127

Find the cost of paving:

1. A walk $5' \times 75'$, at $15¢$ per square foot.
2. A walk $5'\ 6'' \times 90'$, at $22¢$ per square foot.
3. A walk $5'\ 8'' \times 96'$, at $24¢$ per square foot.
4. A walk $6' \times 66'\ 6''$, at $25¢$ per square foot.
5. A walk $6'\ 6'' \times 42'\ 8''$, at $30¢$ per square foot.
6. A walk $6'\ 8'' \times 68'\ 9''$, at $35¢$ per square foot.
7. A walk $6' \times 32'$, using bricks laid on edge, the upper surface then being $2'' \times 8''$, the bricks being worth \$8 per M, the labor costing \$18.50, and the cost of preparing the bed for the bricks being \$5.

Find the number of square yards to be covered in street pavements of the following dimensions:

8. $38' \times 350'$.
9. $41' \times 625'$.
10. $37' \times 420'$.
11. $42' \times 375'$.
12. $36'\ 6'' \times 250'$.
13. $38'\ 8'' \times 325'$.
14. $41'\ 4'' \times 425'$.
15. $42'\ 2'' \times 365'$.
16. How much will it cost to pave a street 42 ft. wide and 625 ft. long, at \$2.75 per square yard?
17. A city wishes to put a 6-foot sidewalk around the outside of a public square 175 ft. long and 125 ft. wide. How much will it cost, at \$2.25 per square yard?
18. A man wishes to put a 5-foot concrete sidewalk on two sides of his corner lot. The lot is 75 ft. long and 35 ft. wide. How much will it cost, at $75¢$ per square yard?

239. Dressmaking. The question of cutting out garments varies so much with the pattern that only a few general problems can be given.

EXERCISE 128

1. If a dressmaker has $94\frac{1}{2}$ yd. of linen for 9 coat suits, how many yards does this allow to a suit?

2. If it takes 150 inches of material for a skirt, how many yards will it take for 4 such skirts?

3. If it takes 4 yd. of taffeta at \$1.10 a yard for a waist, and $3\frac{1}{2}$ yd. of trimming at 60¢ a yard, and the dressmaker charges \$1.25 for making, how much will two such waists cost?

4. If it takes $2\frac{3}{4}$ yd. of lawn at 28¢ a yard for a shirt waist, $5\frac{1}{2}$ yd. of trimming at 22¢ a yard, 8 buttons at 60¢ a dozen, and 10¢ worth of thread, and if the labor costs \$1.15, how much does the waist cost?

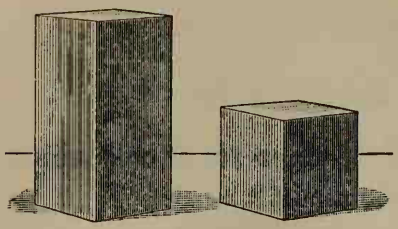
5. How much material is taken up (duplicated) by 32 quarter-inch tucks? If the material is worth \$1.60 a yard, how much must be allowed for the extra cost of material in a piece having these tucks?

6. How wide must a ruffle be cut to be 6 in. deep when completed, if there is a hem $\frac{3}{4}$ in. wide at the bottom, and 8 one-eighth-inch tucks are placed above the hem?

7. If a lady buys a piece of muslin containing 48 yd., and uses 19 yd. for sheets, and 18 yd. for pillow cases, how many skirts can she make from the remainder, allowing 3 yd. 24 in. to a skirt?

8. A lady buys a piece of cloth containing 48 yd. at 22¢ a yard. She uses 33 yd. for various purposes, and the rest for making 4 skirts. What did the cloth cost per skirt?

240. Rectangular Solid. A solid bounded by six rectangles is called a *rectangular solid.*

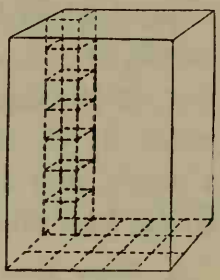


The six rectangles are called the *faces* of the solid, and their sides form the *edges* of the solid.

A rectangular solid has length, breadth, and thickness, and these are called the three *dimensions* of the solid.

241. Cube. A rectangular solid whose faces are squares is called a *cube* (§ 172).

242. Volume of a Rectangular Solid. If this figure represents a rectangular solid 5 in. long, 3 in. wide, and 7 in. high, it is evident that in the column of cubes shown there are 7 cu. in. It is also evident that on the base we can place 3×5 such columns. Hence the volume is $3 \times 5 \times 7$ cu. in., or 105 cu. in. Hence, if the dimensions are all expressed in the same unit,



The number of units of volume in a rectangular solid equals the product of the numbers representing the three dimensions.

EXERCISE 129

Find the volumes of rectangular solids having the following dimensions:

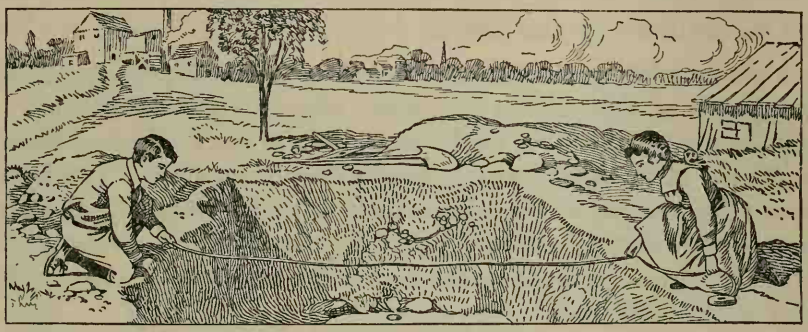
1. 17″, 19″, 32″.
2. 16″, 22″, 27″.
3. 15″, 16″, 34″.
4. 17″, 19″, 29″.
5. 15′, 29′, 12′.
6. 127′, 6′, $1\frac{1}{2}$′.
7. 124 ft., 6 ft., 1 ft. 9 in.
8. 135 ft., 5 ft., 2 ft. 2 in.
9. 275 ft., 4 ft., 1 ft. 10 in.
10. 2 ft. 6 in., 3 ft., 4 ft. 8 in.
11. 6 ft. 2 in., 2 ft., 3 ft. 9 in
12. 4 ft. 3 in., 7 ft., 6 ft. 2 in.

13. What is the volume of a cube 16 in. on an edge?

14. How many cubic feet of air are there in a schoolroom 28 ft. long, 20 ft. wide, and 14 ft. high?

15. What is the total area of the six faces of a rectangular solid 8 in. by 7 in. by 5 in.?

16. What is the total area of the six faces of a rectangular solid 3 ft. 6 in. by 2 ft. 4 in. by 1 ft. 9 in.?



17. These children are measuring an excavation, and find it to be 6 ft. 2 in. wide, 8 ft. 4 in. long, and 4 ft. 3 in. deep. Assuming it to be practically rectangular, how many cubic feet of earth have been removed?

18. What is the weight of the water in an aquarium 1 ft. 4 in. long and 1 ft. wide, the water being 9 in. deep, if a cubic foot of water weighs $62\frac{1}{2}$ lb.?

19. What is the weight of a block of marble 4 ft. 6 in. long, 2 ft. wide, and 1 ft. 4 in. thick, marble being 2.7 times as heavy as water, and a cubic foot of water weighing $62\frac{1}{2}$ lb.?

20. A rectangular solid is 4 ft. 6 in. long and 2 ft. 8 in. wide, and the volume is 72 cu. ft. What is the height?

21. A rectangular solid is 6 ft. 4 in. long and 3 ft. 9 in. wide, and the volume is 285 cu. ft. What is the height?

22. A rectangular solid is 2 ft. 6 in. high and the volume is 240 cu. in. What is the area of the base?

243. Brick Work and Stone Work. The number of bricks required for walls is estimated by the thousand, and 22 bricks of ordinary size laid in mortar are allowed to each cubic foot of wall.

A brick of ordinary size is $2'' \times 4'' \times 8''$, and its volume is therefore 64 cu. in., or $\frac{1}{27}$ cu. ft. Allowing for mortar, however, it is reckoned as $\frac{1}{22}$ cu. ft., as stated.

Stone work is reckoned by the cubic foot or cubic yard, and sometimes by the perch.

A perch is $16\frac{1}{2}' \times 1\frac{1}{2}' \times 1'$, and therefore equals $24\frac{3}{4}$ cu. ft. Practically, however, it is taken as 25 cu. ft., although this varies.

In estimating the labor for brick and stone work, masons measure the outside of the walls, thus reckoning the corners twice. This is considered fair because of the extra trouble in laying a corner. Openings are generally not considered, although sometimes an allowance for half of the openings is made, the gain to the mason being offset by the extra labor in building around them.

In estimating the material they will need, however, masons consider the corners and openings. The custom varies in these matters.

For example, find the number of bricks required for a wall $42' \times 25'$ and 1 ft. thick.

$$42 \times 25 \times 1 \times 22 \text{ bricks} = 23{,}100 \text{ bricks}.$$

How many cubic feet should be allowed in estimating the labor on a cellar wall $40' \times 30'$, $6'$ high, and $2'$ thick?

Perimeter of cellar, outside the wall, is $2 \times (40 + 30)$ ft., or 140 ft. Therefore the number is $6 \times 2 \times 140$ cu. ft., or 1680 cu. ft.

EXERCISE 130

Find the number of cubic feet in walls of these dimensions:

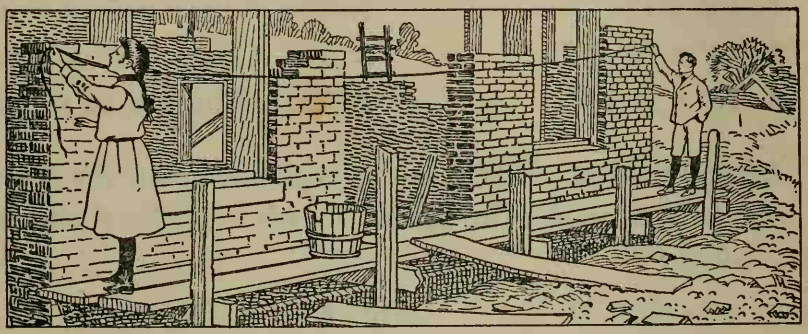
1. 62 ft., 4 ft., 1 ft.
2. 78 ft., 5 ft., 1 ft. 6 in.
3. 49 ft., 5 ft., 1 ft. 8 in.
4. 75 ft., 6 ft., 2 ft.
5. 82 ft., 6 ft., 2 ft. 2 in.
6. 96 ft., 6 ft. 6 in., 2 ft.

Find the number of cubic yards in walls of these dimensions:

7. 4 rd., 6 ft., 15 in.
8. 120 ft., 2 yd., 1 ft.
9. 4 rd., 5 ft., 18 in.
10. 150 ft., 6 ft., 1 ft. 6 in.

11. How many cubic feet of stone in a wall 52 ft. long, 6 ft. high, and 1 ft. 6 in. thick?

12. At $8 a thousand, what will the bricks cost for a wall 120 ft. long, 5 ft. high, and 1 ft. thick?



13. These children measure this house and find it to be 30 ft. by 20 ft. They learn that the brick walls are to be 20 ft. high and 1 ft. thick, and the openings are to amount to 250 cu. ft. How many bricks will be required?

14. How many cubic feet of stone in the cellar walls of a house $42' \times 25'$, if the walls are $18''$ thick and $6'$ high?

15. How many cubic feet of stone in the cellar walls of a house $36' \times 28'$, if the walls are $1\frac{1}{2}'$ thick and $6'$ high?

16. How many bricks will be required for the walls of a flat-roofed house $42' \times 36' \times 21'$, if the walls are 1 ft. thick and if 400 cu. ft. is deducted for doors and windows?

17. How many bricks will be required for the walls of a flat-roofed building $100' \times 60' \times 22'$, if the walls are $1\frac{1}{2}$ ft. thick, and if 800 cu. ft. is deducted for doors and windows?

244. Coal, Sand, and Gravel. A short ton of hard coal measures from $34\frac{1}{2}$ cu. ft. to 37 cu. ft.; of soft coal, about 42 cu. ft. A cubic yard of earth (including sand and gravel) is called a load.

EXERCISE 131

Allowing 35 cu. ft. to a ton of hard coal, find the number of tons that can be placed in bins of the following dimensions:

1. $10' \times 7' \times 6'$.
2. $12' \times 10' \times 7'$.
3. $13' \times 7' \times 5'$.
4. 9 ft. 6 in. $\times$ 6 ft. $\times$ 5 ft.
5. 8 ft. 4 in. $\times$ 7 ft. $\times$ 6 ft.
6. 10 ft. 6 in. $\times$ 6 ft. $\times$ 6 ft.

Allowing 42 cu. ft. to a ton of soft coal, find the number of tons that can be placed in bins of the following dimensions:

7. $10' \times 9' \times 7'$.
8. $12' \times 8' \times 6'$.
9. $14' \times 7' \times 5'$.
10. 8 ft. 4 in. $\times$ 7 ft. $\times$ 5 ft.
11. 9 ft. 2 in. $\times$ 8 ft. $\times$ 6 ft.
12. 7 ft. 3 in. $\times$ 9 ft. $\times$ 4 ft.

13. Taking a bushel as $1\frac{1}{4}$ cu. ft., and a ton of hard coal as measuring 35 cu. ft., how much should a bushel of hard coal weigh?

14. Taking a ton of soft coal as measuring 42 cu. ft., how much should a bushel of soft coal weigh?

15. How many loads in a rectangular cut for a mill flume 300 ft. long, 15 ft. wide, and 10 ft. deep?

16. How many loads of sand in a gravel train of 24 cars, each car being $15' 6'' \times 5' 8''$, and the sand being $2'$ deep?

17. How many loads will be removed in preparing the ground for macadamizing a street $1500'$ long, $40'$ wide, and $2\frac{1}{2}'$ deep?

245. Wood Measure. Wood is sold by the cord. The standard cord is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, and therefore contains 128 cu. ft.

The cord varies, often being considered to be a pile 8 ft. long and 4 ft. high, the price then varying with the length of the stick.

For example, how many cords of wood in a pile 24 ft. long, 4 ft. wide, and 6 ft. high?

Here we have $\frac{\overset{3}{\cancel{24}} \times \cancel{4} \times \overset{3}{\cancel{6}}}{\underset{2}{\cancel{8}} \times \cancel{4} \times \cancel{4}}$ cd. $= 4\frac{1}{2}$ cd.

EXERCISE 132

Find the number of cords of wood in piles 4 ft. wide, whose length and height are as follows:

1. 10 ft., 4 ft.
2. 12 ft., 6 ft.
3. 15 ft., 5 ft.
4. 16 ft., 5 ft.
5. 18 ft., 4 ft.
6. 20 ft., 6 ft.
7. 24 ft., 4 ft.
8. 26 ft., $5\frac{1}{2}$ ft.
9. 32 ft., 5 ft. 8 in.

10. How much are $7\frac{1}{2}$ cd. of wood worth at \$2.50 a cord?

11. How much are $16\frac{3}{4}$ cd. of wood worth at \$2.25 a cord?

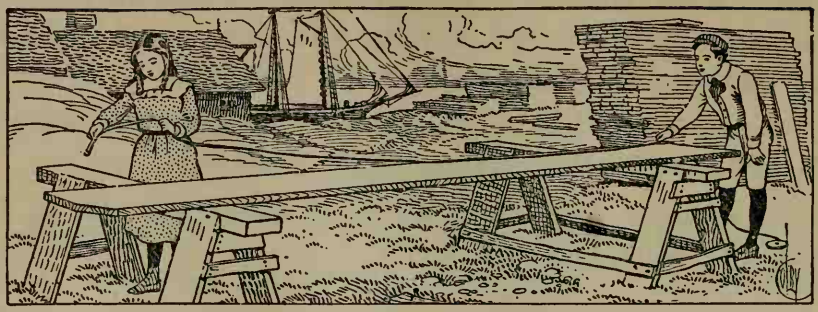
12. If wood is worth \$2.40 a cord, how many cords can be bought for \$42.60?

13. What is the cost of a pile of wood 22 ft. long and 5 ft. high, when wood of that length is selling for \$3.50 a cord? (Take the cord as a pile 8 ft. long and 4 ft. high.)

14. What is the cost of a pile of wood 36 ft. long and 6 ft. high, when wood of that length is selling for \$3.75 a cord?

15. A pile of wood 32 ft. long and 6 ft. high sells for \$24. What is the price of wood of that length per cord?

16. When wood of a certain length is worth \$3.80 a cord, a pile 4 ft. high sells for \$11.40. How long is the pile?



246. Board Measure. Boards one inch or less in thickness are sold by the square foot.

Boards more than one inch in thickness, and all squared lumber, are sold by the number of square feet of boards one inch in thickness to which they are equal.

Thus, a board 1 ft. long, 1 ft. wide, and 1 in. thick contains 1 *board foot*, or 1 ft. board measure. A board 12 ft. long, 1 ft. wide, and 1 in. or less in thickness contains 12 ft. board measure; but if the board is $1\frac{1}{4}$ in. thick, it contains $1\frac{1}{4} \times 12$ ft., or 15 ft.

Therefore, *to find the board measure of lumber an inch or less in thickness, take the number of square feet on one side.*

To find the board measure of lumber more than an inch in thickness, express the length and width in feet, and the thickness in inches. The product of these three numbers will be the number of feet, board measure.

In practice, the width of a board, unless sawed to order, is reckoned only to the next smaller half inch. Thus, a width of $11\frac{3}{8}$ in. is reckoned 11 in.; a width of $13\frac{5}{8}$ or $13\frac{3}{4}$ in. is reckoned $13\frac{1}{2}$ in.

If a board tapers, the average width is used, which the marker finds by taking the width at a point he considers about the average width.

How many feet in a $1\frac{7}{8}$-inch plank 16 ft. long and 14 in. wide?

$$14 \text{ in.} = 1\tfrac{1}{6} \text{ ft.}$$

$$1\tfrac{7}{8} \times 1\tfrac{1}{6} \times 16 \text{ ft.} = 35 \text{ ft., board measure.}$$

In all such cases employ cancellation.

EXERCISE 133

Find the number of feet, board measure, in inch boards of length and width as follows:

1. 16 ft., 4 in.
2. 14 ft., 6 in.
3. 12 ft., 8 in.
4. 16 ft., 9 in.
5. 18 ft., 8 in.
6. 16 ft., 10 in.
7. 18 ft., $9\frac{1}{2}$ in.
8. 16 ft., $6\frac{1}{2}$ in.
9. 14 ft., $8\frac{1}{2}$ in.

Find the number of feet, board measure, in the following:

10. 8 planks 2 in. thick, 10 ft. long, 12 in. wide.
11. 12 planks 3 in. thick, 14 ft. long, 10 in. wide.
12. 16 planks $2\frac{1}{2}$ in. thick, 16 ft. long, 10 in. wide.
13. 16 planks $1\frac{3}{4}$ in. thick, 10 ft. long, 9 in. wide.
14. 2 sticks of timber 18 ft. long, 1 ft. wide, 8 in. thick.
15. 5 sticks of timber 16 ft. long, 10 in. wide, 9 in. thick.
16. 25 joists 3″ by 4″ and 16 ft. long.
17. 30 joists 2″ by 4″ and 14 ft. long.
18. 50 joists 3″ by 8″ and 16 ft. long.
19. 8 beams 10″ by 12″ and 12 ft. long.
20. 30 boards $\frac{3}{4}$ in. thick, 8 in. wide, and 12 ft. long.
21. 50 boards $\frac{7}{8}$ in. thick, 10 in. wide, and 16 ft. long.
22. 80 boards 1 in. thick, 8 in. wide, and 14 ft. long.
23. 115 planks 2 in. thick, 10 in. wide, and 18 ft. long.
24. What is the cost of 8 spruce joists, $2'' \times 6''$, 12 ft. long, at \$24 per M (that is, per 1000 ft. board measure)?
25. What is the cost of 24 spruce joists, $2'' \times 8''$, 16 ft. long, at \$22.50 per M?
26. What is the cost of 6 spruce beams, $8'' \times 10''$, 18 ft. long, at \$27 per M?

247. Capacity of Bins. The following measurements are used in computing the capacity of bins:

1 bu. = 2150.42 cu. in. = nearly $1\frac{1}{4}$ cu. ft.
1 cu. ft. = nearly $\frac{4}{5}$ bu.

Therefore, *to find the approximate capacity, in bushels, of a bin, multiply $\frac{4}{5}$ bu. by the number of cubic feet in the bin.*

For example, how many bushels in a bin $10' \times 8' \times 5'$?

$$10 \times 8 \times 5 \times \tfrac{4}{5} \text{ bu.} = 320 \text{ bu.}$$

To find the approximate space, in cubic feet, required for a given number of bushels of grain, multiply $\frac{5}{4}$ of a cubic foot by the number of bushels.

For example, how many cubic feet will be required for 1000 bu.?

$$1000 \times \tfrac{5}{4} \text{ cu. ft.} = 1250 \text{ cu. ft.}$$

EXERCISE 134

Find the number of bushels in bins of the following dimensions:

1. $12' \times 6' \times 5'$.
2. $12' \times 7' \times 6'$.
3. $15' \times 9' \times 5'$.
4. $16' \times 10' \times 6'$.
5. $18' \times 10' \times 8'$.
6. $18' \times 12' \times 7'$.
7. $12\frac{1}{2}' \times 6' \times 4'$.
8. $15\frac{1}{2}' \times 8\frac{1}{2}' \times 5'$.
9. $16\frac{3}{4}' \times 9\frac{1}{4}' \times 5\frac{1}{2}'$.

Find the number of cubic feet in bins of the following capacities:

10. 600 bu.
11. 850 bu.
12. 975 bu.
13. 1500 bu.
14. 1625 bu.
15. 1860 bu.
16. 3200 bu.
17. 5600 bu.
18. 7500 bu.
19. 16,000 bu.
20. 22,320 bu.
21. 28,000 bu.

22. At 95¢ a bushel, how much is the wheat worth that fills a bin $10' \times 9' \times 4'$?

23. At \$1.02 a bushel, how much is the wheat worth that fills a bin $30' \times 20' \times 10'$?

248. A Cylinder. A solid formed by the revolution of a rectangle about one of its sides is called a *cylinder.*

The two circles that are thus formed are called the *bases*, and the curved surface is called the *lateral surface* of the cylinders.

The distance between the two bases is called the *altitude* of the cylinder.

249. Volume of a Cylinder. We can think of 1 cu. in. on each square inch of the base. Therefore, if the base contains 4 sq. in., and if the cylinder is 1 in. high, the volume will be 4×1 cu. in., or 4 cu. in., and if the cylinder is 5 in. high, the volume will be 5×4 cu. in., or 20 cu. in.

Therefore, to find the volume of a cylinder,

Multiply the number of cubic units corresponding to the number of square units of the base, by the number of units of altitude.

That is, if the base has 9 sq. in., and the altitude is 12 in., the volume is 12×9 cu. in., or 108 cu. in.

If the base has a diameter of 2 in., and the altitude is 5 in., then the area of the base is 3.1416×1 sq. in. (§ 231), and the volume is $5 \times 3.1416 \times 1$ cu. in., or 15.708 cu. in.

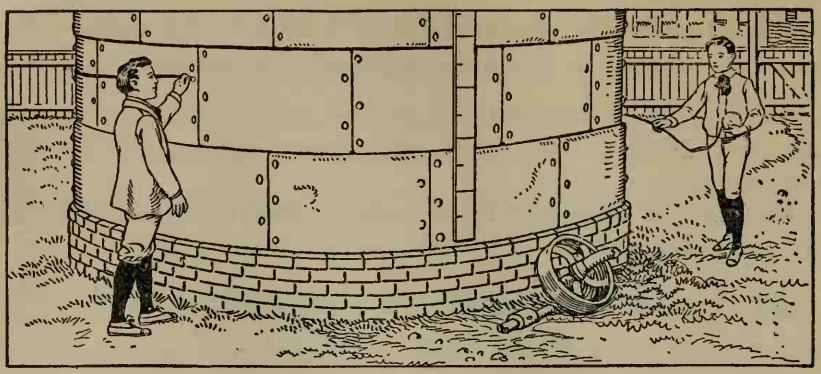
250. Lateral Area of a Cylinder. We may think of the surface of a cylinder as unrolled, like a paper reaching once around a pencil. In this case it would unfold into a rectangle, one side of which would be the circumference of the base and the other the altitude.

Therefore, to find the lateral area of a cylinder,

Multiply the number of units in the circumference by the number of units in the altitude.

For example, if the circumference is 6 in. and the altitude 4 in., the number of square units of lateral area is 4×6, or 24. We may express the work thus:

$$4 \times 6 \text{ sq. in.} = 24 \text{ sq. in.}$$



EXERCISE 135

Find the volumes of cylinders whose altitudes and areas of base are as follows:

1. 13 in., 27 sq. in.
2. 17 in., 36 sq. in.
3. 38 in., 125 sq. in.
4. 3 ft. 7 in., $8\frac{1}{2}$ sq. in.
5. 4 ft. 6 in., $7\frac{1}{2}$ sq. in.
6. 8 ft. 7 in., 4 sq. ft. 24 sq. in.

Find the altitudes of cylinders whose volumes and areas of base are as follows:

7. 20 cu. ft. 324 cu. in., 8 sq. ft. 72 sq. in.
8. 190 cu. ft. 1305 cu. in., 30 sq. ft. 75 sq. in.

Find the areas of base of cylinders whose volumes and altitudes are as follows:

9. 324 cu. ft., 18 ft.
10. 323 cu. ft., 17 ft.
11. 28 cu. ft. 288 cu. in., 2 ft. 2 in.
12. 81 cu. ft. 576 cu. in., 5 ft. 4 in.

Find the volumes of cylinders whose altitudes and radii of base are as follows:

13. 27 in., 4 in.
14. 25 in., 6 in.
15. 32 in., 8 in.
16. 36 in., 9 in.
17. 2 ft. 6 in., 2 in.
18. 3 ft. 4 in., 1 ft.

251. Capacity of Cisterns. The following measurements are used in computing the capacity of cisterns:

1 gal. = 231 cu. in.
1 cu. ft. = 7.48 + gal. = nearly $7\frac{1}{2}$ gal.
$31\frac{1}{2}$ gal. = 1 bbl. = nearly $4\frac{1}{5}$ cu. ft.

In Exercise 136 use $7\frac{1}{2}$ gal. for 1 cu. ft., $4\frac{1}{5}$ cu. ft. for 1 bbl., and $3\frac{1}{7}$ as the multiplier in cases requiring the area of base, given the diameter.

Find the number of gallons of water in a cylindrical tank 40 ft. in diameter, the water being 35 ft. deep.

Radius of base $= \frac{1}{2}$ of 40 ft. = 20 ft.
Area of base $= 3\frac{1}{7} \times 20 \times 20$ sq. ft. (§ 231).
Volume $= 35 \times 3\frac{1}{7} \times 20 \times 20$ cu. ft. (§ 249)
$= 35 \times 3\frac{1}{7} \times 20 \times 20 \times 7\frac{1}{2}$ gal.
= 330,000 gal.

EXERCISE 136

Find the number of gallons in these rectangular tanks:

1. $4' \times 2' \times 1'$.
2. $5' \times 2' \times 1'$.
3. $6' \times 1\frac{1}{2}' \times \frac{3}{4}'$.
4. $6' \times 2\frac{1}{4}' \times 1'$.
5. $12\frac{1}{2}' \times 12\frac{1}{2}' \times 6'$.
6. $15\frac{1}{2}' \times 12\frac{1}{2}' \times 6\frac{1}{2}'$.

Find the number of barrels in the following tanks:

7. $12' \times 11' \times 9'$.
8. $15' \times 14' \times 8'$.
9. $15\frac{1}{2}' \times 12\frac{1}{2}' \times 5\frac{3}{4}'$.

Find the number of gallons in cylindrical tanks of the following diameter and depth of water respectively:

10. 6′, 10′.
11. 15′, 12′.
12. 16′, 14′.
13. 35′, 40′.
14. $48\frac{1}{2}'$, 56′.
15. $48\frac{3}{4}'$, $45\frac{1}{2}'$.

Find the number of gallons of water in wells of the following diameter and depth of water respectively:

16. 3 ft., 8 ft.
17. 3 ft., 6 ft.
18. 4 ft., 8 ft.
19. $3\frac{1}{2}$ ft., 6 ft.
20. $3\frac{3}{4}$ ft., $5\frac{1}{4}$ ft.
21. $3\frac{3}{4}$ ft., $8\frac{3}{4}$ ft.

EXERCISE 137

Problems without Numbers

1. Given the length of each side of a triangle, how do you find the perimeter?

2. Given the length of one side of a square, how do you find the perimeter?

3. Given the lengths of two adjacent sides of a rectangle, how do you find the perimeter?

4. Given the lengths of two adjacent sides of a rectangle, how do you find the area?

5. Given the side of a square, how do you find the area?

6. What two lengths must you know in order to find the area of a parallelogram?

7. What two lengths must you know in order to find the perimeter of a parallelogram?

8. Given the base and altitude of a triangle, how is the area found?

9. Given the area and base of a rectangle, how is the altitude found?

10. Given the area and base of a triangle, how is the altitude found?

11. Given the diameter of a circle, how is the circumference found?

12. Given the circumference of a circle, how is the radius found?

13. Given the circumference and radius of a circle, how is the area found?

14. Given the radius or the diameter of a circle, how is the area found?

15. How would you find the number of rolls of paper needed to cover the walls and ceiling of this room?

16. How would you find the number of yards of carpet needed to cover a floor of the size of this one?

17. How would you find the number of square yards of plastering needed for the walls and ceiling of this room?

18. How would you find the number of square feet of lumber needed to cover this floor with boards of the width of those now used?

19. How would you find the number of square yards of lathing needed for this room?

20. If you knew the cost of shingles by the bunch, how would you find the cost of shingles for a roof of which you knew the dimensions?

21. How would you proceed to find the cost of a sidewalk to be laid in front of the schoolhouse?

22. How would the girls in the class proceed to compute the cost of material and labor for making a dress for school wear?

23. Given the length, breadth, and thickness of a rectangular solid, how would you find the volume?

24. Given the same dimensions, how would you find the total area of the faces?

25. Given the volume of a rectangular solid, and two dimensions, how would you find the third dimension?

26. How would you estimate the number of thousand bricks needed to build a wall?

27. How would you estimate the number of loads of dirt that would have to be removed to excavate for a cellar?

28. How would you estimate the number of tons of hard or soft coal that a bin of given dimensions would contain?

29. How would you find how many cords of wood a pile of given dimensions will contain?

30. How would you find how many bushels of wheat a bin of given dimensions will hold?

31. Given the capacity of a bin in bushels, how would you find its capacity in cubic feet?

32. Given the number of cubic feet in a bin, how would you find the weight of the wheat necessary to fill it?

33. Given the weight of wheat in a bin, how would you find the number of cubic feet of wheat?

34. Given the altitude and the area of the base of a cylinder, how would you find the volume?

35. Given the volume and the area of the base of a cylinder, how would you find the altitude?

36. Given the altitude and the radius of the base of a cylinder, how would you find the volume?

37. Given the volume and the radius of the base of a cylinder, how would you find the altitude?

38. Given the altitude and the circumference of the base of a cylinder, how would you find the lateral area?

39. Given the altitude and the radius of the base of a cylinder, how would you find the lateral area?

40. Given the diameter of a cylindrical water tank and the depth of the water, how would you find the volume of the water?

41. Given the diameter of a cylindrical water tank and the depth of the water, how would you find the number of gallons of water?

42. Given the dimensions of a rectangular tank and the weight of a cubic foot of water, how would you find the weight of water necessary to fill the tank?

CHAPTER XIII

PERCENTAGE

252. Per Cent. Another name for hundredths is *per cent.*

Thus 0.01 or $\frac{1}{100}$ is the same as 1 per cent; $0.00\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{100}$, is the same as $\frac{1}{2}$ per cent; 0.06 is the same as 6 per cent.

That part of arithmetic which treats of per cent is called *percentage.*

253. Symbol for Per Cent. The symbol for per cent is written thus: %.

We may read 0.06 either "6 hundredths" or "6 per cent." In the same way, we may think of 6% either as "6 per cent" or "6 hundredths," although it is read "6 per cent."

The expression 800% means $\frac{800}{100}$ and equals the whole number 8; 225% equals the mixed number 2.25, or $2\frac{1}{4}$; $\frac{1}{2}$% means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$, and is read either "$\frac{1}{2}$ per cent" or, quite commonly, "$\frac{1}{2}$ of 1%."

254. Relation to Fractions. Since 6% means $\frac{6}{100}$, which equals 0.06, or $\frac{3}{50}$, we see that we may express per cent as a decimal fraction or as a common fraction.

It is often convenient to use one form, and often another. Thus if we are multiplying by 13.5%, it is more convenient to think of the multiplier as 0.135, but if we are multiplying by $33\frac{1}{3}$% it is better to think of it as $\frac{1}{3}$ instead of $0.33\frac{1}{3}$.

255. Per Cents as Common Fractions. Since $62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$, therefore

To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.

EXERCISE 138

Reduce to a common fraction, integer, or mixed number:

1. 4%.	11. 100%.	21. $6\frac{1}{4}$%.	31. $3\frac{1}{3}$%.
2. 5%.	12. 200%.	22. $12\frac{1}{2}$%.	32. $6\frac{2}{3}$%.
3. 6%.	13. 500%.	23. $37\frac{1}{2}$%.	33. $33\frac{1}{3}$%.
4. 10%.	14. 125%.	24. $62\frac{1}{2}$%.	34. $66\frac{2}{3}$%.
5. 15%.	15. 150%.	25. $87\frac{1}{2}$%.	35. $16\frac{2}{3}$%.
6. 20%.	16. 175%.	26. $18\frac{3}{4}$%.	36. $83\frac{1}{3}$%.
7. 25%.	17. 120%.	27. $31\frac{1}{4}$%.	37. $112\frac{1}{2}$%.
8. 30%.	18. 140%.	28. $43\frac{3}{4}$%.	38. $133\frac{1}{3}$%.
9. 50%.	19. 160%.	29. $56\frac{1}{4}$%.	39. $166\frac{2}{3}$%.
10. 75%.	20. 180%.	30. $68\frac{3}{4}$%.	40. $187\frac{1}{2}$%.

41. How much is $\frac{1}{25}$ of 75? 4% of 75?

42. How much is $\frac{1}{4}$ of 640? 25% of 640?

43. How much is $\frac{1}{2}$ of 1728? 50% of 1728?

44. How much is $\frac{3}{4}$ of 8440? 75% of 8440?

45. How much is $1\frac{1}{4} \times 864$? 125% of 864?

46. Instead of multiplying 64 by $12\frac{1}{2}$%, what fractional part of 64 may we take? Do this.

47. Instead of multiplying 720 by $37\frac{1}{2}$%, what fractional part of 720 may we take? Do this.

48. Take $87\frac{1}{2}$% of 80 in the shortest way.

49. Take $33\frac{1}{3}$% of 66 in the shortest way.

Find the following:

50. $16\frac{2}{3}$% of 66.	53. $112\frac{1}{2}$% of 176.	56. $133\frac{1}{3}$% of 729.
51. $16\frac{2}{3}$% of 738.	54. $137\frac{1}{2}$% of 256.	57. $133\frac{1}{3}$% of 801.
52. $12\frac{1}{2}$% of 736.	55. $162\frac{1}{2}$% of 312.	58. $166\frac{2}{3}$% of 708.

256. Per Cents as Decimals. Since 25.5% and 0.255 have the same value,

Therefore, *to express as a decimal a number written with the per cent sign, omit the sign and move the decimal point two places to the left.*

Since per cent means hundredths, when we omit the per cent sign we must indicate the hundredths in some other way, as by moving the decimal point two places to the left.

Thus $1\frac{1}{2}\% = 0.01\frac{1}{2}$, or 0.015; $125\% = 1.25$; $0.6\% = 0.006$.

EXERCISE 139

Express as decimals:

1. 6%.	7. $16\frac{2}{3}\%$.	13. 0.9%.	19. 675%.
2. 10%.	8. $33\frac{1}{3}\%$.	14. $0.1\frac{1}{2}\%$.	20. 375%.
3. 25%.	9. $37\frac{1}{2}\%$.	15. $6.2\frac{1}{2}\%$.	21. 100%.
4. 50%.	10. $66\frac{2}{3}\%$.	16. 200%.	22. 1000%.
5. 75%.	11. 0.5%.	17. 150%.	23. 0.005%.
6. $12\frac{1}{2}\%$.	12. 0.7%.	18. 125%.	24. 0.065%.

25. Express $\frac{1}{8}\%$ as a decimal; as a common fraction

26. Express $\frac{7}{8}\%$ as a decimal; as a common fraction.

27. Express $333\frac{1}{3}\%$ as a decimal; as an improper fraction; as a mixed number.

28. How much is $\frac{1}{100}$ of \$200? 1% of \$200?

29. How much is 0.06 of \$100? 6% of \$100?

30. How much is 0.15 of \$300? 15% of \$300?

31. How much is $1\frac{1}{4} \times \$650$? $1.25 \times \$650$? 125% of \$650?

32. How much is $2.50 \times \$460$? 250% of \$460?

33. If 2% of the pupils of this school are absent to-day, how many are absent out of every 100? out of every 50?

257. Decimals as Per Cents. Since per cent means hundredths, to express a decimal as per cent we have to consider only how many hundredths the decimal represents.

For example, express 0.3 as per cent.

Here $0.3 = 0.30$, which is 30 hundredths, or 30%.

Express 0.375 as per cent.

Here $0.375 = 37.5$ hundredths, or $37\frac{1}{2}$ hundredths. Hence it equals 37.5% or $37\frac{1}{2}$%.

Express 0.00125 as per cent.

Here $0.00125 = 0.00\frac{125}{10000} = 0.00\frac{1}{8} = \frac{1}{8}$%.

Express $6.5\frac{1}{2}$ as per cent.

Here $6.5\frac{1}{2} = \frac{655}{100} = 655$%.

Therefore, *to express a decimal as per cent, write the per cent sign after the number of hundredths.*

EXERCISE 140

Express as per cents:

1. 0.5.	6. 0.25.	11. 0.125.	16. $0.2\frac{1}{2}$.	21. $0.3\frac{1}{3}$.
2. 0.7.	7. 0.57.	12. 0.275.	17. $0.5\frac{1}{4}$.	22. $0.333\frac{1}{3}$.
3. 0.6.	8. 0.34.	13. 0.375.	18. $0.6\frac{3}{4}$.	23. $6.6\frac{2}{3}$.
4. 6.2.	9. 0.29.	14. 0.758.	19. $5.2\frac{1}{8}$.	24. 0.00375.
5. 7.5.	10. 7.65.	15. 9.637.	20. $7.3\frac{3}{8}$.	25. 0.000125.

26. A pint, being 0.5 qt., is what per cent of a quart?
27. A peck, being 0.25 bu., is what per cent of a bushel?
28. A quart, being 0.125 pk., is what per cent of a bushel?
29. Express 0.007 of a mile as per cent of a mile.
30. What per cent of a dollar is 37¢? is 37 dimes?
31. Express 1 gill as per cent of a gallon.
32. Express 1 quart as per cent of a bushel.

258. Common Fractions as Per Cents. Since per cent means hundredths, to express a common fraction as per cent we have only to reduce it to hundredths.

For example, $\frac{4}{5} = \frac{80}{100}$, and this is the same as 0.80 or 80%. Likewise $\frac{2}{25} = \frac{8}{100} = 8\%$, and $2\frac{1}{2} = \frac{5}{2} = \frac{250}{100} = 250\%$.

Therefore, *to express a common fraction as per cent, reduce it to hundredths, omit the denominator, and write the numerator followed by the per cent sign.*

This is easily done by reducing to a decimal, and writing this in the per cent form (§ 257).

It is often more convenient to express per cents decimally than with the symbol %. Thus, in the case of $25\frac{2}{3}$ we have $25.66\frac{2}{3}$, a more convenient form than $2566\frac{2}{3}\%$.

EXERCISE 141

Express as per cents:

1. $\frac{1}{2}$.	6. $\frac{3}{5}$.	11. $\frac{9}{20}$.	16. $\frac{1}{3}$.	21. $\frac{1}{16}$.	26. $2\frac{1}{4}$.
2. $\frac{1}{4}$.	7. $\frac{4}{5}$.	12. $\frac{1}{25}$.	17. $\frac{2}{3}$.	22. $\frac{5}{16}$.	27. $3\frac{3}{5}$.
3. $\frac{3}{4}$.	8. $\frac{1}{10}$.	13. $\frac{11}{25}$.	18. $\frac{1}{8}$.	23. $\frac{1}{32}$.	28. $7\frac{5}{8}$.
4. $\frac{1}{5}$.	9. $\frac{7}{10}$.	14. $\frac{1}{50}$.	19. $\frac{3}{8}$.	24. $\frac{15}{32}$.	29. $6\frac{1}{3}$.
5. $\frac{2}{5}$.	10. $\frac{1}{20}$.	15. $\frac{37}{50}$.	20. $\frac{5}{8}$.	25. $\frac{5}{24}$.	30. $75\frac{7}{8}$.

31. A foot is what per cent of a yard?

32. An inch is what per cent of a foot?

33. A pint is what per cent of a gallon?

34. What per cent of a gallon is 3 qt.?

35. What per cent of a pound is 4 oz.?

36. What per cent of a pound is 7 oz.?

37. If a man spends $\frac{5}{6}$ of his income, what per cent of his income does he spend?

38. If you have increased $\frac{1}{10}$ in weight during the past year, what per cent have you gained in weight?

259. Terms used in Percentage. The number of which some per cent is to be taken is called the *base*.

The number of hundredths of the base is called the *rate*.

For example, in 25% of \$300, \$300 is the base and 25% is the rate.

Sometimes 25 is called the *rate per cent*, 25% being called the *rate*, but these two terms are commonly used to mean the same thing.

The result found by taking a certain per cent of the base is called the *percentage*. Therefore,

The percentage is the product of the base and the rate.

260. Important Per Cents. Certain per cents are used so frequently that their equivalent common fractions should be remembered. These are as follows:

$50\% = \frac{1}{2}$,	$37\frac{1}{2}\% = \frac{3}{8}$,	$16\frac{2}{3}\% = \frac{1}{6}$,	$20\% = \frac{1}{5}$,
$25\% = \frac{1}{4}$,	$62\frac{1}{2}\% = \frac{5}{8}$,	$33\frac{1}{3}\% = \frac{1}{3}$,	$40\% = \frac{2}{5}$,
$12\frac{1}{2}\% = \frac{1}{8}$,	$87\frac{1}{2}\% = \frac{7}{8}$,	$66\frac{2}{3}\% = \frac{2}{3}$,	$60\% = \frac{3}{5}$,
$6\frac{1}{4}\% = \frac{1}{16}$,	$3\frac{1}{8}\% = \frac{1}{32}$,	$83\frac{1}{3}\% = \frac{5}{6}$,	$80\% = \frac{4}{5}$.

To take $87\frac{1}{2}\%$ of 648 is, therefore, the same as to take $\frac{7}{8}$ of 648.

EXERCISE 142

Find:

1. 50% of \$274.
2. 25% of \$372.
3. $12\frac{1}{2}\%$ of \$512.
4. $6\frac{1}{4}\%$ of \$3376.
5. $37\frac{1}{2}\%$ of \$17.76.
6. $62\frac{1}{2}\%$ of \$11.92.
7. $87\frac{1}{2}\%$ of \$34.48.
8. $3\frac{1}{8}\%$ of \$739.20.
9. $16\frac{2}{3}\%$ of \$34.26.
10. $33\frac{1}{3}\%$ of \$77.16.
11. $66\frac{2}{3}\%$ of \$80.07.
12. $83\frac{1}{3}\%$ of \$71.10.
13. 20% of \$735.15.
14. 40% of \$611.25.
15. 60% of \$734.15.
16. $112\frac{1}{2}\%$ of 72.
17. $137\frac{1}{2}\%$ of 96.
18. $116\frac{2}{3}\%$ of 72.
19. $133\frac{1}{3}\%$ of 81.
20. $166\frac{2}{3}\%$ of 84.

261. To find Some Per Cent of a Number. Finding some of the more important per cents of numbers, by reducing to common fractions, has already been considered on page 256. The more general case, that in which common fractions cannot be used to advantage, will now be considered.

For example, required to find $23\frac{3}{4}\%$ of 275.

$$23\tfrac{3}{4}\% = 0.23\tfrac{3}{4}.$$

Multiplying by $0.23\frac{3}{4}$, we have $65.31\frac{1}{4}$.

If we wish, we may write $23\frac{3}{4}\%$ as 0.2375 and then multiply. We may also, if we choose, write the product 65.3125.

$$\begin{array}{r} 275 \\ .23\frac{3}{4} \\ \hline 206\frac{1}{4} \\ 825 \\ 550 \\ \hline 65.31\frac{1}{4} \end{array}$$

Therefore, *to find a required per cent of a number, multiply the number by the given rate.*

EXERCISE 143

Find, by using decimals:

1. 22% of 75.
2. 35% of 86.
3. 27% of 32.
4. 41% of 78.
5. 62% of 93.
6. 26% of 3.4.
7. 43% of 6.7.
8. 39% of 5.3.
9. 28% of 0.7.
10. 35% of 0.63.
11. $2\frac{3}{4}\%$ of 8.4.
12. 0.5% of 5.7.
13. 1.9% of 6.8.
14. 235% of 742.
15. 622% of 0.72

Find, by using common fractions:

16. 25% of 1728.
17. 50% of 6235.
18. 20% of 7341.
19. 30% of 2790.
20. 75% of 6448.
21. $12\frac{1}{2}\%$ of 9616.
22. $6\frac{1}{4}\%$ of 24,328.
23. $33\frac{1}{3}\%$ of 9051.
24. $66\frac{2}{3}\%$ of 7116.
25. $87\frac{1}{2}\%$ of 35,360.
26. $16\frac{2}{3}\%$ of \$13,806.
27. $37\frac{1}{2}\%$ of 25,040.
28. $83\frac{1}{3}\%$ of 22,350.
29. 125% of 948.
30. $133\frac{1}{3}\%$ of 729.
31. $166\frac{2}{3}\%$ of 816.

32. A man borrowed \$175 and had to pay 6% of that sum for the use of it for a year. How much did he pay in addition to the sum borrowed?

33. If a manufacturer sells at a profit of 15% shoes that cost him \$2.33$\frac{1}{3}$ a pair to make, how much is his profit on 1000 pairs of shoes?

34. If a shop manufactures 276 locomotives and sells 75% of them for \$11,125 each and the rest for \$9825 each, how much is received for all?

35. A farmer can have his land poorly plowed for \$3.48 an acre, but it will cost him 66$\frac{2}{3}$% more to have a good job done. How much will it cost to have 60 acres well done?

36. How much butter fat is there in 375 lb. of milk brought to a creamery, the creamery test showing that 3.9% of the weight of the milk is butter fat?

37. A farmer takes 340 lb. of milk to a creamery and is paid 26¢ a pound for the butter fat. The tests show that the butter fat is 3.8% of the weight of this milk. How much does he receive?

38. One cow in a dairy gives 28 lb. of milk a day, which tests 3.1% of butter fat; another gives 24 lb., testing 3.7%. Butter fat being worth 23¢ a pound, which cow is the more profitable, and how much more?

39. A farmer has 75 trees on an acre of woodland, of which he decides to cut 60%. If wood is worth \$5.75 a cord, and he can cut 3 cords from 5 trees, how much will he receive for the wood?

40. A dressmaker bought 37$\frac{1}{2}$ yd. of chiffon velvet at \$4.10 a yard, and received a reduction of 8% for paying cash. She sold the velvet at \$4.25 a yard. How much did she make on the transaction?

262. Given the Percentage and Rate to find the Base. In accordance with the definitions in § 259, finding a number of which a given per cent is known may be spoken of as finding the base when the percentage and rate are given.

In these examples we let 100% of a number equal the number. This is legitimate, because $100\% = \frac{100}{100} = 1$, so that we merely let once the number stand for the number. This is often convenient.

(1) If 15% of a number is 9165, what is the number?

Since 15% of the number = 9165,
therefore 1% of the number = $\frac{1}{15}$ of 9165, or 611,
and the number = 100 × 611, or 61,100.

(2) If $33\frac{1}{3}\%$ of a number is 247, what is the number?

Since $33\frac{1}{3}\%$, or $\frac{1}{3}$, of a number is 247, the number is 3 × 247, or 741.

(3) 275 is $66\frac{2}{3}\%$ more than what number?

Since 100% of the number = the number,
therefore $66\frac{2}{3}\%$ of the number = the increase,
and $166\frac{2}{3}\%$, or $\frac{5}{3}$, of the number = 275.
Therefore $\frac{1}{3}$ of the number = $\frac{1}{5}$ of 275, or 55,
and the number = 3 × 55, or 165.

(4) 616 is 12% less than what number?

Since 100% of the number = the number,
therefore 12% of the number = the decrease,
and 88% of the number = 616.
Therefore 1% of the number = $\frac{1}{88}$ of 616, or 7,
and the number = 100 × 7, or 700.

In each of these examples it will be noticed that we practically divide the percentage by the rate. Therefore,

The base equals the percentage divided by the rate.

This is also evident because the percentage equals the base multiplied by the rate (§ 261). Therefore we have the product (percentage) and one factor (rate) to find the other factor (base), and this we find by dividing (§ 49).

EXERCISE 144

Of what numbers are the following the given per cents?

1. 72, 12%.
2. 165, 15%.
3. 252, 21%.
4. 578, 34%.
5. 7999, 25%.
6. 48.72, $33\frac{1}{3}$%.
7. 96.54, $66\frac{2}{3}$%.
8. \$114, 6%.
9. \$3113, $5\frac{1}{2}$%.
10. 32.86, 106%.
11. 45.10, 110%.
12. 55.55, 125%.

The following numbers are the given per cents more than what numbers?

13. $34\frac{1}{3}$, 3%.
14. 26, 4%.
15. 42, 5%.
16. 259.7, 6%.
17. 342.4, 7%.
18. 459, 8%.
19. 800, 25%.
20. 520, 30%.
21. $1087\frac{1}{2}$, 50%.
22. 9.72, $33\frac{1}{3}$%.
23. 6.37, $16\frac{2}{3}$%.
24. 15.6, $66\frac{2}{3}$%

The following numbers are the given per cents less than what numbers?

25. 24, 4%.
26. 38, 5%.
27. $31\frac{1}{3}$, 6%.
28. 251.1, 7%.
29. 294.4, 8%.
30. 682.5, 9%.
31. 573, 25%.
32. 140, 50%.
33. 180, 75%.
34. 6.3, $12\frac{1}{2}$%.
35. 20, $37\frac{1}{2}$%.
36. 3.3, $62\frac{1}{2}$%.

37. \$435 is 6% of what sum of money?

38. \$19.75 is 5% of what sum of money?

39. \$38.25 is $4\frac{1}{2}$% of what sum of money?

40. What is the number of which 14.7 is $66\frac{2}{3}$%?

41. What is the sum of which \$41.25 is 15%?

42. What is the sum of which $33\frac{1}{3}$% is \$2.25?

43. A farmer sold his milk to a factory where he received credit for 1045 lb. of butter fat. If his milk tested 3.8% butter fat, how many pounds of milk did he sell?

263. Given the Base and Percentage to find the Rate. Since the percentage is the product of the base and rate (§ 259), therefore the rate may be found by dividing the percentage by the base (§ 49). Consider, for example, the following:

(1) What per cent of 5 is 4?

Here 4 is the product of some number and 5. Therefore the required per cent may be found by dividing 4 by 5 (§ 49); and $4 \div 5$, or $\frac{4}{5}$, $= 0.80 = 80\%$.

(2) What per cent of 245 is 29.4?

Here 29.4 is the product of some number and 245. Therefore the required per cent (§ 49), $29.4 \div 245$, $= 0.12 = 12\%$.

Therefore, *the rate equals the percentage divided by the base.*

This case and the two cases considered in § 261 and § 262 cover all important questions in percentage.

EXERCISE 145

Find what per cent the second number is of the first:

1. 72, 3.6.	7. 28.6, 6.006.	13. 785.6, 392.8.
2. 97, 6.79.	8. 23.4, 10.53.	14. 48.36, 8.06.
3. 387, 15.48.	9. 33.3, 10.989.	15. 542.1, 361.4.
4. 426, 38.34.	10. 1671, 50.13.	16. 21.36, 8.01.
5. 121, 13.31.	11. 1555, 373.2.	17. 72.48, 45.3.
6. 29.1, 4.656.	12. 2035, 447.7.	18. 2.248, 1.967.

19. A dressmaker bought a 20-yard silk dress pattern for \$45 less 20%. She sold it for \$45. What per cent did she gain on what she paid for it?

20. Allowing 8 oz. of Paris green to 50 gal. of water in preparing a spray against leaf-eating insects, and considering a gallon of water to weigh 8.4 lb., the weight of the Paris green is what per cent of the weight of the water?

264. Commercial Discount. A reduction from the list price of an article, from the amount of a bill of goods, or from the amount of a debt is called *commercial discount.*

Commercial discount is also called *trade discount.*

Discounts are usually reckoned at some common fraction of, or at some rate per cent of, the amount from which the discount is made.

The amount of the bill after the discount has been made is called the *net amount,* or the *net price.*

For example, what is the net amount of a bill of $275 after a discount of 20% is made?

Amount of bill	$275
Less 20%, or $\frac{1}{5}$, of $275	55
Net amount	$220

Instead of speaking of "20% off," merchants often speak of "$\frac{1}{5}$ off." So they speak of "$\frac{1}{4}$ off," and "$\frac{1}{3}$ off," but it is not so convenient to use common fractions for such discounts as 12%, 7%, 15%, etc.

EXERCISE 146

Find the net prices of goods billed as follows, with discounts as stated:

1. $27, 6%.
2. $32, 7%.
3. $46, 5%.
4. $75, 8%.
5. $64, 4%.
6. $86, 9%.
7. $68, 10%.
8. $82, 12%.
9. $96, 15%.
10. $252, 8%.
11. $175, 9%.
12. $25.50, 6%.
13. $48.75, 8%.
14. $62.30, 10%.
15. $73.40, 10%.
16. $68.75, 12%.
17. $86.40, 15%.
18. $37.90, 10%.
19. $48.50, 12%.
20. $52.50, 14%.
21. $68.20, 15%.
22. $47.20, 15%.
23. $125.50, 4%.
24. $375.50, 8%.
25. $426.50, 10%.
26. $382.60, 10%.
27. $296.80, 15%.
28. $492.50, 12%.
29. $598.20, 15%.
30. $275.00, 11%.
31. $826.50, 12%
32. $962.20, 15%.
33. $896.50, 14%

34. The discount on some goods listed at \$460 is \$115. What is the rate of discount?

35. The discount at 8% on some goods is \$41.60. What is the list price?

36. The discount at $12\frac{1}{2}\%$ on some goods is \$2.05. What is the list price?

37. A bicycle was marked \$45, but was sold at 12% discount. What was the net price?

38. A piano was listed at \$550, but was sold at 30% discount. What was the selling price?

39. A jobber bought some goods listed at \$2450. He was allowed a discount of 22%. What was the net price?

40. A merchant bought 850 yards of muslin at 6¢ a yard, less 18%. How much did it all cost?

41. A merchant bought 975 yards of lawn for \$146.25, less 20%. How much did he pay per yard?

42. A grocer bought 36 doz. cans of soup listed at \$1.75 a dozen. He was allowed a discount of 25%. What was the net price?

43. A dealer sold some boxes of soap, 60 cakes to the box, for \$2.25 a box. He allowed a discount of 20%. How much did he receive per cake?

44. A grocer bought 160 bags of flour, $12\frac{1}{4}$ lb. to the bag, listed at 4¢ a pound. He was allowed a discount of 15%. What did the flour cost him?

45. A grocer bought 24 doz. pound packages of macaroni for \$31.20, less 15%. He sold it for 15¢ per package. How much did he gain in all?

46. After the holidays a man bought some silver goods that had been marked \$280 but were now subject to a discount of $12\frac{1}{2}\%$. How much did they cost him?

265. Discount Series. The changes in cost of production from time to time, and the variation due to the credit of the buyer, and the quantity of goods purchased, give rise frequently to several discounts known as a discount series.

Goods that are selling at a discount of 20% may be produced at a lower cost owing, for example, to an unusually large cotton crop. An additional discount of 10% may then be allowed. Likewise a person whose credit is exceptional, who buys an especially large quantity, or who pays cash, may be entitled to a still further discount of 5%.

In a discount series the first denotes the discount from the list price, the second denotes the discount from the remainder, and so on.

Thus \$100 less 20%, 10%, and 5%, means that 20% is taken from \$100, leaving \$80; then 10% from \$80, leaving \$72; then 5% from \$72, leaving \$68.40. The order is, however, immaterial.

When there is a fraction of a cent in a remainder, the seller usually counts it a whole cent in his favor.

The regular discount is quoted first, and then the special discounts.

Advantages of a discount series are that it saves reprinting elaborate catalogues, allows of easy change, enables the dealer easily to offer different discounts to different customers, and allows for several options to purchasers.

If a bill is due at once it usually bears the words "Terms cash." If due in 60 days without discount, it may bear the marks "N/60" or "Net 60 da." If due in 60 days, but with a special discount of 3% if paid in 10 days, it may read "3/10, N/60," or "3% 10 da., net 60 da."

EXERCISE 147

Given these list prices and discounts, find the net prices:

1. \$225, 20% and 6%.
2. \$368, 25% and 5%.
3. \$480, $12\frac{1}{2}$% and 3%.
4. \$350, 15% and 4%.
5. \$560, 25% and 10%.
6. \$270, $33\frac{1}{3}$% and 10%.
7. \$480, 25%, 10%, 5%.
8. \$560, 25%, 20%, 3%.
9. \$729, $33\frac{1}{3}$%, 10%, 10%.
10. \$280, 25%, 10%, 10%.
11. \$2760, $33\frac{1}{3}$%, 20%, 10%.
12. \$5550, $33\frac{1}{3}$%, 25%, 15%.

13. How much will 240 ft. of iron pipe cost if listed at \$1.50 a foot, a discount of 40% and 10% being allowed?

14. How much will a gross of jars of mixed pickles cost at \$6 a dozen, a discount of 20% and 10% being allowed?

15. The list price of 2-pound cartons of prunes is \$24 per 100. A discount of 20% and 10% is allowed. What is the net price of 300 cartons?

16. A book dealer can buy an encyclopedia for \$60 less 20% and 10%, or for the same price less 15% and 15%. Which is the better for him, and how much better?

17. A grocer buys 100 dozen jars of table salt listed at \$1.50 a dozen, a discount of 25% and 8% being allowed. If he sells the salt at 12¢ a jar, how much does he gain?

18. A dry goods dealer buys 960 yd. of silk listed at \$1.50 a yard, a discount of $33\frac{1}{3}\%$ and 5% being allowed. If he sells it at \$1.50 a yard, how much does he gain?

19. A dealer buys 36 doz. pocket knives listed at \$8.40 a dozen, a discount of 25% and 10% being allowed. If he sells them at 75¢ apiece, how much does he gain?

20. A furniture dealer buys 16 bedroom sets listed at \$36 each, a discount of $16\frac{2}{3}\%$ and 10% being allowed. If he sells the furniture at \$36 a set, how much does he gain?

21. A hardware dealer bought a shipment of stoves. After deducting a discount of $33\frac{1}{3}\%$ and 25%, and paying \$9.75 freight and \$2.50 cartage, the stoves cost him \$252.25. What was the list price of the stoves?

22. A dry goods dealer bought a shipment of cloth. After deducting a discount of 25% and 10%, and paying \$8.50 for freight and cartage, the cloth cost him \$410. What was the list price?

266. Bills Discounted. The following is a common form of a bill of goods purchased from a wholesale dealer.

Chicago, March 15, 19.....

Mr. R. D. Scanlan, Springfield, Ill.

Bought of **Lane & Company, Jewelers**
1072 Wabash Avenue

Terms *20%, 10%*

Feb. 10.	*8 doz. spoons No. 487 @ $14.75*	*$118.00*
	7 doz. plated forks No. 511 @ $4.20	*29.40*
		$147.40
	Less 20%, 10%	*41.27*
		$106.13

EXERCISE 148

Make out bills for the following:

1. 36 doz. files at $6.25; 3 doz. saws at $17.50. Discounts 30%, 20%.

2. 960 yd. silk at $1.40; 840 yd. velvet at $1.60. Discounts 25%, 10%.

3. 4 doz. pr. hinges at $4.60; 25 doz. table knives at $8.40. Discounts 20%, 10%.

4. 12 doz. locks at $4.20; 6 doz. mortise locks at $4.60. Discounts 25%, 8%.

5. 360 yd. taffeta at 98¢; 4 gross pompons at $144; 3 doz. pieces braid at $20.40. Discounts 10%, 5%.

6. 60 lb. brads at 6¢; 200 bolts at $4.80 per C; 6 doz. knives at $7.20. Discounts 40%, 20%, 10%.

7. 480 yd. silk at $1.60; 640 yd. lawn at 22¢; 960 yd. taffeta at 96¢. Discounts 10%, 5%, 5%.

267. Single Discount equal to a Series. It is often convenient to find a single discount equal to a series.

For example, find the single discount equal to the discount series 20%, 10%, and 10%.

Let the list price be represented by 100 % of the list price.
Then deducting 20% of this, or 20 % of the list price,
we have 80 % of the list price.
Deducting 10% of this, or 8 % of the list price,
we have 72 % of the list price.
Deducting 10% of this, or 7.2% of the list price,
we have 64.8% of the list price.

Then 100% − 64.8% = 35.2%, the single discount.

EXERCISE 149

Find the single discount equal to the discount series:

1. 10%, 8%.
2. 15%, 6%.
3. 18%, 3%.
4. 14%, 9%.
5. 20%, 10%.
6. 30%, 10%.
7. 25%, $12\frac{1}{2}$%.
8. 10%, 10%, 10%.
9. 25%, 15%, 10%.
10. 25%, 20%, 10%.
11. 30%, 20%, 10%.
12. $33\frac{1}{3}$%, 10%, 5%.
13. $33\frac{1}{3}$%, 10%, 10%.
14. $16\frac{2}{3}$%, 10%, 10%.
15. 20%, $12\frac{1}{2}$%, 10%.
16. 25%, $12\frac{1}{2}$%, 10%.

17. Compare the single discount equal to the series 10% and 5%, with the sum of 10% and 5% minus the product of 10% and 5%.

18. By the method suggested in Ex. 17, find the single discount equal to the series 20%, 5%.

19. How does the single discount equal to the discount series 20%, 10%, and 5%, compare with that equal to the discount series 10%, 5%, and 20%? Prove it.

268. Gain and Loss. In business transactions gain and loss is often computed as a certain per cent of the cost of the property sold. The following examples may be taken as typical problems:

(1) For how much must a merchant sell goods that cost \$275, so as to gain 20% ?

If he gains 20% of \$275 he gains $\frac{1}{5}$ of \$275, or \$55. Therefore he must sell the goods for \$275 + \$55, or \$330.

(2) If goods marked \$75 are sold at a bargain sale for 15% off, at what price are they sold ?

If the goods are sold at 15% off the marked price, they are sold for \$75 less 15% of \$75, or \$75 − \$11.25, or \$63.75.

(3) If some goods are damaged so that they are sold for \$120, or 25% below cost, how much did they cost ?

100% of the cost = the cost.
25% of the cost = the loss.
75% of the cost = the selling price = \$120.
1% of the cost = $\frac{1}{75}$ of \$120.
100% of the cost = $\frac{100}{75}$ (or $\frac{4}{3}$) of \$120, or \$160.

(4) A dealer sold a hat that cost \$2.40 so as to gain 25%. The selling price was 20% less than the marked price. What was the marked price ?

He gained 25% (or $\frac{1}{4}$) of \$2.40, or \$0.60. Therefore the selling price was \$2.40 + \$0.60, or \$3.

100% of the marked price = the marked price.
20% of the marked price = amount taken off.
80% of the marked price = the selling price = \$3.
1% of the marked price = $\frac{1}{80}$ of \$3.
100% of the marked price = $\frac{100}{80}$ (or $\frac{5}{4}$) of \$3, or \$3.75.

We may prove this result or any of the preceding results by working backward. Here \$3.75 less 20% (or $\frac{1}{5}$) of \$3.75 = \$3.75 − \$0.75 = \$3, the selling price. Also \$2.40 + 25% of \$2.40 = \$3, the selling price, and the two results agree.

EXERCISE 150

For how much must a merchant sell goods that he bought at the price here given, so as to gain the per cent specified?

1. \$25, 12%.
2. \$36, 14%.
3. \$65, 15%.
4. \$72, 18%.
5. \$85, 25%.
6. \$175, $8\frac{1}{2}$%.
7. \$190, $9\frac{1}{4}$%.
8. \$250, 12%.
9. \$375, 15%.
10. \$735, 18%.
11. \$172.80, $12\frac{1}{2}$%.
12. \$373.20, $16\frac{2}{3}$%.
13. \$45,550, $17\frac{1}{2}$%.
14. \$27,475, 22%.
15. \$75,250, 25%.

If goods marked as here shown are sold at a bargain sale for the given per cent off, find the selling price:

16. \$60, 15%.
17. \$85, 18%.
18. \$72, 22%.
19. \$176, $6\frac{1}{4}$%.
20. \$712, $12\frac{1}{2}$%.
21. \$972, $16\frac{2}{3}$%.
22. \$225.40, 25%.
23. \$27,250, 20%.
24. \$22,400, $17\frac{1}{2}$%.

If goods are sold for the sum specified, which is the given per cent below cost, find the cost:

25. \$71.25, 5%.
26. \$72.16, 12%.
27. \$58.48, 14%.
28. \$140, 20%.
29. \$207.20, 30%.
30. \$382.50, 15%.
31. \$226.20, 13%.
32. \$265.60, 17%.
33. \$358.80, 22%.

34. A merchant sold some goods that cost \$175 so as to gain 20%. The selling price was 20% less than the marked price. What was the marked price?

35. A dealer sold some furniture for which he had paid \$240, so as to gain $12\frac{1}{2}$%. The selling price was 25% less than the marked price. What was the marked price?

36. A merchant sold some goods at 10% below cost, this being 20% below his marked price. The goods cost \$300. What was the marked price?

Given the cost and rate of gain as follows, find the amount of gain:

37. \$275, 15%.
38. \$365, 12½%.
39. \$4723, 14%.
40. \$3965, 16%.
41. \$7287, 22%.
42. \$275.40, 15%.
43. \$346.50, 16%.
44. \$492.75, 18%.
45. \$575.50, 24%.
46. \$812.50, 16%.
47. \$4275.50, 8%.
48. \$2936.40, 12%.
49. \$3275.75, 16%.
50. \$4206.03, 33⅓%.
51. \$3742.20, 16⅔%

Given the cost and rate of loss as follows, find the amount of loss:

52. \$275, 6%.
53. \$345, 4%.
54. \$463, 8%.
55. \$362.50, 4%.
56. \$427.50, 8%.
57. \$732.25, 12%.
58. \$4725.25, 8%.
59. \$2632.50, 12%.
60. \$4170.00, 15%.

61. How much is 33⅓% profit on goods costing \$637.20?

62. A dealer sold a sewing machine at a gain of 25%. It cost him \$31.40. Find the selling price.

63. A man bought a horse for \$150 and sold it at a loss of 8%. Find the selling price.

64. A merchant bought 85 yd. of silk at \$1.25 a yard and sold it at a loss of 8%. How much was his loss?

65. A jeweler bought a dozen clocks at \$4.75 apiece and sold them at an advance of 16%. How much was his gain?

66. A man bought a horse for \$140 and sold it for \$161. What was his gain per cent?

67. A dealer bought a piano for \$320 and sold it for \$400. What was his gain per cent?

68. A grocer bought 500 lb. of coffee at 32¢ a pound. He sold it at an advance of 6¼%. How much did he gain?

69. A dealer sold a piano for \$280, thereby losing 12½%. How much did the piano cost him?

70. A wholesale dealer paid \$2.50 per barrel for apples. He sold them to a retailer at a gain of 8%, and the retailer sold them at a gain of $16\frac{2}{3}$%. What was the retail price?

71. A clothier marked a suit of clothes that cost him \$13.20, so as to gain 40%. He discounted the marked price $16\frac{2}{3}$%. What was the actual selling price?

72. A man bought a horse for \$150 and a carriage for \$120. He sold them, gaining 8% on the horse and losing 12% on the carriage. Find his net gain or loss.

73. A farmer received \$160 for some produce. It passed through the hands of a shipper, a commission merchant, and a retailer, who gained respectively $12\frac{1}{2}$%, $3\frac{1}{3}$%, and 20%. What was the retail price?

74. Some goods cost a merchant \$100. At what price must he mark them so that he can take off 20% and still make a profit of 20%?

75. Some goods cost a merchant \$240. At what price must he mark them so that he can take off 20% and still make a profit of 20%?

76. At what per cent above cost must a merchant mark his goods so that he can take off 20% and still make a profit of 20%?

77. At what per cent above cost must a merchant mark his goods so that he can take off 25% and still make a profit of 25%?

78. A man sold two houses for \$9600 each. On one he gained 20% and on the other he lost 20%. Did he gain or lose on the whole transaction, and how much?

79. A man sold two houses for \$9600 each. He sold one at a profit of 25% and the other at a loss of 25%. Did he gain or lose on the whole transaction, and how much?

269. Commission or Brokerage. A sum paid by one person to another for transacting business is called *commission* or *brokerage.*

270. Principal and Agent. The person for whom another transacts business is called the *principal.* The person who transacts the business is called the *agent.*

271. Commission Merchant and Broker. An agent who has actual possession of the goods of his principal is called a *commission merchant.* An agent who makes sales or purchases for his principal without having actual possession of the goods is called a *broker.*

Commission and brokerage are usually reckoned at a certain per cent of the money involved in the transaction. If goods are bought, the commission is a certain per cent of the cost; if goods are sold, the commission is a certain per cent of the amount received; if a collection is made, the commission is a certain per cent of the amount collected.

Brokerage is often a fixed sum per bushel (as of wheat purchased), a fixed sum per 100 bales (as of cotton), a fixed sum per tierce (as of lard), and so on.

In all problems, unless otherwise directed, discard any fraction of a cent less than $\frac{1}{2}$ in a final result; when a fraction of a cent is $\frac{1}{2}$ or more, count it as a full cent.

EXERCISE 151

Find the commissions on sales or purchases of the following amounts, at the rates specified:

1. \$275, 2%.
2. \$345, 3%.
3. \$675, 4%.
4. \$725, 5%.
5. \$875, 6%.
6. \$2750, $1\frac{1}{2}$%.
7. \$3275, $2\frac{1}{2}$%.
8. \$2760, $3\frac{1}{2}$%.
9. \$4175, $4\frac{1}{2}$%.
10. \$6375, $3\frac{1}{3}$%.
11. \$8000, $\frac{1}{8}$%.
12. \$7500, $\frac{1}{4}$%.
13. \$6200, $\frac{1}{2}$%.
14. \$3520, $\frac{3}{8}$%.
15. \$4800, $1\frac{1}{8}$%.

Find the net amounts remaining after deducting commissions on sales as follows, at the rates specified:

16. \$4800, $\frac{1}{8}\%$.
17. \$6400, $\frac{1}{4}\%$.
18. \$3500, $\frac{1}{5}\%$.
19. \$2800, $2\frac{1}{4}\%$.
20. \$5600, $2\frac{1}{8}\%$.
21. \$6400, $3\frac{1}{2}\%$.
22. \$275.50, 4%.
23. \$612.50, 8%.
24. \$721.50, 6%.

25. What is the commission at $3\frac{1}{2}\%$ on a sale of potatoes to the amount of \$1270?

26. A broker bought 750 tierces of lard for a customer, charging $2\frac{1}{2}¢$ a tierce. A tierce being 340 lb., how many pounds did he buy, and what was the brokerage?

27. Lard selling at \$7.03 per tierce, how much will 1250 tierces cost? How many pounds are there? What is the brokerage on the purchase at $2\frac{1}{2}¢$ per tierce?

28. A commission merchant sells for a factory 6900 lb. of cheese at 12¢ a pound, charging 5% commission. What sum should he remit to his principal?

29. A commission merchant sold 1700 dozen eggs for a dealer, charging him 5% on the sale, and remitting \$339.15. At what price per dozen did he sell the eggs?

30. A commission merchant sold 160 tubs of butter, 50 lb. to the tub, for \$1920, charging 5% commission. What did the commission amount to per pound?

31. A broker buys 1750 bags of coffee for a customer, at 8.43¢ per pound. There being 130 lb. to the bag and the broker charging \$10 per 250 bags, how many pounds are bought, and what is the total cost, including the brokerage?

32. A broker bought 700 bales of cotton for a customer, at 12.1¢ per pound. Taking the bales as weighing 500 lb. each, and the brokerage at \$5 per hundred bales, find the number of pounds purchased and the total cost including the brokerage.

272. Insurance. An agreement by which, for a lawful consideration, a company agrees to pay an indemnity for loss by fire or other specified cause is called *insurance.*

273. Policy. The written contract of insurance is called a *policy.* The sum agreed to be paid by the terms of an insurance policy is called the *face* of the policy. The consideration, or sum paid for insurance, is called the *premium.*

There are several kinds of insurance. There are companies that insure people's lives (life insurance); that insure against loss by fire (fire insurance), by accidents to ships (marine insurance), by accidents to people (accident insurance), by burglary (burglary insurance), and so on. Those who insure are often called *underwriters.*

274. Fire Insurance. Insurance against loss or damage by fire is called *fire insurance.*

In fire insurance the premium is sometimes stated as a certain sum on $100, and sometimes as a certain rate per cent.

EXERCISE 152

Find the premiums on these policies at the rates specified:

1. $1500, $1\frac{1}{4}\%$.
2. $1500, $1\frac{1}{2}\%$.
3. $1750, $1\frac{3}{4}\%$.
4. $2250, $2\frac{1}{4}\%$.
5. $3500, $1\frac{1}{2}\%$.
6. $2600, $2\frac{3}{4}\%$.
7. $3800, $2\frac{1}{2}\%$.
8. $4500, $1\frac{3}{4}\%$.
9. $12,500, 1%.
10. $13,500, 2%.
11. $15,000, $1\frac{1}{4}\%$.
12. $12,750, $1\frac{3}{4}\%$.
13. $15,250, $1\frac{1}{2}\%$.
14. $16,500, $2\frac{1}{4}\%$.
15. $17,250, $2\frac{1}{2}\%$.
16. $18,750, $2\frac{3}{4}\%$.
17. $5200, 2.1%.
18. $6500, 1.2%.
19. $3750, 1.6%.
20. $2550, 1.8%.
21. $2750, 2.2%.
22. $3500, 2.5%.
23. $4800, 2.8%.
24. $5500, 3.25%.

Find the faces of policies, given the premiums and rates:

25. $65, 1%.
26. $48, 2%.
27. $19, $\frac{1}{2}\%$.
28. $24, $\frac{3}{4}\%$.
29. $19.75, 1%.
30. $41.25, $2\frac{1}{2}\%$.

Find the rates, given the faces of policies and premiums:

31. \$1800, \$18. 33. \$2100, \$31.50. 35. \$2800, \$56.
32. \$1650, \$41.25. 34. \$3200, \$56. 36. \$3500, \$52.50.

Find the premiums on the following policies, given the premiums per \$100:

37. \$2750, \$1.25. 39. \$4800, \$1.75. 41. \$15,000, \$2.50.
38. \$3250, \$1.50. 40. \$5600, \$2.25. 42. \$27,500, \$2.75.

43. At \$1.10, what is the premium on a \$3750 policy?

44. At \$1.30, what is the premium on a \$5500 policy?

45. A building worth \$4500 is insured for $\frac{2}{3}$ of its value at $1\frac{3}{4}\%$. What is the premium?

46. The premium for insuring some property at \$1.75 is \$96.25. What is the face of the policy?

47. Some property worth \$2700 is insured for $\frac{2}{3}$ of its value, the annual premium being \$27. What is the rate?

48. Merchandise worth \$3600 is insured for 80% of its value at \$1.50 on \$100. What is the premium?

49. If a 3-year policy for \$2500 costs \$60, what is the rate of premium per year?

50. A building is insured at 60¢ per \$100, the annual premium being \$14.40. If the face of the policy is 80% of the value of the building, what is the value?

51. A dealer insured his stock of goods for 80% of its value at \$1.60 per \$100. The premium was \$57.60. What was the value of the stock?

52. A factory is insured for \$7000 in one company and \$5000 in another. The factory is partly destroyed by fire, the insurance adjusters fixing the damage at \$8000. How much should each company pay?

275. Life Insurance. Insurance whereby some person known as the beneficiary receives a certain sum on the death of the insured, or when the insured reaches a certain age, is called *life insurance.*

The beneficiary may be the estate of the insured, or one or more persons named in the policy.

There are various forms of life insurance, but for the purposes of this arithmetic and of explaining the general principles of the subject, the standard form known as *ordinary life* is sufficient.

The rate is always given as the cost of $1000 worth of insurance, the rate $27.75 meaning that the annual premium on a policy for $1000 is $27.75. This subject is not, therefore, an application of percentage, but is properly considered here in connection with the other insurance problems.

EXERCISE 153

Find the premiums on these policies at the given rates:

1. $5000, $21.75.
2. $7500, $22.65.
3. $3500, $28.45.
4. $4500, $26.36.
5. $7000, $27.75.
6. $4500, $22.35.
7. $3500, $24.65.
8. $12,000, $25.62.
9. $14,000, $26.38.
10. $15,500, $29.46.

11. If a man takes out a policy for $7500 at $26.36 per thousand, how much premium will he pay in 20 payments?

12. A man takes out a $4500 policy in one company at $24.30 per thousand and a $3500 policy in another company at $24.75 per thousand. What do the two premiums together amount to in a year?

13. A man takes out a $6000 policy at $26.65 per thousand. He dies just after eight annual premiums have been paid. How much does the beneficiary receive? How much does he receive in excess of the premiums paid?

276. Taxes. Money levied by a government for its support and for public purposes is called a *tax.*

277. Kinds of Taxes. There are two general kinds of taxes, direct and indirect. A *direct* tax is a tax levied on a person, or on the value of his property or business. An *indirect* tax is a tax levied in other ways, as on goods imported into the country, on tobacco, or on alcoholic liquors.

The expenses of the United States government are, in general, met by indirect taxes, including duties on imports, and excise duties on tobacco and liquor. There is also some revenue from the sale of public lands.

The expenses of the several states, counties, cities, towns, and villages are, in general, met by direct taxes, although some of these corporations levy a few indirect taxes.

278. Poll Tax. A tax levied upon a person is called a *poll tax.*

The old word for head was *poll;* hence this name. In some places the poll tax is no longer collected.

279. Property Tax. A tax levied upon property is called a *property tax.*

This may be upon land and buildings (real estate), or upon money, securities, and movable property generally (personal property).

280. Licenses. A permit to engage in some specified act or to enjoy some special privilege is called a *license.*

For example, there are licenses to run automobiles, to own dogs, to sell liquor, and so on, all of which are forms of taxes.

281. Assessing Taxes. Taxes are usually assessed against the individual by officers called *assessors.* They decide how much each person's property is worth and the rate of tax.

The law provides for methods of reviewing the action of assessors in case it is not considered fair.

The *tax rate* is sometimes expressed as a rate per cent, but more often as the number of mills on a dollar.

282. Collecting Taxes. Taxes are collected by an officer known as the *collector*.

The collector sometimes receives a fixed salary, sometimes a commission on the total amount of tax collected, and sometimes a commission on the net amount paid by each taxpayer. In the latter case this is added to the tax, thus: If the tax to be paid is \$66.56 and the collector's commission is 1%, the total tax plus the commission is \$66.56 + \$0.67, or \$67.23.

For example, a tax of \$43,600 is to be raised by a town which contains 1200 polls, taxed at \$1 each, and which has taxable property assessed at \$3,200,000. The town receives from the state \$7200 as its share of the railroad tax, and other corporation taxes collected by the state. Find the rate of taxation, and the tax paid by Mr. Brown, if his property is assessed at \$5960 and he pays for 1 poll.

Total tax		\$43,600
Deduct poll taxes	\$1200	
Amount received from state	7200	
Total to be deducted	\$8400	8,400
Net property tax		\$35,200

Tax on \$1 = \$35,200 ÷ 3,200,000 = \$0.011.
That is, the tax is 11 mills on \$1, or \$11 on \$1000, or 1.1%.
Therefore Mr. Brown will pay as follows:

Property tax, 1.1% of \$5960,	\$65.56
Poll tax	1.
Property and poll tax	\$66.56

In practice, in a case like that of the above example, the assessors would probably make the tax rate 12 mills, or 1.2%, in order to provide for contingencies such as reduction of assessments and abatement of taxes.

If the assessors make the tax rate a little too high there will be a little larger balance in the treasury at the end of the year.

In the examples in Exercise 154 consider the collector's commission only when told to do so.

283. Tax Table. Assessors usually prepare a table like the following, which is arranged for $5\frac{1}{2}$ mills on a dollar:

TAX TABLE. RATE $5\frac{1}{2}$ MILLS ON $1

	0	1	2	3	4	5	6	7	8	9
0	0000	0055	0110	0165	0220	0275	0330	0385	0440	0495
1	0550	0605	0660	0715	0770	0825	0880	0935	0990	1045
2	1100	1155	1210	1265	1320	1375	1430	1485	1540	1595
3	1650	1705	1760	1815	1870	1925	1980	2035	2090	2145
4	2200	2255	2310	2365	2420	2475	2530	2585	2640	2695
5	2750	2805	2860	2915	2970	3025	3080	3135	3190	3245
6	3300	3355	3410	3465	3520	3575	3630	3685	3740	3795
7	3850	3905	3960	4015	4070	4125	4180	4235	4290	4345
8	4400	4455	4510	4565	4620	4675	4730	4785	4840	4895
9	4950	5005	5060	5115	5170	5225	5280	5335	5390	5445

Here the first figure of the number of dollars assessed is given at the left, and the second one at the top. Thus, at $5\frac{1}{2}$ mills on $1, the tax on $100 is $0.55; on $140, $0.77; on $2500, $13.75.

For example, find by the above table the tax of James Johnson, who pays no poll tax and has his property assessed at $9765, the collector's commission being 1%.

Tax on $9700 (to the right of 9, and under 7)	$53.35
Tax on $65 (to the right of 6, and under 5)	.36 (.3575)
Property tax	$53.71
Collector's commission, 1% of $53.71,	.54
Total tax	$54.25

EXERCISE 154

Using the above tax table, find the tax on the following amounts, adding the collector's commission of 1%:

1. $250.	4. $4000.	7. $7550.	10. $11,500.
2. $850.	5. $3650.	8. $9000.	11. $11,250.
3. $900.	6. $4550.	9. $12,000.	12. $39,750.

Make out a tax table for the tax rate of 9 mills on a dollar. Then find the tax on the following amounts, adding the collector's commission of 1% :

13. $350.	17. $5000.	21. $5550.	25. $12,500.
14. $550.	18. $3550.	22. $8000.	26. $26,500.
15. $700.	19. $4750.	23. $13,000.	27. $48,750.
16. $3600.	20. $3750.	24. $35,000.	28. $82,350.

29. The tax rate is 11 mills. How much tax must be paid on property assessed at $4500 ?

30. The tax rate is 13 mills. How much tax must be paid on property assessed at $12,500 ?

31. If a man's tax is $97.50 and the rate is 13 mills on a dollar, what is the assessed valuation of his property ?

32. If a man's property is assessed at $6500, and his tax is $78, what is the rate of tax ?

33. If the assessed valuation of the property in a town is $486,500, what must be the tax rate to raise $5838 ?

34. The tax rate is 14 mills. A man pays a tax of $54.64, of which $2 is poll tax. What is the assessed valuation of his property ?

35. If the state tax is 1 mill, the county tax $3\frac{1}{2}$ mills, and the town tax 4 mills, how much tax must a man pay on a house assessed at $5000 ?

36. A man died, leaving $17,500, of which $17,000 was willed to his children and $500 to his brother. Under the laws of his state there is an inheritance tax of 1% to be paid by children and 4% by brothers or sisters. How much tax must be paid on these bequests ?

37. If $49,760.73 is $101\frac{1}{4}\%$ of the net amount of a tax, what is $1\frac{1}{4}\%$ of the net amount ?

284. Indirect Taxes. Indirect taxes have already been defined in § 277. One of the most important is the tax on goods imported into this country. This tax is paid to the United States government.

285. Duties or Customs. Taxes levied by the government on imported goods are called *duties* or *customs.*

Duties are collected at customhouses situated in large cities or in important seaports or places on the border between different countries as between the United States and Canada or Mexico.

286. Kinds of Duty. There are two kinds of duty, specific and ad valorem. A *specific duty* is a definite sum levied on each unit by which an article is measured or weighed. An *ad valorem* duty is a certain per cent on the value of the goods.

287. Tariff. A schedule of duties on different articles, such as are imposed by our Congress, is called a *tariff.*

Goods on which duty is not levied are placed on the *free list.*

EXERCISE 155

Find the ad valorem duty on each of the following invoices of goods at the rate specified:

1. \$475, 30%.	6. \$1500, 25%.	11. \$12,450, 20%.
2. \$625, 35%.	7. \$1650, 30%.	12. \$42,375, 30%.
3. \$750, 40%.	8. \$2700, 35%.	13. \$29,362, 35%.
4. \$835, 45%.	9. \$3650, 40%.	14. \$32,275, 50%.
5. \$975, 60%.	10. \$7575, 50%.	15. \$75,295, 60%.

16. Find the specific duty on a gross of articles at 17¢ each.

17. Find the specific duty on 275 articles at \$1.25 each.

18. The duty paid on a certain invoice of goods was \$4500, at 60% ad valorem. What was the valuation on which the duty was reckoned?

19. Find the specific duty on 756 lb. of a certain material at 55¢ a pound.

20. An importer pays $952.50 ad valorem duty on goods valued at $6350. What is the rate of duty?

21. Ad valorem duty is levied on goods valued at $2750, making the total cost $3850. What is the rate of duty?

22. If the duty at 45% is $3723.75, what is the valuation on which the duty is reckoned?

23. If a duty of $3237.50 ad valorem is levied on goods valued at $9250, what is the rate of duty?

24. The duty on certain farm products is $4 a ton. If a dealer imports $82\frac{3}{4}$ tons, what is the duty?

25. Some goods cost the importer $10,875, including the duty of 45%. How much was the duty?

26. Some goods cost the importer $3712.50, including the duty of 35%. What was the original cost?

27. The cost of certain goods plus the duty of $33\frac{1}{3}$% was $8800. Required to find the original cost.

28. The duty on certain goods is 22¢ a pound, plus 30% ad valorem. If the goods weigh 275 lb. and are worth $360, what is the total duty?

29. The duty on a certain product is 60¢ a pound, plus 45% ad valorem. If the product weighs 168 lb. and is worth $380, what is the total duty?

30. A man imported some goods on which he paid a specific duty of $385. If the duty is 22¢ a pound, how many pounds did he import?

31. A man imported 475 lb. of goods on which he paid a specific duty of 6¢ a pound and an ad valorem duty of 25%. The total duty was $216. What was the valuation on which the duty was reckoned?

EXERCISE 156

Review Problems

Find separately the following per cents:

1. 0.25%, $2\frac{1}{2}$%, 25%, and 250% of 4884.
2. $0.12\frac{1}{2}$%, $1\frac{1}{4}$%, $12\frac{1}{2}$%, and 125% of 9768.
3. $\frac{1}{3}$%, $3\frac{1}{3}$%, $33\frac{1}{3}$%, $333\frac{1}{3}$%, and $333.33\frac{1}{3}$% of 7365.
4. $\frac{1}{6}$%, $1\frac{2}{3}$%, $16\frac{2}{3}$%, $166\frac{2}{3}$%, and $166.66\frac{2}{3}$% of 9732.
5. $\frac{2}{3}$%, $6\frac{2}{3}$%, $66\frac{2}{3}$%, $666\frac{2}{3}$%, and $666.66\frac{2}{3}$% of 7596.
6. $\frac{3}{4}$%, $7\frac{1}{2}$%, 75%, 750%, and 7500% of 13,748.
7. $\frac{3}{8}$%, $3\frac{3}{4}$%, $37\frac{1}{2}$%, 375%, and 3750% of 11,168.
8. $\frac{5}{8}$%, $6\frac{1}{4}$%, $62\frac{1}{2}$%, 625%, and 6250% of 12,344.
9. $\frac{7}{8}$%, $8\frac{3}{4}$%, $87\frac{1}{2}$%, 875%, and 8750% of 37,072.
10. A man bought a horse for \$125 and sold it for \$147.50. What per cent did he gain?
11. A man bought a house for \$6800 and sold it for \$5780. What per cent did he lose?
12. A real estate agent sold a lot for \$1775 and received \$71 commission. What per cent did he receive?
13. A real estate agent sold a farm, his commission at 3% being \$145.50. What was the selling price?
14. A man sold a piece of land for \$1320, thereby gaining 10% on the cost. How much did he gain?
15. An agent received \$14 for selling some property on a commission of 2%. What was the selling price?
16. A dealer sold some fruit for $12\frac{1}{2}$% less than it cost, thereby losing \$70. What was his selling price?
17. An agent sold some property on a commission of 5%. After deducting his commission from the selling price he remitted \$2137.50. What was the selling price?

18. The premium for insuring a house at $1\frac{1}{4}\%$ was \$75. What was the face of the policy?

19. A man paid \$42 for insuring his house at $\frac{3}{4}\%$ on 80% of its value. What was its value?

20. A man sold his farm for \$975 less than it cost him, which was a loss of 15%. Find the cost.

21. A merchant bought hats at \$2.75 each, and sold them at \$3.50. Find his gain per cent.

22. A merchant's assets amount to \$4063.50, and this sum will pay 54% of his debts. How much does he owe?

23. A dealer sells cloth at \$2.53 a yard, gaining 15%. What per cent would he gain by selling it at \$2.64 a yard?

24. What was the amount of a bill upon which a collector received \$22.75 for collecting, his commission being $3\frac{1}{2}\%$?

25. A merchant sold cloth at \$1.40 a yard and gained 12%. What did the cloth cost him?

26. A merchant sold cloth that cost him \$1.23 a yard and gained $33\frac{1}{3}\%$. At what price did he sell it?

27. What must be the marked price of a hat that cost \$2.50, so as to make 12% after discounting the price 30%?

28. A man bought a farm at \$62.50 an acre and sold it at \$75 an acre. What per cent did he gain?

29. A workman's wages, amounting to \$1050 a year, was increased 10%. After a year it was decreased 10%. What was it then?

30. Two men own a mill together. The first paid 45% of the purchase price and the second paid the balance, \$9350. What was the cost of the mill?

31. A bank clerk started work at a salary of \$900 a year. The second year his salary was raised 20%, the third year 25%, and the fourth year $33\frac{1}{3}\%$. What was his salary then?

32. A merchant sold a hat for \$3.30, thereby gaining 25%. Find the cost.

33. A man sold a horse at a gain at \$32.50, which was $33\frac{1}{3}\%$ of the cost. Find the selling price.

34. At what price must you mark a hat, costing \$1.20, so as to discount the price 25% and still make 20% ?

35. The premium for insuring a \$7000 house for 80% of its value was \$70. What was the rate of insurance ?

36. Two men own a boat. One paid \$132 and the other paid 45% of the cost. How much did the boat cost ?

37. What is the per cent of attendance in a school of 35 pupils, if there are 21 days of absence in 5 school weeks ?

38. Two men are in partnership. One owns $47\frac{1}{2}\%$ of the business. If the business is worth \$5000, what is the value of the share of each ?

39. A merchant's sales in October amounted to \$1552.50, which was 9% of the sales for the year. What were the total sales for the year ?

40. A dry goods dealer sold a piece of cloth at \$1.40 a yard, thereby gaining $16\frac{2}{3}\%$. At what price should he have sold it in order to gain 20% ?

41. A dry goods dealer sold a piece of cloth and gained $12\frac{1}{2}\%$. If he had sold it at 90¢ a yard, he would have gained 25%. What was his selling price ?

42. An agent collected rents, charging 3% on all sums collected. The rents in one month amounted to \$675. How much did he remit to the owner ?

43. A workman in a factory received \$1350 the first year. The second year his wages was advanced 10%, and the third year 10%. The fourth year it was reduced 10%. How much did he then receive ?

44. The discount from the list price of a sewing machine, at $33\frac{1}{3}\%$, was \$13.50. What was the list price?

45. A man paid \$60 for insuring his \$6000 house at 80% of its value. Find the rate of insurance.

46. An agent collected rents, charging 3% on all sums collected. He remitted to the owner \$271.60. How much rent did he collect?

47. A dry goods dealer sold a piece of cloth at 72¢ a yard, thereby gaining 20%. What would have been his gain per cent if he had sold it at 75¢ a yard?

48. A man was unsuccessful in business and failed, owing \$2376.50. He was able to pay his creditors only 40¢ on a dollar. What was the total loss to his creditors?

49. A man paid \$5500 for a farm. He rents it for \$425 a year and pays \$60.50 for taxes, \$25 for insurance, and \$37 for repairs. What is the per cent of net income?

50. A merchant bought a bill of goods, and paid \$27.49 for freight, \$15 for insurance, and \$4.75 for cartage. These expenditures represented 4% of the purchase price. What was the purchase price? What was the total cost?

51. A sugar factory pays \$4.60 per ton for all beets testing 14% sugar, and 22¢ per ton for each additional 1%. A farmer grows $13\frac{1}{2}$ tons of beets per acre, from which 2.025 tons of sugar are extracted. How much per acre does he receive for his crop?

52. Bordeaux mixture for plant diseases consists of 4 lb. of unslaked lime, 6 lb. of copper sulphate, and 25 gal. of water. Taking the weight of a gallon of water as 8.4 lb., what per cent of the weight of the mixture is the weight of the lime? what per cent is the weight of the copper sulphate? what per cent is the weight of the water?

288. Interest. Money paid for the use of money is called *interest.*

289. Principal. Money loaned on which interest is paid is called the *principal.*

The sum of the principal and interest is called the *amount.*

290. Rate of Interest. The rate per cent of the principal paid in interest for one year is called the *rate of interest.*

291. Illustrative Problems.

(1) Find the interest on \$840 at 5% for 2 yr.

Interest for 1 yr. = 5% of \$840 = \$42.
Interest for 2 yr. = 2 × \$42 = \$84.

(2) Find the interest on \$750 at $5\frac{1}{2}\%$ for $3\frac{1}{2}$ yr.

Interest for 1 yr. = $5\frac{1}{2}\%$ of \$750 = \$41.25.
Interest for $3\frac{1}{2}$ yr. = $3\frac{1}{2}$ × \$41.25 = \$144.38.

Therefore, *to find the interest, given the principal, rate, and time, multiply the principal by the rate, and the product by the number that expresses the time in years.*

EXERCISE 157

Given the principal, rate, and time, find the interest:

1. \$275, 5%, 2 yr.
2. \$350, 4%, 3 yr.
3. \$460, 6%, 2 yr.
4. \$525, 5%, 3 yr.
5. \$265, 4%, 2 yr.
6. \$425, 5%, 3 yr.
7. \$635, 6%, 4 yr.
8. \$875, 6%, 2 yr.
9. \$925, 5%, 5 yr.
10. \$975, 4%, 2 yr.
11. \$975, $5\frac{1}{2}\%$, 2 yr.
12. \$725, $4\frac{1}{2}\%$, 3 yr.
13. \$835, $3\frac{1}{2}\%$, 2 yr.
14. \$425.50, 6%, $2\frac{1}{2}$ yr.
15. \$375.75, 6%, $3\frac{1}{2}$ yr.
16. \$420.50, 5%, $2\frac{1}{4}$ yr.
17. \$2450, 4%, $3\frac{3}{4}$ yr.
18. \$3575, 5%, $2\frac{3}{4}$ yr.

292. Interest when Time is expressed in Days. The case considered in § 291 involved time expressed in years. If the time involves days we may first reduce this to a fraction of a year and proceed as before.

For example, find the interest on \$240 for 2 yr. 3 mo. 10 da. at 6%.

$$10 \text{ da.} = \tfrac{1}{3} \text{ mo.}$$

$$3\tfrac{1}{3} \text{ mo.} = \tfrac{10}{3} \text{ of } \tfrac{1}{12} \text{ yr.} = \tfrac{5}{18} \text{ yr.}$$

$$2\tfrac{5}{18} \times 6\% \text{ of } \$240 = \frac{41 \times 6 \times \$240}{18 \times 100} = \$32.80.$$

It is the custom in computing interest, when tables (see § 297) are not used, to take 30 da. to the month, which means 360 da. to the year.

EXERCISE 158

Given the principal, rate, and time, find the interest:

1. \$250, 5%, 4 mo. 10 da.
2. \$520, 6%, 2 yr. 9 mo. 25 da.
3. \$760, 5%, 4 yr. 6 mo. 15 da.
4. \$875, 6%, 3 yr. 7 mo. 16 da.
5. \$1250, 5%, 1 yr. 10 mo. 5 da.
6. \$2500, 5½%, 2 yr. 10 mo. 7 da.
7. \$2275, 5½%, 3 yr. 11 mo. 2 da.
8. \$3525, 4½%, 1 yr. 11 mo. 3 da.

Given the principal, rate, and time, find the amount:

9. \$175, 6%, 6 mo.
10. \$225, 6%, 8 mo. 3 da.
11. \$625, 5%, 4 mo. 3 da.
12. \$1250, 5%, 2 yr. 3 mo. 8 da.
13. \$2275, 5½%, 3 yr. 10 mo. 14 da.

293. Problems in Interest. Of the four elements used in computing interest,—principal, rate, time, and interest (or amount, which is only the sum of the principal and interest),—we can find any one if the other three are known.

For, since the interest is the product of three factors (principal, rate, and time), if it is divided by the product of any two of these factors the quotient is the third.

The following examples will illustrate:

(1) What principal will, in 3 yr. 6 mo., yield \$52.50 interest at 6%?

Since the interest for $3\frac{1}{2}$ yr. is \$52.50,
therefore the interest for 1 yr. is \$52.50 ÷ $3\frac{1}{2}$, or \$15.

Since 6% of the principal is \$15,
therefore 1% of the principal is \$2.50,
and 100% of the principal is \$250.

(2) What principal will amount to \$570 in 2 yr. 4 mo. at 6%?

The amount of \$1 for 2 yr. 4 mo. at 6% is \$1.14.

Since \$1.14 is the amount of \$1,

\$570 is the amount of $\frac{\$570}{1.14}$, or \$500.

(3) In what time will \$540 yield \$109.80 interest at 6%?

The interest on \$540 at 6% for 1 yr. is \$32.40.

Since \$32.40 is the interest for 1 yr.,

\$109.80 is the interest for $\frac{109.80}{32.40}$ yr., or $3\frac{7}{18}$ yr.

$3\frac{7}{18}$ yr. = 3 yr. $4\frac{2}{3}$ mo. = 3 yr. 4 mo. 20 da.

(4) At what rate will \$600 yield \$81 interest in 3 yr.?

The interest on \$600 at 1% for 3 yr. is \$18.

Since \$18 is the interest at 1%,

\$81 is the interest at $\frac{81}{18}$%, or $4\frac{1}{2}$%.

EXERCISE 159

Find the principal that will yield the following interest:

1. \$20, in 2 yr. 8 mo., at 6%.
2. \$93$\frac{1}{3}$, in 3 yr. 4 mo., at 5%.
3. \$9.59, in 3 yr. 6 mo., at 4%.
4. \$40.47, in 1 yr. 7 mo., at 6%.
5. \$3.64, in 5 mo. 18 da., at 6%.
6. \$45.90, in 4 yr. 3 mo., at 4$\frac{1}{2}$%.

Find the principal that will amount to the following:

7. \$482.40, in 3 yr. 4 mo., at 6%.
8. \$1151.41, in 2 yr. 4 mo., at 4$\frac{1}{2}$%.
9. \$1210.50, in 2 yr. 5 mo., at 5%.

Find the time, given the principal, rate, and interest:

10. \$840, 5%, \$147.
11. \$625, 4%, \$56.25
12. \$350, 6%, \$49.
13. \$1250, 6%, \$187.50.

Find the rate, given the principal, time, and interest:

14. \$330, 1 yr. 2 mo., \$19.25.
15. \$68.50, 3 yr. 6 mo., \$9.59.
16. \$87.50, 2 yr. 8 mo. 24 da., \$14.35.
17. How long will it take \$1 to yield \$1 interest at 6% ?
18. How long will it take \$1 to double itself at 5% ?
19. What sum must be put at interest at 6% so that it shall amount to \$1000 in 16$\frac{2}{3}$ yr. ?
20. What sum must be put at interest at 5% so that it shall amount to \$1000 in 20 yr. ?

294. Difference between Dates. The time between two dates may be found as here shown. For example, required to find the time from April 16, 1909, to February 5, 1911.

yr.	mo.	da.
1911	2	5
1909	4	16
1	9	19

Since February is the second month of the year, and April the fourth, the work is arranged as here shown. We then subtract as with other compound numbers; $5 - 16$ is impossible, but $35 - 16 = 19$. Proceeding in the usual manner, the result is 1 yr. 9 mo. 19 da. In such cases allow 30 da. to the month.

EXERCISE 160

Find the difference in time between these dates:

1. Oct. 2, 1909, Nov. 15, 1911.
2. Sept. 5, 1909, Dec. 25, 1912.
3. Sept. 7, 1909, July 4, 1913.
4. Dec. 10, 1910, Feb. 5, 1914.
5. Aug. 20, 1909, Jan. 6, 1912.
6. What is the interest on \$375 from July 4, 1910, to May 1, 1913, at 6% ?
7. What is the interest on \$650 from June 20, 1910, to April 5, 1913, at 5% ?
8. What is the interest on \$825 from May 15, 1910, to Feb. 10, 1914, at $4\frac{1}{2}$% ?
9. The interest on \$360 from April 5, 1910, to Feb. 3, 1912, is \$39.48. What is the rate ?
10. The interest at 5% on \$480 from March 7, 1910, to a certain other date is \$68. Required to find the second date.
11. Upon what sum is the interest at 6%, from July 4, 1910, to Oct. 4, 1913, equal to \$39 ?
12. What is the amount of \$650 at interest at 6% from Oct. 27, 1911, to April 12, 1914 ?

295. Six Per Cent Method. There is a very convenient method of computing the interest at 6%, depending upon the fact that if the rate for 1 yr. is 6%, the rate for 2 mo. is $\frac{1}{6}$ of 6%, or 1%.

Required the interest on \$720 for 5 mo. 17 da. at 6%.

The interest for	2 mo. is 1% of \$720 =	\$7.20
The interest for	2 mo. more =	7.20
The interest for	1 mo. is $\frac{1}{2}$ of \$7.20 =	3.60
The interest for	15 da. is $\frac{1}{2}$ of \$3.60 =	1.80
The interest for	1 da. is $\frac{1}{15}$ of \$1.80 =	.12
The interest for	1 da. more =	.12
The interest for	5 mo. 17 da. =	\$20.04

Therefore, *to find the interest for 60 da., move the decimal point two places to the left; for 6 da., three places to the left. From these results find the interest for the required time.*

In most cases in which months and days are involved, this is the simplest of all methods. Thus, to find the interest on \$720 for 10 mo. 10 da. at 6%, we have \$7.20 for 2 mo., 5 × \$7.20, or \$36, for 10 mo., $\frac{1}{6}$ of \$7.20, or \$1.20, for 10 da. Hence the total interest is \$37.20.

Of all the forms of the six per cent method this is the most easily applied. It is particularly convenient because most bank notes run for 30, 60, or 90 days.

EXERCISE 161

Find the interest at 6% on the following:

1. \$240, 4 mo. 6 da.
2. \$360, 3 mo. 3 da.
3. \$750, 3 mo. 3 da.
4. \$475, 3 mo. 6 da.
5. \$280, 8 mo. 6 da.
6. \$520, 5 mo. 12 da.
7. \$450, 2 mo. 15 da.
8. \$585, 2 mo. 20 da.
9. \$1275, 2 mo. 6 da.
10. \$3200, 3 mo. 7 da.
11. \$4500, 4 mo. 9 da.
12. \$3500, 2 mo. 15 da.
13. \$2700, 2 mo. 19 da.
14. \$3350, 8 mo. 17 da.
15. \$4950, 10 mo. 10 da.
16. \$6250, 10 mo. 18 da.

296. Six Per Cent Method for Other Rates. We may use this method to find interest at the other rates in common use.

Thus, since 5 is $\frac{1}{6}$ less than 6, the interest at 5% is $\frac{1}{6}$ less than the interest at 6%.

Similarly, the interest at $4\frac{1}{2}$% is $\frac{1}{4}$ less than the interest at 6%, the interest at 4% is $\frac{1}{3}$ less than the interest at 6%, and so on.

Find the interest on \$1240 for 3 mo. 3 da. at 5%.

Interest for 2 mo. at 6%	\$12.40
Interest for 1 mo. at 6%	6.20
Interest for 3 da. ($\frac{1}{10}$ mo.) at 6%	.62
Total interest at 6%	\$19.22
Deduct $\frac{1}{6}$	3.20
Interest at 5%	\$16.02

EXERCISE 162

Find the interest at 5% on the following:

1. \$780, 3 mo. 6 da.
2. \$860, 2 mo. 12 da.
3. \$940, 1 mo. 18 da.
4. \$1200, 2 mo. 3 da.
5. \$1500, 3 mo. 3 da.
6. \$2200, 4 mo. 15 da.

Find the interest at 4% on the following:

7. \$250, 2 mo.
8. \$320, 2 mo. 6 da.
9. \$420, 3 mo. 12 da.
10. \$1250, 4 mo.
11. \$1575, 3 mo. 5 da.
12. \$2500, 2 mo. 12 da.

Find the interest at $5\frac{1}{2}$% on the following:

13. \$275, 2 mo.
14. \$575, 4 mo. 3 da.
15. \$1575, 3 mo.
16. \$2550, 3 mo. 8 da.

Find the interest at $4\frac{1}{2}$% on the following:

17. \$150, 2 mo.
18. \$975, 3 mo. 10 da.
19. \$1650, 3 mo.
20. \$1850, 4 mo. 5 da.

297. Table of Difference of Time. Bankers always find the difference in time between two dates, and also the interest on any sum, by means of tables. The following is one form of the table of difference of time:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April . . .	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December . .	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of any month to the corresponding day of any month within a year is found opposite the first month and under the second. For example, from August 7 to December 7 is 122 da.; from November 16 to April 23 is 158 da., since to April 16 it is 151 da., and to April 23 it is 7 da. more, or 158 da. in all.

EXERCISE 163

Find from the table the number of days from:

1. May 7 to Aug. 7.
2. Apr. 6 to Sept. 6.
3. Jan. 5 to Aug. 20.
4. Feb. 7 to Sept. 15.
5. July 4 to Dec. 25.
6. Apr. 15 to Oct. 20.
7. May 20 to Sept. 1.
8. Aug. 15 to Oct. 2.
9. Nov. 16 to July 17.
10. Sept. 20 to Mar. 25.
11. Oct. 21 to Feb. 20.
12. Nov. 25 to Apr. 15.
13. Aug. 27 to Feb. 5.
14. Aug. 19 to Jan. 7.
15. Oct. 7 to Apr. 16.
16. May 13 to Jan. 3.

298. Use of the Table of Time. In computing interest, the table of time (§ 297) is commonly used. We then allow 30 da. to the month and 360 da. to the year.

For example, required the interest on $420 from July 7 to Nov. 4 at 6%.

The difference of time (§ 297) is 123 da. − 3 da., or 120 da.

Interest at 6% for 60 da. (§ 295) = $4.20.

Interest at 6% for 120 da. = 2 × $4.20 = $8.40.

EXERCISE 164

Using the table on p. 294, find the interest at 6% on the following sums from the first date to the second:

1. $250, Apr. 6, Nov. 6.
2. $320, May 4, Nov. 10.
3. $220, Jan. 5, Sept. 2.
4. $625, June 20, Dec. 17.
5. $250, Aug. 16, Feb. 18.
6. $375, Sept. 25, Mar. 24.
7. $525, Oct. 17, Apr. 4.
8. $475, Feb. 4, May 5.
9. $240, Mar. 6, Nov. 1.
10. $560, Apr. 15, Jan. 10.
11. $355, May 1, July 5.
12. $750, Aug. 5, Oct. 1.
13. $800, Sept. 5, Jan. 2.
14. $725, May 7, Oct. 3.

Also on the following at 5%:

15. $125, Sept. 5, Feb. 10.
16. $326, Oct. 10, Apr. 3.
17. $450, July 7, Jan. 20.
18. $540, Aug. 15, Nov. 27.
19. $315, July 24, Apr. 3.
20. $430, Oct. 7, Jan. 16.
21. $585, Nov. 24, Mar. 7.
22. $695, Jan. 16, Aug. 10.
23. $843, Apr. 22, Oct. 7.
24. $925, Mar. 9, June 15.

Also on the following at 4%:

25. $250, Jan. 1, Sept. 15.
26. $375, Mar. 4, Nov. 18.
27. $840, May 7, Aug. 13.
28. $765, Apr. 12, Oct. 4.

299. Interest Tables. The most common interest tables are based on 360 days to the year, although the government and some banks use tables based on 365 days as being more fair, although yielding less interest. The following shows part of a page of an interest table:

3 MONTHS, 6%

Total Days	1000	2000	3000	4000	5000	6000	7000	8000	9000	Days over 3 Months
90	15.00	30.00	45.00	60.00	75.00	90.00	105.00	120.00	135.00	0
91	15.17	30.33	45.50	60.67	75.83	91.00	106.17	121.33	136.50	1
92	15.33	30.67	46.00	61.33	76.67	92.00	107.33	122.67	138.00	2
93	15.50	31.00	46.50	62.00	77.50	93.00	108.50	124.00	139.50	3
94	15.67	31.33	47.00	62.67	78.33	94.00	109.67	125.33	141.00	4
95	15.83	31.67	47.50	63.33	79.17	95.00	110.83	126.67	142.50	5
. . .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. . . .

For example, find from the table the interest on \$2750 for 94 da. at 6%.

Interest on \$2000 = \$31.33
Interest on 700 = 10.97
Interest on 50 = .78
Interest on \$2750 = \$43.08

EXERCISE 165

Using the above table, find the interest at 6%:

1. \$2670, 91 da.
2. \$3170, 93 da.
3. \$4250, 90 da.
4. \$2575, 92 da.
5. \$3250, 95 da.
6. \$4050, 92 da.
7. \$5075, 91 da.
8. \$3775, 93 da.
9. \$15,750, 90 da.

Using the tables of § 297, § 299, find the interest at 6%:

10. \$2300, from Mar. 7 to June 7.
11. \$3550, from Nov. 6 to Feb. 9.
12. \$4250, from Dec. 15 to Mar. 18.

300. Exact Interest. When interest is computed for exact days, and at 365 days to the year, it is called *exact interest.*

For example, required the exact interest on $1241 from Dec. 21 to Feb. 9 at 5%.

The time, found by the table (§ 297), is 50 da. Then $\frac{50}{365} \times 5\%$ of $1241 is required. Canceling,

$$\frac{\overset{73}{\cancel{50}} \times \cancel{5} \times \overset{17}{\cancel{\$1241}}}{\underset{73}{\cancel{365}} \times \underset{2}{\cancel{100}}} = \$8.50.$$

The common interest is a little greater than the exact interest.

EXERCISE 166

Find the exact interest on the following:

1. $250 for 75 da. at 6%.
2. $275 for 85 da. at 5%.
3. $325 from Apr. 2 to July 1 at 5%.
4. $350 from May 7 to Sept. 9 at 5%.
5. $425 from Jan. 6 to Aug. 4 at 6%.
6. $460 from Feb. 9 to July 9 at 6%.
7. $500 from May 15 to Dec. 16 at 6%.
8. $219 from Aug. 14 to Jan. 11 at 5%.
9. $292 from Sept. 27 to Jan. 25 at 5%.
10. $438 from July 15 to June 15 at 5%.
11. $730 from Aug. 17 to Apr. 14 at 4%.
12. $1460 from May 23 to Mar. 19 at 4%.
13. $1533 from Feb. 1 to Dec. 28 at $5\frac{1}{2}$%.
14. $7300 from Mar. 3 to Oct. 4 at $5\frac{1}{2}$%.
15. $3650 from May 23 to Jan. 23 at $4\frac{1}{2}$%.
16. $1825 from Nov. 19 to June 17 at $4\frac{1}{2}$%.
17. $5475 from Dec. 21 to June 19 at $3\frac{1}{2}$%.

301. Periodic Interest. Simple interest on the principal and on each installment of interest from the time each interest is due until settlement is called *periodic interest.*

Periodic interest in which the installments are due annually is called *annual interest.*

For example, find the interest on \$400 for 4 yr. 7 mo. 20 da. at 5%, payable annually.

Simple interest on \$400 for 4 yr. 7 mo. 20 da. at 5% = \$92.78.

Interest due the 1st year, \$20, draws interest 3 yr. 7 mo. 20 da.
Interest due the 2d year, \$20, draws interest 2 yr. 7 mo. 20 da.
Interest due the 3d year, \$20, draws interest 1 yr. 7 mo. 20 da.
Interest due the 4th year, \$20, draws interest 7 mo. 20 da.
Interest upon the interest = interest on \$20 for 8 yr. 6 mo. 20 da.

Interest on \$20 for 8 yr. 6 mo. 20 da. at 5% = \$8.56.
The total interest = \$92.78 + \$8.56 = \$101.34.

The law does not generally allow periodic interest, but in some parts of the country it is common to give notes not only for the principal but for each interest installment, all bearing interest.

EXERCISE 167

Find the interest upon the following, payable annually:

1. \$500, 4 yr., 6%
2. \$250, 3 yr., 6%.
3. \$475, 5 yr., 5%.
4. \$750, 6 yr., 5%.
5. \$850, $2\frac{1}{2}$ yr., 6%.
6. \$725, $3\frac{1}{2}$ yr., 5%.
7. \$650, $2\frac{1}{4}$ yr., 5%.
8. \$900, $2\frac{1}{3}$ yr., 5%.
9. \$350, 3 yr. 6 mo. 6 da., 6%.
10. \$1250, 4 yr. 8 mo. 10 da., $5\frac{1}{2}$%.

Find the amount of the following, interest due annually but remaining unpaid until settlement:

11. \$140, 3 yr., 6%.
12. \$375, 4 yr., 6%.
13. \$1250, $2\frac{3}{4}$ yr., 6%.
14. \$3500, $2\frac{2}{3}$ yr., $5\frac{1}{2}$%.

302. Compound Interest. When interest is added to the principal and becomes a part of it at specified intervals, the total interest is called *compound interest.*

In compound interest, the interest draws interest every time it is added to the principal; in periodic interest it draws interest only once.

Interest is compounded annually, semiannually, or quarterly, according to agreement, but it cannot be enforced legally.

Most savings banks allow compound interest at a low rate, the interest usually being compounded quarterly or semiannually. Since the interest is deposited when it becomes due, the problem is merely one of computing simple interest each time.

For example, find the amount of \$60 at compound interest for 3 yr. at 5%, the interest being compounded annually. Find also the compound interest.

Interest for 1st year = 5% of \$60 = \$3. Amount = \$63.
Interest for 2d year = 5% of \$63 = \$3.15. Amount = \$66.15.
Interest for 3d year = 5% of \$66.15 = \$3.31. Amount = \$69.46.

\$69.46 (compound amt.) − \$60 (principal) = \$9.46 (compound int.).

If the time were $3\frac{1}{2}$ years, we should find the amount at compound interest for 3 years and add to it the interest upon itself for six months.

If the interest is 5% compounded semiannually for 3 years, it amounts to the same as $2\frac{1}{2}\%$ compounded annually for 6 years.

EXERCISE 168

Find the amount at 4%, interest compounded annually:

1. \$250, 3 yr.
2. \$375, 4 yr.
3. \$425, 3 yr.
4. \$650, 5 yr.
5. \$875, 6 yr.
6. \$1250, $2\frac{1}{2}$ yr.
7. \$1500, $1\frac{1}{2}$ yr.
8. \$2650, $3\frac{1}{2}$ yr.
9. \$2575, $4\frac{1}{4}$ yr.

Find the amount at 3%, compounded semiannually:

10. \$100, 4 yr.
11. \$250, 3 yr.
12. \$575, 2 yr.
13. \$650, 3 yr.
14. \$980, 5 yr.
15. \$1350, $2\frac{1}{2}$ yr.
16. \$2500, $3\frac{1}{2}$ yr.
17. \$1650, $4\frac{1}{2}$ yr.
18. \$1700, $4\frac{1}{2}$ yr.

303. Promissory Notes. A written promise to pay a specified sum of money on demand or at a specified time is called a *promissory note*, or simply a *note*.

304. Parties to a Note. The person who signs a note is called the *maker*. The person to whom a note is payable is called the *payee*. The lawful owner of a note is called the *holder*.

A note may be made payable to *bearer*, in which case it is lawful to pay it to any one who presents it to the maker.

It may be made payable to some person named in the note, in which case he or his legal representatives must present it. Such a note is said to be *non-negotiable*.

It may be made payable to the payee, *or order*, in which case the payee may transfer it to another person by writing his own name on the back, thus *indorsing* the note. Such a note is said to be, like one payable to bearer, *negotiable*.

An indorser of a note is responsible for its payment if the maker does not pay it, unless he writes the words "without recourse" above his signature.

305. Form of a Note. A common form of note is as follows:

$375.25. *Boston, Mass., June 1, 1910*

Six months after date, I promise to pay Frank Mills, or order, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received.

James Lang

Here the maker is James Lang, and the payee is Frank Mills.

If the payee writes across the back,

Payable to the order of John B. Brown,
Frank Mills

he becomes an indorser, and John B. Brown becomes the holder.

If Frank Mills writes simply his name across the back, the note becomes payable to bearer, that is, to any one who may have it.

This note is due on Dec. 1, 1910, which is called the *day of maturity*.

The words "with interest," without specifying the rate, carry the legal rate of the state in which the note is made.

EXERCISE 169

Find the day of maturity and amount due, having given:

	Face of Note	*Date of Note*	*Time*	*Rate of Interest*
1.	$350	July 7, 1911	60 da.	6%
2.	$570	Jan. 4, 1910	6 mo.	5%
3.	$460	Feb. 8, 1910	1 yr.	$4\frac{1}{2}$%
4.	$328.45	Mar. 7, 1910	3 mo.	6%
5.	$536.25	Apr. 15, 1911	90 da.	6%
6.	$275.50	May 16, 1910	30 da.	5%

Write notes for the following, and compute the interest:

7.	$480	June 6, 1910	6 mo.	6%
8.	$640	Nov. 9, 1911	4 mo.	5%
9.	$750	Jan. 7, 1910	3 mo.	6%

Find the amount due at maturity on the following notes:

10. $250. CHICAGO, ILL., Oct. 20, 1910

Sixty days after date, I promise to pay George A. Wood, or order, Two Hundred Fifty Dollars, with interest at 5%. Value received. JAMES SINCLAIR

11. $1250. SPRINGFIELD, MO., Mar. 7, 1910

Ninety days after date, I promise to pay James Goodwin, or order, Twelve Hundred Fifty Dollars, with interest at 6%. Value received. JAMES HILTON

12. $275.50. CLEVELAND, OHIO, Apr. 6, 1910

One year after date, I promise to pay Francis Warren, or order, Two Hundred Seventy-five and $\frac{50}{100}$ Dollars, with interest at 5%. Value received. ALFRED GIBSON

306. Partial Payments. Payments of a part of a note are called *partial payments.*

There are several methods of computing interest upon such notes, all recognizing the principle that interest should stop upon the amount paid.

307. Merchants' Rule. This is a convenient rule, although one not generally recognized in law, and is commonly used when settlement is made within a year. It is as follows:

(1) *Find the amount of the note at the date of settlement without regarding payments.*

(2) *Find the amount of each payment with interest from the date of payment to the date of settlement.*

(3) *Subtract the sum of the payment amounts from the total amount.*

For example, a man holds a note of $380, dated Aug. 16, 1913, on which the following payments are indorsed: Nov. 4, 1913, $120; Mar. 18, 1914, $70; May 1, 1914, $145. Settlement is made June 25, 1914. Find the balance due, interest at 6%.

Time from Aug. 16 to June 25 is 10 mo. 9 da.	Int. on \$1 = \$0.0515.
Time from Nov. 4 to June 25 is 7 mo. 21 da.	Int. on \$1 = \$0.0385.
Time from Mar. 18 to June 25 is 3 mo. 7 da.	Int. on \$1 = \$0.016⅙.
Time from May 1 to June 25 is 1 mo. 24 da.	Int. on \$1 = \$0.009.

Int. on \$380 = 380 × \$0.0515 = \$19.57.	Amount is	\$399.57
Int. on \$120 = 120 × \$0.0385 = \$4.62.	Amount is	\$124.62
Int. on \$70 = 70 × \$0.016⅙ = \$1.13.	Amount is	71.13
Int. on \$145 = 145 × \$0.009 = \$1.31.	Amount is	146.31
Total payment amounts		342.06
Balance due June 25, 1914		\$57.51

The Merchants' Rule is used by bankers and business men on notes running only a few months, or what are called *short-time notes.*

In the following examples find the difference in time in each case by subtraction of compound numbers, as above, and solve by the Merchants' Rule.

EXERCISE 170

Find the balance due:

1. Note of $230 given May 9, 1910, bearing interest at 6%. Payments: July 1, 1910, $40; Sept. 16, 1910, $75; Jan. 2, 1911, $50. Settlement Feb. 24, 1911.

2. Note of $375 given Mar. 3, 1911, bearing interest at 5%. Payments: May 8, 1911, $30; Aug. 16, 1911, $65; Nov. 20, 1911, $110. Settlement Jan. 1, 1912.

3. Note of $600 given May 16, 1912, bearing interest at 5%. Payments: June 1, $50; July 3, $25; July 10, $50. Settlement Nov. 12, 1912.

4. Note of $1000 given July 18, 1910, bearing interest at 6%. Payments: Sept. 10, $200; Dec. 20, $140; April 21, 1911, $350. Settlement June 2, 1911.

5. Note of $2760 given Oct. 1, 1909, bearing interest at 5%. Payments: Nov. 1, $250; Jan. 1, 1910, $300; Mar. 1, 1910, $750. Settlement July 16, 1910.

6. Note of $1000 given Aug. 4, 1909, bearing interest at 6%. Payments: Sept. 10, $200; Dec. 8, $200; Feb. 2, 1910, $250. Settlement May 3, 1910.

7. Note of $1500 given April 5, 1910, bearing interest at 5%. Payments: June 1, $500; Aug. 2, $300; Dec. 1, $200. Settlement Jan. 4, 1911.

8. Note of $1250 given July 5, 1911, bearing interest at $5\frac{1}{2}$%. Payments: Sept. 1, $275; Dec. 14, $325; Feb. 8, 1912, $280. Settlement May 17, 1912.

9. Note of $80 given Nov. 1, 1912, bearing interest at 6%. Payments: $10 on the first day of each month following. How much interest must be paid with the last payment?

308. United States Rule. The legal rule for partial payments in most of our states is that laid down by the Supreme Court of the United States. It is as follows:

(1) *Find the amount of the principal to the time when the payment, or sum of the payments, is equal to or greater than the interest.*

(2) *From this amount deduct the payment, or sum of the payments.*

(3) *Consider the remainder as a new principal, and proceed as before.*

For example, consider the following promissory note on which partial payments have been made:

$760. BOSTON, MASS., Oct. 16, 1911

Three months after date, I promise to pay Henry W. Street, or order, Seven Hundred Sixty Dollars, with interest at 6%. Value received. JAMES JOHNSON

Payments indorsed on the back of the note: Jan. 10, 1912, $200; Aug. 4, 1914, $75; May 1, 1915, $568. What is due Oct. 7, 1916?

Face of note	$760.00
Int., Oct. 16, 1911, to Jan. 10, 1912, 2 mo. 24 da.	10.64
Amount due Jan. 10, 1912	$770.64
Deduct payment	200.00
New principal	$570.64
Int., Jan. 10, 1912, to Aug. 4, 1914, 2 yr. 6 mo. 24 da., is $87.88, exceeding the payment.	
Int. Jan. 10, 1912, to May 1, 1915, 3 yr. 3 mo. 21 da.	113.27
Amount due May 1, 1915	$683.91
Deduct sum of payments, $75 + $568, . . .	643.00
New principal	$40.91
Int. May 1, 1915, to Oct. 7, 1916, 1 yr. 5 mo. 6 da.	3.52
Amount due Oct. 7, 1916	$44.43

EXERCISE 171

1. Note of $425. Date, Oct. 10, 1910. Interest at 6%. Payments: Mar. 14, 1911, $100; July 8, 1911, $85; Feb. 24, 1912, $125. What was due Oct. 10, 1912?

2. Note of $360. Date, Jan. 10, 1911. Interest at 6%. Payments: Sept. 16, 1911, $75; Oct. 8, 1912, $20; Dec. 2, 1912, $160. What was due Jan. 10, 1913?

3. Note of $350. Date, May 1, 1911. Interest at 6%. Payments: Dec. 25, 1911, $50; June 29, 1912, $5; Aug. 22, 1912, $15; June 4, 1913, $100. What was due Apr. 5, 1914?

4. $375. INDIANAPOLIS, IND., May 20, 1910

Six months after date, I promise to pay Oliver Higgins Three Hundred Seventy-five Dollars, with interest at 5%. Value received. WILLIAM JULIAN

Indorsements: Nov. 20, 1910, $25; Apr. 6, 1911, $75; Aug. 23, 1911, $140. What was due Mar. 12, 1912?

5. $250. OMAHA, NEBR., Aug. 24, 1910

One year after date, I promise to pay, for value received, Ira Hoyt, or order, Two Hundred Fifty Dollars, with interest at 6%. JAMES PATTERSON

Payments: Aug. 24, 1911, $100; Nov. 15, 1911, $40; Feb. 3, 1912, $80; May 10, 1912, $25. Paid in full, Apr. 16, 1913. What was the last payment?

6. $620. AUSTIN, TEX., Nov. 1, 1910

Six months after date, I promise to pay J. R. Roberts, or order, Six Hundred Twenty Dollars, with interest at 6%. Value received. W. J. CAMPBELL

Indorsements: Oct. 6, 1911, $61.07; Mar. 4, 1912, $89.03; Dec. 11, 1912, $107.77; July 23, 1913, $200.50. What was due Oct. 15, 1913?

EXERCISE 172

Review of Interest

1. Find the interest on \$725 for 187 da. at 5%.

2. A man has \$14,600 at interest at 6%. What is his average daily income from this investment?

3. Find the interest on \$475 from Aug. 14, 1913, to Feb. 2, 1915, at 4%.

4. Find the interest on \$7550 at 6% for 60 da.; for 30 da.; for 90 da.; for 93 da.

In Exs. 5, 6, 7, find the difference in time by § 294.

5. Find the interest on \$660 from Oct. 14 to the following July 1, at 5%.

6. Find the interest on \$720 from Aug. 17 to the following April 15, at $5\frac{1}{2}$%.

7. Find the amount of principal and interest on \$1440 from Sept. 20 to the following July 1, at 4%.

In Exs. 8, 9, 10, find the difference in time by the table in § 297.

8. Find the interest on \$960 from Nov. 16 to the following Aug. 27, at 6%.

9. Find the interest on \$7200 from July 20 to the following April 7, at 5%.

10. Find the amount of principal and interest on \$420 from June 7 to the following Dec. 1, at 6%.

11. Find the exact interest at 6% on \$650 from Sept. 21, 1911, to Jan. 1, 1912.

12. Find the exact interest at 5% on \$730 from Oct. 15 to the following Aug. 10.

13. Find the exact interest at 4% on \$9125 from Nov. 1 to the following Aug. 5.

14. Find the amount of principal and exact interest on \$1095 from Aug. 9 to the following June 17, at 6%.

15. The interest at 6% for two years on a certain sum is \$93. What is the sum at interest?

16. The interest at 5% for 1 yr. 6 mo. on a certain sum is \$18. What is the sum at interest?

17. The interest at 4% for 2 yr. 3 mo. on a certain sum is \$31.50. What is the sum at interest?

18. What is the sum that will amount, with interest at 6%, to \$517.50 in 2 yr. 6 mo.?

19. What is the sum that will amount, with interest at 5%, to \$793.75 in 1 yr. 2 mo.?

20. What is the sum that will amount, with interest at $5\frac{1}{2}$%, to \$1006.32 in 3 yr. 7 mo. 6 da.?

21. At $3\frac{1}{2}$% interest for a certain time, \$240 amounts to \$281.44. Find the time.

22. At $4\frac{1}{2}$% interest for a certain time, \$850 amounts to \$977.50. Find the time.

23. At $5\frac{1}{2}$% interest for a certain time, \$940 amounts to \$1022.72. Find the time.

24. At $3\frac{1}{2}$% interest for a certain time, \$480 amounts to \$610.90. Find the time.

25. How long will it take \$1200 to earn \$370.15 interest at $5\frac{1}{2}$%?

26. At what rate will \$1500 yield \$84 interest in 1 yr. 10 mo. 12 da.?

27. At what rate will \$5400 yield \$344.25 interest in 1 yr. 5 mo.?

28. At what rate will \$4200 yield \$784 interest in 3 yr. 1 mo. 10 da.?

EXERCISE 173

Problems without Numbers

1. How do you find the discount on the list price of some goods, given the rate of discount?

2. If there is a discount series, how do you find the net price of some goods?

3. If you know the net price of some goods and the single rate of discount, how do you find the list price?

4. If you know the net price of some goods and the two rates of discount in a series, how do you find the list price?

5. If a merchant buys a bill of goods and wishes to mark the goods so as to realize a certain per cent of profit, how does he find the marked price?

6. Explain how a merchant may sell goods at a certain per cent below the marked price and still make a profit.

7. If you know the marked price of some goods and the per cent of profit, how do you find the cost?

8. How do you find a commission merchant's commissions on the sale of produce sent him to sell?

9. If you know the rate and amount of a commission merchant's commission, how do you find the selling price?

10. If you know the price at which a commission merchant sells some goods, and the amount of his commission, how do you find the rate?

11. On what is an insurance premium reckoned? How is the rate stated?

12. If you know the face of an insurance policy, and the premium, how do you find the rate?

13. If you know the premium on an insurance policy, and the rate, how do you find the face of the policy?

14. What is a tax table, and how would you prepare one on the basis of 12 mills on a dollar?

15. If you know the assessed valuation of a man's property, and the rate of taxation, how do you find his tax?

16. If you know the assessed valuation of a man's property, and his tax, how do you find the rate?

17. If you know the tax paid by a man, and the rate of taxation, how do you find the assessed valuation of his property?

18. If you know the amount of tax to be raised and the total assessed valuation, how do you find the tax rate?

19. If you know the number of units of some imported goods, and the rate of specific duty, how do you find the amount of the duty?

20. If you know the value of some imported goods, and the rate of duty ad valorem, how do you find the amount of duty?

21. Given the principal, rate, and time, how do you find the interest?

22. Given the principal, rate, and interest, how do you find the time?

23. Given the principal, interest, and time, how do you find the rate?

24. Given the rate, time, and interest, how do you find the principal?

25. How would you proceed to compute an interest table, given the rate?

26. If you have an example in partial payments, all the dates within a single year, how do you proceed to solve?

27. If you have an example in partial payments extending beyond a year, how do you proceed to solve?

28. How do you express a given per cent as a common fraction?

29. If you are to find a certain fractional part of a number, how do you express this as a per cent?

30. If you have a certain rate per cent expressed with the per cent sign, how do you express it as a decimal fraction?

31. If you have a decimal fraction and wish to express the same number with a per cent sign, how do you proceed?

32. If you know what a given per cent of a number is, how do you find the number?

33. How do you find what per cent one number is of another? Illustrate.

34. A man's salary two years ago was increased a certain per cent. Last year it was decreased the same per cent. Is his salary at present greater than or less than it was two years ago?

35. A merchant marks his goods a certain per cent above cost. Owing to hard times, he sells them at the same per cent below the marked price. Does he sell them for more than or less than cost?

36. Which is the greater, the common interest on a certain sum or the exact interest? Why is this?

37. What are the common methods of finding the difference between two given dates? Which one is the more exact?

38. State the easiest way of finding the interest on a given sum of money for sixty days at 6%.

39. How would you find the compound interest on a given sum of money for a certain number of years at a given rate?

CHAPTER XIV

RATIO AND PROPORTION

309. Ratio. The relative magnitude of two numbers, as expressed by the fraction which has the first number for the numerator and the second for the denominator, is called the *ratio* of the first number to the second.

The ratio of 2 to 3, or $\frac{2}{3}$, is commonly written 2 : 3.

310. Antecedent and Consequent. The first term of a ratio is called the *antecedent*, and the second term the *consequent*.

For example, $\frac{5 \text{ ft.}}{6 \text{ ft.}} = \frac{5}{6} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{antecedent}}{\text{consequent}}$.

311. Ratios always Abstract. The two terms of a ratio always being like numbers, as in 2 : 3, \$4 : \$5, 6 ft. : 7 ft., the quotient is always abstract.

Therefore, *a ratio is always abstract, and its terms may be written as abstract numbers.*

Instead of writing $\frac{2 \text{ ft.}}{3 \text{ ft.}}$ we may therefore write simply $\frac{2}{3}$.

It is better to use the fractional form at first, and then to use the other form, 2 : 3.

312. Reduction of Ratios. Since a ratio may be expressed as a fraction, therefore

If the terms of a ratio are both multiplied by or both divided by the same number, the value of the ratio is not altered.

Thus, if both terms of the ratio $2\frac{1}{2} : 3\frac{1}{3}$ are multiplied by 6, the resulting ratio is 15 : 20, and these two ratios are equal.

Again, since $\frac{15}{20} = \frac{3}{4}$, the simplest expression for $2\frac{1}{2} : 3\frac{1}{3}$ is 3 : 4.

313. Separating a Number in a Given Ratio. Required to separate \$63 in the ratio of 3 to 4.

Since there must be \$3 in the first part to every \$4 in the second, there must be \$7 every time two such amounts are taken out. Therefore the first part must contain $\frac{3}{7}$ of the total and the second $\frac{4}{7}$.

But $\frac{3}{7}$ of \$63 is \$27, and $\frac{4}{7}$ of \$63 is \$36.

These are the results, because $\$27 : \$36 = \frac{27}{36} = \frac{3}{4}$.

EXERCISE 174

Express the following ratios in simplest form:

1. $\frac{34}{51}$.
2. $\frac{57}{76}$.
3. $\frac{46}{115}$.
4. $\frac{2}{\frac{3}{4}}$.
5. $\frac{\frac{4}{5}}{6}$.
6. $\frac{\frac{1}{2}}{\frac{7}{8}}$.
7. $\frac{13}{3\frac{1}{4}}$.
8. $\frac{2\frac{1}{2}}{10}$.
9. 58 : 145.
10. 93 : 124.

Find the missing term in each of the following ratios:

11. $\frac{(?)}{4} = 2$.
12. $\frac{6}{(?)} = 2$.
13. $\frac{(?)}{9} = \frac{1}{3}$.
14. $\frac{5}{(?)} = \frac{1}{4}$.
15. $\frac{(?)}{10} = \frac{3}{5}$.
16. $\frac{7}{(?)} = \frac{1}{5}$.
17. $(?) : 9 = 2$.
18. $7 : (?) = \frac{1}{2}$.

Separate the following numbers in the given ratios:

19. 28, 3 : 4.
20. 45, 4 : 5.
21. 260, 5 : 8.
22. $31\frac{1}{2}$, $2\frac{1}{2}$: 5.
23. \$72, 1 : 8.
24. \$96, 3 : 5.
25. \$78, 3 : 10.
26. 341, 15 : 16.
27. 492, $5\frac{1}{2}$: 15.

28. Two farmers pay \$15 together for some threshing, one having 350 bu. and the other 400 bu. of wheat. What is the share of each?

29. A man leaves an estate of \$13,470, giving \$1 to his widow for every \$2 to his children. How much did the children receive?

EXERCISE 175

Practical Uses of Ratio

1. The ratio of water to solid matter in apples is 423 : 77. How many pounds of water in 250 lb. of apples?

2. In a sack of rough rice, containing 162 lb., the ratio of hulls to cleaned rice is 1 : 4. How many pounds of cleaned rice in a sack?

3. A pineapple grower in Florida received in one year \$15 for plants to every \$7.50 for fruit. If his total receipts were \$3375, how much did he receive for the fruit?

4. In fertilizing land for growing peanuts a Virginia farmer used 30 lb. of cotton-seed meal to 13 lb. of cotton-hull ashes. How many pounds of each in 215 lb. of fertilizer?

5. Two partners invest respectively \$2750 and \$3500. They divide their profits, \$3750, in the ratio of their investments. What is the share of each?

6. A recipe for fertilizer for oats requires $12\frac{1}{2}$ lb. of nitrate of soda to 15 lb. of other ingredients. How many pounds of nitrate of soda in $412\frac{1}{2}$ lb. of fertilizer?

7. A recipe for grafting wax calls for 4 parts of resin to $1\frac{1}{2}$ parts of beeswax and 1 part of linseed oil. How many pounds of resin in 26 lb. of the wax?

8. Two rice planters expended \$2240 on an irrigating plant. One owned 80 acres of rice and the other 120 acres. If they paid in the ratio of their acreage, what was the share of each?

9. A New Jersey gardener estimated that the ratio of the cost of planting and caring for asparagus, to the cost of cutting and bunching, was 1 : 2. If the cost of both for his crop was \$84, what was the cost of each?

10. Two men rent a pasture for $115. One puts in 9 cattle and the other 14. How much rent should each pay ?

11. If the ratio of a man's income to his expenses is 7 : 6, and his expenses are $1800 a year, how much is his income ?

12. Taking the ratio of the diagonal to the side of a square as 7 : 5, how long is the diagonal of a square 35 ft. on a side ?

13. Taking the ratio of the circumference of a circle to the diameter as 22 : 7, how long is the circumference of a circle whose diameter is 4 ft. ?

14. If the ratio of cream to the rest of the milk of a certain quality is 1 : 5, how many pounds of cream in 375 lb. of milk of that quality ?

15. If the ratio of butter fat to the rest of the milk of a certain quality is 1 : 24, how many pounds of butter fat in 350 lb. of milk of that quality ?

16. If the ratio of solid matter to the rest of the milk of a certain quality is 33 : 217, how many pounds of solid matter in 750 lb. of milk of that quality ?

17. Two shippers charter a canal boat for $84 to carry some freight to a certain city. One ships 75 tons and the other 135 tons. How much should each pay?

18. Two partners having the same amount of money in their business divide the profits in the ratio of the time they devote to it. One gives 5 months in the year to the business and the other 7 months, and the profits are $5400. What is the share of each ?

19. The ratio of copper sulphate and unslaked lime to water in Bordeaux spraying mixture is 1 : 39. How much copper sulphate and unslaked lime in 160 lb. of the mixture ? How much of each, if equal amounts are used?

314. Proportion. An expression of equality between two ratios is called a *proportion.*

Thus $\frac{2}{3} = \frac{10}{15}$ and $2 : 3 = 10 : 15$ are proportions, and are read "2 is to 3 as 10 is to 15," or "the ratio of 2 to 3 equals the ratio of 10 to 15."

315. Terms of a Proportion. The first and last numbers of a proportion are called the *extremes*, and the two middle numbers are called the *means.* The extremes and means together are called the *terms* of the proportion.

Thus in the proportion $2 : 3 = 10 : 15$, the extremes are 2 and 15, and the means are 3 and 10.

316. Test of a Proportion. In the proportion $\frac{2}{3} = \frac{10}{15}$ we shall still have an equality if we multiply these equals by the same number. If we multiply by 3×15 we have

$$\frac{\cancel{3} \times 15 \times 2}{\cancel{3}} = \frac{3 \times \cancel{15} \times 10}{\cancel{15}}, \text{ or } 15 \times 2 = 3 \times 10.$$

Therefore, *in any proportion the product of the extremes equals the product of the means.*

317. Finding a Missing Term in a Proportion. The product of the extremes equals the product of the means (§ 316).

Therefore, *either extreme equals the product of the means divided by the other extreme;*

Either mean equals the product of the extremes divided by the other mean.

Hence if three terms are given, the fourth may be found.

(1) Find the missing term of the proportion $(?) : 6 = 9 : 27$.

Dividing the product of the means by the given extreme, we have

$$\frac{9 \times 6}{27} = 2.$$

(2) Find the missing term of the proportion $3 : 7 = (?) : 35$.

Dividing the product of the extremes by the given mean, we have

$$\frac{3 \times 35}{7} = 15.$$

EXERCISE 176

Find the missing term:

1. $\frac{(?)}{12} = \frac{24}{16}$.
2. $\frac{(?)}{15} = \frac{21}{35}$.
3. $(?) : 15 = 32 : 6$.
4. $(?) : 8 = 15 : 20$.
5. $15 : (?) = 10 : 8$.
6. $55 : (?) = 22 : 10$.
7. $12 : (?) = 18 : 10$.
8. $18 : (?) = 27 : 45$.
9. $12 : 18 = (?) : 27$.
10. $25 : 15 = (?) : 21$.
11. $21 : 49 = (?) : 63$.
12. $18 : 45 = (?) : 35$.
13. $10 : 45 = 14 : (?)$.
14. $24 : 54 = 28 : (?)$.
15. $35 : 49 = 45 : (?)$.
16. $\$5 : \$7 = (?) : 63$.
17. $\$9 : (?) = 189 : 231$.
18. $(?) : \$6 = 85 : 102$.
19. $\$15 : \$17 = 75 : (?)$.
20. $21 \text{ ft.} : 5 \text{ ft.} = 147 : (?)$
21. $35 \text{ ft.} : (?) = 315 : 99$.
22. $2\frac{1}{2} : 7 = 100 : (?)$.
23. $3\frac{1}{3} : 2\frac{1}{2} = (?) : 30$.
24. $12\frac{1}{2} : 16\frac{2}{3} = (?) : 80$.
25. $12\frac{1}{2} : (?) = 33\frac{1}{3} : 160$.
26. $37.5 : (?) = 250 : 80$.
27. $62.5 : 12.5 = 75 : (?)$.
28. $(?) : 66\frac{2}{3} = 90 : 600$.
29. $(?) : 87\frac{1}{2} = 33\frac{1}{3} : 2\frac{1}{3}$.
30. $(?) : 1.6 = 3.1416 : 1$.

31. Show that 29, 435, 17, and 255 are in proportion.

32. Show that 217, 2821, 311, and 3421 are not in proportion.

33. What number has the same ratio to 21 that 16 has to 35?

34. If the ratio of 7 to 16 equals the ratio of 63 to some number, what is that number?

35. Show that 401 and 5287 are the extremes and 311 and 6817 are the means of a proportion. Write the proportion in three different ways.

318. Directly Proportional. If one ratio equals another, the terms of the first are said to be *directly proportional* to those of the second.

Since 2 : 3 and 4 : 6 are equal ratios, 2 and 3 are said to be directly proportional to 4 and 6.

In general, the following are among the cases of directly proportional quantities :

Cost of goods is directly proportional to the quantity.
Weight of substances is directly proportional to the volume.
Income from labor is directly proportional to time given.
Distance traveled is directly proportional to time.

319. Inverse Ratio. The result of interchanging the terms of a ratio is called the *inverse ratio* of the given ratio.

Thus 3 : 2 is the inverse of 2 : 3, just as $\frac{3}{2}$ is the inverse of $\frac{2}{3}$.

320. Inversely Proportional. If one ratio equals the inverse of another, the terms of the first are said to be *inversely proportional* to those of the second.

Since 2 : 3 equals the inverse of 6 : 4, 2 and 3 are said to be inversely proportional to 6 and 4; that is, 2 : 3 = 4 : 6.

As an example of inverse proportion, the time required for some work is inversely proportional to the number of laborers. That is, if we take twice as many workmen, it will take half as long for the work.

321. Commercial Applications. Consider the following problem : If 5 T. of hay cost \$87.50, what will 21 T. cost ?

Since the ratio of cost is evidently equal to the ratio of quantity, we have

$$\$\ (?) : \$87.50 = 21\ \text{T.} : 5\ \text{T.}$$

Since we cannot multiply dollars by tons in getting the product of the means, and since we wish to preserve the dollars for the result, we write the second ratio as 21 : 5 (§ 311).

Then (§ 317), $\$\ (?) = \dfrac{21 \times \$87.50}{5} = \$367.50.$

We may write the proportion so that the unknown number will come in any term we wish, but the result is the same. Teachers formerly preferred to put it in the fourth term, and this may be done if desired.

If 8 men can do a piece of work in 9 days, how long will it take 12 men to do it?

Since it takes correspondingly less time as we increase the number of workmen, the ratio of the number of days equals the inverse ratio of the number of men.

Therefore (?) da. : 9 da. = 8 men : 12 men,

or (?) da. : 9 da. = 8 : 12 (§ 311).

Therefore $$(?)\text{ da.} = \frac{8 \times 9\text{ da.}}{12} = 6\text{ da.}$$

In proportion the words "at the same rate" are always understood.

EXERCISE 177

1. If 75 bu. of wheat cost \$72, what will 260 bu. cost?

2. If 36 yd. of cretonne cost \$9.72, what will 56 yd. cost?

3. If 42 yd. of ribbon cost \$9.24, what will 75 yd. cost?

4. If 18 yd. of braid cost \$2.25, what will 62 yd. cost?

5. If $2\frac{1}{2}$ M feet of lumber cost \$70, what will 7 M feet cost?

6. If a man earns \$16.50 in 6 da., how much will he earn in 25 da.?

7. A manufacturer used 71 yd. of braid on 4 sailor suits. How much would he need for 15 such suits?

8. A merchant bought 640 yd. of muslin for \$36.80. How much did 360 yd. cost?

9. For 10 aprons a manufacturer needs 15 yd. of goods. How many yards will he need for 75 aprons?

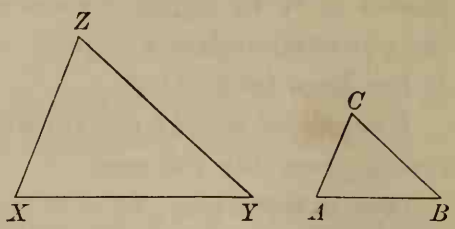
10. How long should it take 16 men to do some work that 52 men can do in 12 da.?

11. If 8 men require 75 da. to do a piece of work, how many men should it take to do it in 40 da.?

12. How far can 36 tons of freight be carried for the money paid for carrying 54 tons 144 miles?

322. Similar Figures. Figures that are of exactly the same shape are called *similar figures.*

For example, these two triangles are similar.



The corresponding sides of similar figures are proportional.

In these figures, $AB : BC = XY : YZ$. Also, $ZX : XY = CA : AB$.

This principle can be used in measuring the height of trees or buildings, as shown in Exs. 3–6 below. It is the principle involved in all map drawing.

EXERCISE 178

1. If $\frac{3}{4}$ in. on a map represents a distance of 375 mi., how many miles will $1\frac{1}{4}$ in. represent?

2. If a distance of 325 mi. is represented on a map by $1\frac{5}{8}$ in., how many inches will represent a distance of 340 mi.?

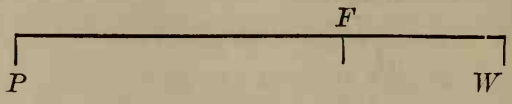
3. If a tree casts a shadow 50 ft. long when a post 4 ft. high casts a shadow 5 ft. long, how high is the tree?

4. If a spire casts a shadow 200 ft. long when a post $4\frac{1}{2}$ ft. high casts a shadow 5 ft. long, how high is the spire?

5. A class wishing to find the height of the school building, one of the boys stood a yard measure upright and another lay on the ground where he could just see the top of the building in line with the top of the stick. The distance from his eye to the foot of the stick was 4 ft. 6 in., and to the building 45 ft. How high was the building?

6. A man whose eye is 5 ft. 6 in. above the ground sights over the top of a 12-foot pole and just sees the top of a tree. If he is 7 ft. from the pole and 63 ft. from the tree, how high is the tree?

323. The Lever. If we have a lever as shown in this illustration, where F represents the fulcrum on which it rests, we may lift a weight placed at W by applying power enough at P. It has been found that



Power and weight are inversely proportional to their distances from the fulcrum.

Thus, if $WF = 8$ in., $PF = 16$ in., and weight $= 7$ lb., we have

$$\text{Power} : 7 \text{ lb.} = 8 \text{ in.} : 16 \text{ in.}$$

Therefore $$\text{Power} = \frac{8 \times 7 \text{ lb.}}{16} = 3\tfrac{1}{2} \text{ lb.}$$

That is, it would take only $3\frac{1}{2}$ lb. at P to lift 7 lb. at W.
Other simple forms of the lever may be considered if desired.

EXERCISE 179

Given the following distances from the fulcrum, find the power necessary to raise the weight specified:

	PF	*WF*	*Weight*		*PF*	*WF*	*Weight*
1.	2 ft.	6 in.	17 oz.	6.	6 ft.	1 ft.	360 lb.
2.	13 in.	5 in.	23 oz.	7.	5 ft.	6 in.	375 lb.
3.	17 in.	8 in.	12 lb.	8.	4 ft.	8 in.	900 lb.
4.	32 in.	9 in.	25 lb.	9.	5 ft.	4 in.	750 lb.
5.	48 in.	8 in.	100 lb.	10.	$5\frac{1}{2}$ ft.	6 in.	1 T.

Given the following distances from the fulcrum, find the greatest weight that can be raised by the power specified:

	PF	*WF*	*Power*		*PF*	*WF*	*Power*
11.	3 ft.	6 in.	10 lb.	14.	$4\frac{1}{2}$ ft.	6 in.	25 lb.
12.	4 ft.	4 in.	30 lb.	15.	$5\frac{1}{2}$ ft.	8 in.	125 lb.
13.	5 ft.	8 in.	100 lb.	16.	$6\frac{1}{2}$ ft.	8 in.	150 lb.

EXERCISE 180

Vocational Problems

1. If a shipment of 5100 lb. of cattle, live weight, sold for $225.42, what would 3500 lb. sell for at the same rate?

2. If a Louisiana farmer paid $75 for 3 T. 1500 lb. of cotton-seed meal for fodder, how much would he have to pay for 5000 lb.?

3. If in 225 lb. of milk there are 8.1 lb. of butter fat, how many pounds of milk will be required to produce 27 lb. of butter fat?

4. A farmer had 26 acres planted to potatoes. The crop from 7 acres amounted to 1260 bu. At the same rate how many bushels did he receive from the whole field?

5. To irrigate a farm at the rate of $\frac{1}{3}$ in. in depth every day requires the flow of 210 gal. an hour through a certain ditch. What flow would be necessary to irrigate it at the rate of 0.3 in. a day?

6. A Wisconsin farmer built a cylindrical silo 20 ft. in diameter and 32 ft. high, and found that it had a capacity of 200 tons. What must be the height of one of the same diameter that it may have a capacity of 125 tons?

7. If a certain kind of bird eats wild berries and also insects that harm fruit trees, and if out of 14 oz. of food eaten by this bird only $1\frac{2}{5}$ oz. is wild berries, what per cent of the bird's food is insects that would harm fruit trees?

8. An Arkansas farmer sprayed an apple tree and the crop numbered 1656 sound apples and 186 diseased ones. From a tree not sprayed he obtained 142 sound apples and 1278 diseased ones. Are these numbers inversely proportional or not?

9. If a farmer needs 108 lb. of seed for a 2-acre lot, how much will he need for $26\frac{1}{2}$ acres?

10. If the cost of 35 ft. of sidewalk is \$31.50, what will 75 ft. cost, the width being the same?

11. If 2.8 bbl. of lime are required for 75 sq. yd. of plastering, how many barrels will be needed for 275 sq. yd.?

12. A stretch of railway track runs 437 ft. on the same grade, $6\frac{1}{2}$ in. per 100 ft. What is the total difference in level?

13. A storage warehouse stored 675 bbl. of apples for 90 days at the rate of \$6 per 100 bbl. for 30 da. How much was the charge?

14. If a contractor agrees to build a sidewalk 4 ft. 6 in. wide in front of a lot for \$36, how much should he charge if it were 5 ft. wide?

15. A man builds a coal bin 6 ft. wide and finds that it holds $11\frac{1}{2}$ tons. How much would it hold if it had been made a foot wider?

16. An ice dealer can gather 2900 tons of ice from a pond when the ice has frozen 12 in. thick. How many tons can he gather when the ice is 15 in. thick?

17. An excavation has been half completed, 35 men having been employed for 16 days. How long should it take to complete the other half if 5 more men are added?

18. Gun metal is composed of 1 part of tin to $5\frac{1}{2}$ parts of copper by weight. How many pounds of tin must be added to 210 lb. 6 oz. of copper to make gun metal?

19. How many pounds of tin are there in 232 lb. 6 oz. of gun metal such as described in Ex. 18?

20. On January 1 a gas meter registers 42,500 cu. ft., and on January 8 it registers 43,400 cu. ft. If this rate is exactly maintained, how much will it register on January 29?

324. Proportional Parts. Just as we have learned (§ 313) how to separate a number into two parts having a given ratio, so we may separate a number into several parts proportional to given numbers.

For example, separate \$1230 into parts proportional to 2, 3, and 5.

The whole quantity may be represented by $2 + 3 + 5$, or 10.

Therefore the respective parts are $\frac{2}{10}$, $\frac{3}{10}$, and $\frac{5}{10}$ of \$1230. These parts are \$246, \$369, and \$615.

Separate 527 ft. into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$.

Multiply the fractions by 30, the L.C.M. of their denominators. The results are 15, 10, and 6. These numbers may therefore represent the parts, and their sum, 31, will represent the whole.

Therefore the respective parts are $\frac{15}{31}$, $\frac{10}{31}$, and $\frac{6}{31}$ of 527 ft., or 255 ft., 170 ft., and 102 ft.

Two farmers rent a pasture for \$2.55. The first puts in 4 cattle for 2 weeks, and the second 3 cattle for 3 weeks. How much should each farmer pay?

Since 4 cattle in 2 weeks eat as much as 8 cattle in 1 week, and 3 cattle in 3 weeks eat as much as 9 cattle in 1 week, one should pay $\frac{8}{17}$ of \$2.55, or \$1.20, and the other should pay $\frac{9}{17}$ of \$2.55, or \$1.35.

EXERCISE 181

1. Separate 297 into parts proportional to 7, 9, and 11.

2. Separate 253 into parts proportional to $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{1}{4}$.

3. Divide \$2646 between two partners in the ratio of 4 to 5.

4. Divide \$3744 among three partners so that their shares shall be proportional to 3, 4, and 5.

5. Four men invested \$2700, \$3300, \$3500, and \$4000 respectively in a joint business. They gained \$6750. What was each one's share of the profit?

6. Three farmers together pay $225 for irrigation taxes. The first owns 225 acres, the second 190 acres, and the third 335 acres. What is the tax of each?

7. Three men deliver milk at a creamery. The first delivers 776 lb., the second 938 lb., and the third 870 lb. They receive $38.76. What is the share of each?

8. Three men pay $848 for some water power. The first uses 28 horse power, the second 36 horse power, and the third 42 horse power. How much should each pay?

9. Three men rent a yacht for the summer for $540. The first uses it for 4 weeks, the second 5 weeks, and the third $4\frac{1}{2}$ weeks. How much should each pay?

10. Two farmers pasture cattle in a field, the first putting in 24 head for 6 weeks, and the second 36 head for 4 weeks. The bill being $43.20, what is the share of each?

11. Two hotel proprietors arrange to transport their employees from Florida to New York at the end of the season for $2793. The first has 65 employees and the second 82. How much should each proprietor pay?

12. Four boys arrange a three weeks' camping trip. The first stays 18 days; the second comes late and stays 12 days; the third is in camp the last two weeks; the fourth stays all through. The total expense is $32.50. What is the share of each?

13. A contractor employs 16 men 15 days on one job, and 14 men 12 days on another job. The total expense is $1020. How much should be charged to each job?

14. A manufacturer ships 1200 lb. of goods 300 miles, and 900 lb. 400 miles. If he pays according to weight and distance, and the total bill is $24, how much does each shipment cost?

EXERCISE 182

Review Problems

1. These boys find that this post is 3 ft. 6 in. high and casts a shadow 3 ft. 2 in. long. They find that the shadow of the tree is 34 ft. 10 in. long at the same time. Required the height of the tree.

2. If a building casts a shadow 75 ft. long when a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long, how high is the building?

3. If the railway fare for a journey of 75 mi. is $1.50, what will be the fare for 275 mi.?

4. Two boys weighing respectively 100 lb. and 80 lb. sit 9 ft. apart on the ends of a plank. Not counting the weight of the plank, how far from the heavier boy must the fulcrum be placed so that they will just balance?

5. How much pressure will you have to exert on the handles of a pair of shears just 3 in. from the fulcrum (screw or bolt), in order to exert a pressure of 5 lb. at a point 5 in. from the fulcrum?

EXERCISE 183

Problems without Numbers

1. If you double the consequent of a ratio, what else must you do so as not to change the value of the ratio?

2. How do you proceed to separate a number into parts having a given ratio?

3. How do you ascertain whether four quantities are in proportion?

4. Given the extremes and one mean of a proportion, how do you find the other mean?

5. Given the means and one extreme of a proportion, how do you find the other extreme?

6. If you know all of the terms of a proportion except the first, how is that found?

7. If you know all of the terms of a proportion except the antecedent of the second ratio, how is that found?

8. If you know the weight of two volumes of a substance, and one of the volumes, how do you state the proportion?

9. Knowing the number of men required to do a piece of work in a given number of days, how do you find how many men are required to do it in another given number of days?

10. How may you find the height of your school building by means of shadows and the height of a pole?

11. How may you proceed to find the height of a tree by sighting over the top of a pole of known length?

12. How may you tell where to place the fulcrum under a lever so as to lift a given weight by pressing your full weight on one end of the lever?

13. How do you proceed to separate a number into parts proportional to four given numbers?

CHAPTER XV

POWERS AND ROOTS

325. Powers. The result of taking a number any number of times as a factor is called a *power* of the number (§ 45).

326. Square. The result of taking a number twice as a factor is called the *second power* or *square* of the number.

Thus the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

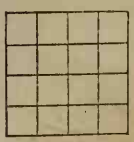
327. Square Root. One of the two equal factors of a number is called the *square root* of the number.

Thus the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Some numbers are *perfect squares*. We also speak of the square root of 5 as 2.236 + because the square of 2.236 is nearly 5.

328. Symbols. The square of 5 is written 5^2; the square root of 5 is written $\sqrt{5}$. The symbol $\sqrt{\ }$ is called a *radical sign*. The small figure 2 in 5^2 is called an *exponent*.

329. Square Roots and Areas. If this square is 4 units long, the area is 16 square units. Therefore, considering all the numbers as abstract,



The side of a square equals the square root of its area.

330. Square Roots by Factoring. In the case of perfect squares we may find the square roots if we can factor.

For example, since $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = (2 \times 2 \times 3) \times (2 \times 2 \times 3)$, therefore $\sqrt{144} = 2 \times 2 \times 3 = 12$.

EXERCISE 184

Find the areas of squares whose sides are as follows:

1. 17 ft.	4. 2.9 in.	7. 0.62 yd.	10. $\frac{4}{5}$ mi.
2. 19 ft.	5. 3.7 in.	8. 1.27 yd.	11. $\frac{7}{8}$ in.
3. 23 ft.	6. 4.3 in.	9. 4.93 yd.	12. $\frac{9}{32}$ ft.

By factoring, find the square roots of the following:

13. 196.	16. 400.	19. 625.	22. 900.	25. 1225.
14. 225.	17. 324.	20. 441.	23. 729.	26. 1089.
15. 256.	18. 484.	21. 784.	24. 1024.	27. 1296.

Find the sides of squares whose areas are as follows:

28. 1.44 sq. in.	31. 2500 sq. ft.	34. 0.09 sq. mi.
29. 1.21 sq. in.	32. 3600 sq. yd.	35. 0.81 sq. ft.
30. 5.76 sq. in.	33. 6400 sq. rd.	36. 0.0144 sq. ft.

Find the perimeters of squares whose areas are as follows:

37. 576 sq. in.	39. 23.04 sq. in.	41. 65.61 sq. ft.
38. 5184 sq. in.	40. 40.96 sq. in.	42. 110.25 sq. ft.

43. A square field is 40 rd. on a side. How many acres does it contain?

44. A square field contains 40 acres. Express the length of the field in rods.

45. A square lot has an area of 169 sq. rd. How many rods of fence will be needed to inclose it?

46. What is the area of the cross section of a square pillar that is $11\frac{7}{8}$ in. on a side?

47. The area of the cross section of a square pillar is 2.89 sq. ft. What is the width of the pillar?

331. Square of the Sum of Two Numbers. Since $47 = 40 + 7$, the square of 47 may be obtained as follows:

$$
\begin{array}{rl}
 & 40 + 7 \\
 & 40 + 7 \\
\hline
 & (40 \times 7) + 7^2 \\
40^2 + & (40 \times 7) \\
\hline
 & 40^2 + 2 \times (40 \times 7) + 7^2 \\
= & 1600 + 2 \times 280 \quad + 49 \\
= & 1600 + 560 \quad + 49 \\
= & 2209.
\end{array}
$$

280	49
1600	280
40	7
t	$+ u$

This relationship is seen in the annexed figure where the side of the square is $40 + 7$.

Every number consisting of two or more figures may be regarded as composed of tens and units.

Therefore, *the square of a number contains the square of the tens, plus twice the product of the tens and units, plus the square of the units.*

This is the most important principle in square root, and should be clearly understood, both from the multiplication and from the illustration, before proceeding.

332. Separating into Periods. The first step in extracting the square root of a number is to separate the figures of the number into groups of two figures each, called *periods.*

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one* or *two* figures is a number of *one* figure; expressed by *three* or *four* figures is a number of *two* figures; and so on.

If, therefore, an integral number is separated into periods of two figures each, from the right to the left, the number of figures in the square root will be equal to the number of the periods of figures. The last period at the left may have one figure or two figures: for example, 22 09, and 7 89 04 81.

333. Extracting the Square Root. The process of extracting the square root of a number not readily factored will now be considered.

For example, required the square root of 2209.

Separating into periods (§ 332), we see that there will be two integral places in the root.

```
 22 09 (47
 16
-----
80  6 09
87  6 09
-----
```

The first period, 22, contains the square of the tens' number of the root. And since the greatest square in 22 is 16, therefore $\sqrt{16}$, or 4, is the tens' figure of the root.

Subtracting the square of the tens, the remainder contains twice the tens × the units, plus the square of the units (§ 331). Therefore if we divide by twice the tens (that is, by 80, which is 2 × 4 tens), we shall find approximately the units. Dividing 609 by 80 (or 60 by 8), we have 7 as the units' figure.

Since twice the tens × the units, plus the square of the units, is equal to (twice the tens + the units) × the units, i.e. $2 \times 40 \times 7 + 7^2 = (2 \times 40 + 7) \times 7$, therefore we add 7 to 80 and multiply the sum by 7. The product is 609, thus completing the square of 47.

To check the work, $47^2 = 2209$.

EXERCISE 185

Extract the square root of:

1. 841.	**4.** 1369.	**7.** 3249.	**10.** 8281.
2. 961.	**5.** 1681.	**8.** 3721.	**11.** 9801.
3. 529.	**6.** 1521.	**9.** 1849.	**12.** 6241.

13. A square lot has an area of 6889 sq. ft. How many feet has it on a side?

14. A square piece of linoleum has an area of 37 sq. ft. 1 sq. in. How many inches has it on a side?

15. The bottom of a square box has an area of 484 sq. in. How long is it?

16. A square lot has an area of 7569 sq. ft. How many feet has it on a side?

334. Explanation by Diagram. It sometimes makes the reasoning in square root more clear if we consider a diagram.

FIG. 1

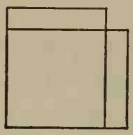


FIG. 2

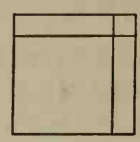


FIG. 3

We have seen (§ 331) a figure showing the square on the lines $40 + 7$. In the explanation in § 333 we first took away from the whole square (as in Fig. 3) the square of the tens, or 1600, which is the square of 40 (represented by Fig. 1).

There then remained the two rectangles and the small square as seen in Fig. 3, the combined area being 609 square units. But these rectangles are as long as Fig. 1, and therefore 40 units long. Dividing their area, plus the area of the small square, by their length, we have approximately their width. (We would get their exact width if it were not for the small square.) This gives the units.

If we now add twice the tens to the units, we shall have the length of the two rectangles and the small square ($80 + 7 = 87$). Multiplying this by the units, the width of the rectangles, we have the area of the two rectangles and the small square ($87 \times 7 = 609$).

We see from the figures that the square is now entirely used; that is, the square of tens (Fig. 1) + twice the product of tens and units (making Fig. 2) + the square of the units (making Fig. 3) uses all the area. Therefore the square root is 47.

335. Square Root of Larger Numbers. The method of § 333 will apply to numbers of more than two periods, by considering *the part of the root already found as so many tens* with respect to the next figure. For example, required the square root of 7,890,481.

```
      7 89 04 81 (2809
      4
40    3 89
48    3 84
5600    5 04 81
5609    5 04 81
```

When the third period, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 560 is not contained in 504. Therefore 0 is placed both in the root and in the divisor, and the next two figures, 81, are brought down.

336. Square Roots of Decimals. If the square root of a number has decimal places, the number itself will have twice as many.

Thus, if 0.25 is the square root of some number, the number will be 0.25^2, or 0.25×0.25, or 0.0625. Hence, if a given number contains a decimal, we separate it into periods of two figures each, beginning at the decimal point and proceeding toward the left for the integral part, and toward the right for the decimal. The last period of the decimal must have two figures, a zero being annexed if necessary.

Extract the square root of 52.2729.

We see at once that the root will have one integral place. Furthermore, if it is a perfect square it can have only two decimal places, since the square of thousandths would be millionths.

```
        52.27 29 (7.23
        49
140     3 27
142     2 84
1440     43 29
1443     43 29
```

EXERCISE 186

Extract the square root of:

1. 190,969.	**6.** 804,609.	**11.** 1036.84.	**16.** 3.9204.
2. 743,044.	**7.** 194,481.	**12.** 82.2649.	**17.** 462.25
3. 401,956.	**8.** 173,056.	**13.** 0.063001.	**18.** 0.003969.
4. 758,641.	**9.** 174,724.	**14.** 1.5129.	**19.** 0.182329.
5. 117,649.	**10.** 509,796.	**15.** 2.6244.	**20.** 0.054756.

Find the side of a square whose area is:

21. 12,321 sq. ft. **22.** 8046.09 sq. in.

23. What is the perimeter of a square whose area is 1944.81 sq. in.?

24. The total area of the sides of a certain cube is found to be 355.74 sq. in. What is the length of an edge? What is the volume of the cube?

337. Approximate Square Roots. If a number is not a perfect square, zeros may be annexed and an approximate value of the root found.

For example, required to extract to three places of decimals the square root of 19.

	19.00 00 00 (4.3588+
	16
80	3 00
83	2 49
860	51 00
865	43 25
8700	7 75 00
8708	6 96 64
87160	78 3600

In this example we proceed in the usual way, annexing pairs of zeros for each decimal place in the root. We carry the work to four decimal places, so as to find the nearest approximation for three places. The result is, therefore, 4.359−; that is, it is nearer 4.359 than 4.358.

338. Summary of Square Root. We may summarize the process of square root as follows:

Separate the number into periods of two figures each, beginning at the decimal point.

Find the greatest square in the left-hand period and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

For a partial divisor double the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished slightly if necessary) will be the next figure of the root.

To this partial divisor add this next figure of the root for a complete divisor. Multiply the complete divisor by this next figure of the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

Proceed in this manner until all the periods have been thus annexed. The result will be the square root required.

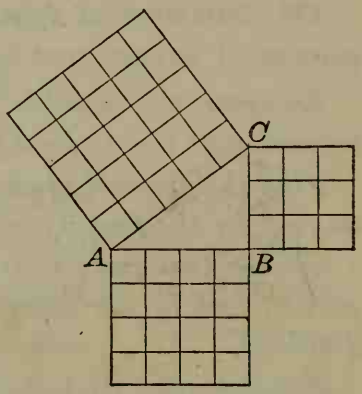
339. Square Roots of Common Fractions. To find the square root of a common fraction we may extract the square roots of the numerator and denominator.

If the denominator is not a perfect square, it is better to multiply both terms of the fraction by such a number as shall make it such, or else to reduce to a decimal and then extract the square root. For example, instead of saying $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{1.414+}$, which requires us to divide by a long divisor, it is better to say $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2} = \frac{1}{2}$ of $1.414+ = 0.707+$, or else to extract the square root of 0.5.

340. Right Triangle. A triangle that contains a right angle is called a *right triangle* (§ 216). It is sometimes called a *right-angled triangle.*

341. Hypotenuse. In a right triangle the side opposite the right angle is called the *hypotenuse.*

In the triangle ABC the hypotenuse is AC, the base is AB, and the perpendicular (or altitude) is BC.



342. Square on the Hypotenuse. By counting the small squares in this figure we see that the square on AC equals the sum of the squares on AB and BC. It is proved in geometry that this is true for all right triangles.

Therefore, *in a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides.*

The hypotenuse equals the square root of the sum of the squares of the other two sides.

The base or the perpendicular equals the square root of the difference of the squares of the hypotenuse and the other side.

EXERCISE 187

Extract the square root of:

1. $\frac{49}{144}$. 3. $\frac{225}{361}$. 5. $\frac{529}{729}$. 7. $\frac{1369}{1681}$. 9. $\frac{2209}{2601}$.
2. $\frac{121}{169}$. 4. $\frac{441}{625}$. 6. $\frac{961}{1089}$. 8. $\frac{841}{1849}$. 10. $\frac{3809}{3025}$.

Extract the square root, to two decimal places:

11. 2. 13. 11. 15. 30. 17. 125. 19. $\frac{5}{9}$.
12. 7. 14. 15. 16. 50. 18. 650. 20. $\frac{7}{12}$.

Reduce to fractions with denominators perfect squares and extract the square root, to two decimal places:

21. $\frac{1}{3}$. 22. $\frac{2}{5}$. 23. $\frac{2}{3}$. 24. $\frac{3}{8}$. 25. $\frac{7}{8}$.

Reduce to decimals and extract the square root, to two decimal places:

26. $\frac{5}{8}$. 27. $\frac{4}{7}$. 28. $\frac{3}{5}$. 29. $\frac{5}{12}$. 30. $\frac{11}{12}$.

Find the hypotenuse, given the other sides as follows:

31. 39 ft., 52 ft. 32. 21 ft., 72 ft. 33. 51 ft., 68 ft.

Also as follows, carrying the results to three decimals:

34. 82 ft., 35 ft. 35. 31 ft., 23 ft. 36. 27 ft., 43 ft.

Given the hypotenuse and one side, find the other side:

37. 10 ft., 6 ft. 38. 17 in., 15 in. 39. 26 in., 10 in.

Also as follows, carrying the results to three decimals:

40. 15 ft., 6 ft. 41. 18 ft., 12 ft. 42. 23 in., 12 in.

Find the diagonal of the square whose side is:

43. 20″. 44. 32″. 45. 45″. 46. 70″. 47. 75″.

343. Area of a Triangle in Terms of the Sides. It is proved in geometry that

To find the area of any triangle given the sides, multiply half the sum of the sides in succession by the three remainders obtained by subtracting each side separately from the half sum of the sides, and take the square root of this product.

For example, find the area of a triangle whose sides are respectively 5 in., 6 in., and 7 in.

The half sum of the sides is $\frac{1}{2}$ of $(5 + 6 + 7)$, or 9. Then $9 - 5 = 4$, $9 - 6 = 3$, and $9 - 7 = 2$. Therefore the area $= \sqrt{9 \times 4 \times 3 \times 2} = \sqrt{216} = 14.696 +$. That is, the area is 14.696 sq. in.

344. Similar Figures. It is also proved in geometry that

The areas of similar figures are to each other as the squares of their corresponding dimensions.

The corresponding dimensions of similar figures are to each other as the square roots of their areas.

That is, if a side of one square is 5 times as long as a side of another, the area is 25 times as much. If one circle has 16 times the area of another, the radius is 4 times as long.

EXERCISE 188

Find the areas (to two decimal places) of the triangles whose sides are:

1. 5″, 7″, 8″.
2. 6″, 8″, 9″.
3. 4″, 3″, 6″.
4. 9″, 10″, 11″.
5. 12″, 13″, 15″.
6. 17″, 21″, 25″.

7. A triangle has an area of 275 sq. ft. What is the area of a similar triangle whose base is only half as long?

8. A rectangular lot has a front of 160 ft. and is worth \$1000. Find the value, at the same rate per square foot, of a similar lot which has twice the front and twice the depth.

345. Finding the Radius. Since (§ 231) the area of a circle equals 3.1416 times the square of the radius, therefore

The radius of a circle equals the square root of the quotient of the area divided by 3.1416.

For example, if the area is 78.54 sq. in., the number of inches in the radius is $\sqrt{\frac{78.54}{3.1416}} = \sqrt{25} = 5$. Therefore the radius is 5 in.

In Exercise 189 carry the square roots to two decimal places.

EXERCISE 189

Find the area of the circle whose radius is:

1. 17″. **2.** 29″. **3.** 37″. **4.** 5.27″. **5.** 6.07′.

Find the area of the circle whose diameter is:

6. 38″ **7.** 46″. **8.** 29″. **9.** 7.54″. **10.** 9.72′.

Find the radius of the circle whose area is:

11. 314.16 sq. in. **12.** 113.0976 sq. in. **13** 250 sq. in.

14. What must be the diameter of a water pipe in order that the area of a cross section shall be 2 sq. in.?

15. What must be the diameter of a water main in order that the area of a cross section shall be 3 sq. ft.?

16. What must be the diameter of the piston head of an engine in order that the area may be 100 sq. in.?

17. A cylindrical water tank is 20 ft. high and will contain 28,000 cu. ft. of water. What is the diameter?

18. A horse tethered by a rope can graze over 1570.8 sq. ft. of ground. How long is the rope? If the rope was twice as long, over how much ground could he graze?

19. A tinsmith wishes to make some cylindrical gallon cans. They are to be 10 in. high. What must be the area of the base? What radius must he use to draw the circle?

346. Cube. The result of taking a number three times as a factor is called the *third power* or *cube* of the number.

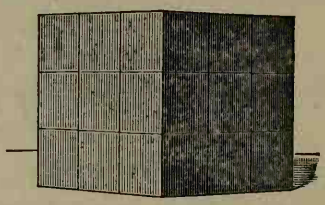
Thus the cubes of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

347. Cube Root. One of the three equal factors of a number is called the *cube root* of the number.

Thus the cube roots of 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

348. Symbols. The cube of 5 is written 5^3; the cube root of 125 is written $\sqrt[3]{125}$.

349. Cube Roots and Volumes. If this cube is 3 units on an edge, we can see that there will be 3×3, or 9, cubic units in the front layer of cubes and that there will be 3 of these layers. The volume is therefore $3 \times 3 \times 3$ cubic units, or 27 cubic units.



Therefore, *the volume of a cube equals the cube of the edge.*

The edge of a cube equals the cube root of the volume.

350. Cube Roots by Factoring. In the case of perfect cubes we may find the cube roots if we can factor.

For example, since $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = (2 \times 2 \times 3) \times (2 \times 2 \times 3) \times (2 \times 2 \times 3)$, therefore $\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$.

EXERCISE 190

Find the value of the following:

1. 37^3.	4. 7.5^3.	7. $\sqrt[3]{1331}$.	10. $\sqrt[3]{5832}$.
2. 49^3.	5. 4.8^3.	8. $\sqrt[3]{3375}$.	11. $\sqrt[3]{9261}$.
3. 59^3.	6. 0.69^3.	9. $\sqrt[3]{4096}$.	12. $\sqrt[3]{10648}$.

13. Find the edge of a cube containing 13,824 cu. in.

351. Cube of the Sum of Two Numbers. Since $47 = 40 + 7$, the cube of 47 may be obtained by multiplying the square of $40 + 7$ (as found in § 331) by $40 + 7$, thus:

$$(40 + 7)^2 = 40^2 + 2 \times (40 \times 7) + 7^2$$

$$40 + 7$$

$$(40^2 \times 7) + 2 \times (40 \times 7^2) + 7^3 \quad \text{product by } 7$$

$$40^3 + 2 \times (40^2 \times 7) + (40 \times 7^2) \quad \text{product by } 40$$

$$40^3 + 3 \times (40^2 \times 7) + 3 \times (40 \times 7^2) + 7^3$$

We therefore see that

The cube of any number composed of tens and units contains four parts:

(1) *The cube of the tens.*

(2) *Three times the product of the square of the tens by the units.*

(3) *Three times the product of the tens by the square of the units.*

(4) *The cube of the units.*

352. Separating into Periods. The first step in extracting the cube root of a number is to separate the figures of the number into groups of three figures each, called *periods.*

Since $1 = 1^3$, $1000 = 10^3$, $1{,}000{,}000 = 100^3$, and so on, it follows that the cube root of any integral number between 1 and 1000, that is, the cube root of any integral number that has *one*, *two*, or *three* figures, is a number of *one* figure; that the cube root of any integral number between 1000 and 1,000,000, that is, the cube root of any integral number that has *four*, *five*, or *six* figures, is a number of *two* figures; and so on.

If, therefore, an integral number is separated into periods of three figures each, from right to left, the number of figures in the root will be equal to the number of periods. The last period to the left may consist of one figure, two figures, or three figures.

For example, the cube root of 42,875 will have two integral places, and the cube root of 34,645,976 will have three.

353. Extracting the Cube Root. The process of extracting the cube root of a number not readily factored will now be considered. For example, required the cube root of 42,875.

We see that there will be two integral places in the root (§ 352). The first period, 42, contains the cube of the tens' number. The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence 3 is the tens' figure of the root.

$$
\begin{array}{rr|l}
 & 42\,875 & (35 \\
 & 27 & \\
\hline
3 \times 30^2 = & 2700 & 15\,875 \\
3 \times (30 \times 5) = & 450 & \\
5^2 = & 25 & \\
\hline
 & 3175 & 15\,875
\end{array}
$$

The remainder, 15,875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Each of these three parts contains the units' number as a factor.

Hence the 15,875 consists of two factors, one of which is the units' number of the root; and the other factor is three times the square of the tens + three times the product of the tens by the units + the square of the units. The largest part of this second factor is three times the square of the tens.

If the 158 hundreds of the remainder is divided by 3×30^2, or 27 hundreds, the quotient will be approximately the units' number of the root. The second factor can now be completed by adding to the 2700 the sum of $3 \times (30 \times 5)$, or 450, and 5^2, or 25.

If this factor, 3175, is now multiplied by 5, the result is 15,875, which completes the cube of 35. There being no remainder, $\sqrt[3]{42875} = 35$.

To check the work, $35^3 = 42{,}875$.

EXERCISE 191

Extract the cube root of:

1. 2197.	4. 13,824.	7. 226,981.	10. 857,375.
2. 4913.	5. 29,791.	8. 132,651.	11. 884,736.
3. 6859.	6. 110,592.	9. 373,248.	12. 941,192.

13. Find the edge of a cube whose volume is 778,688 cu. in.

14. Find the edge of a cube whose volume is 205,379 cu. in.

354. Explanation by Blocks. It sometimes makes the reasoning in cube root more clear if we consider the building up of a cube by blocks.

We have learned that the cube of $35 = 30^3 + 3 \times (30^2 \times 5) + 3 \times (30 \times 5^2) + 5^3$ (§ 351).

The 30^3 may be represented by a cube whose edge is 30 in. (Fig. 1).

The $3 \times (30^2 \times 5)$ may be represented by three equal rectangular solids, each 30 in. long, 30 in. wide, and 5 in. thick, to be added to three adjacent faces of Fig. 1.

The $3 \times (30 \times 5^2)$ may be represented by three equal rectangular solids, 30 in. long, 5 in. wide, and 5 in. thick, to be added to Fig. 2.

The 5^3 may be represented by the small cube required to complete the cube of Fig. 3.

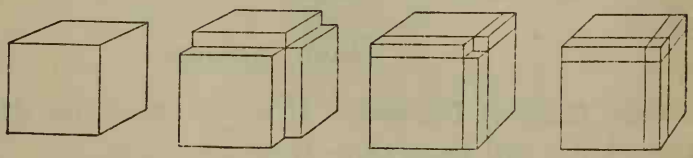


FIG. 1 FIG. 2 FIG. 3 FIG. 4

In extracting the cube root of 42,875, the large cube (Fig. 1) whose edge is 30 in. is first removed.

There remains 42,875 cu. in. − 27,000 cu. in., or 15,875 cu. in.

The greatest part of this is contained in the three rectangular solids, each 30 in. long and 30 in. wide, which are added to Fig. 1.

The thickness of these solids is found by dividing the 15,875 cu. in. by the sum of the three faces, each of which is 30 in. square; that is, by 2700 sq. in. The result is 5 in.

There are also the three rectangular solids which are added to Fig. 2, and which are 30 in. long and 5 in. wide; and a cube which is added to Fig. 3, and which is 5 in. long and 5 in. wide.

Hence the sum of the products of two dimensions of all these solids is

For the larger rectangular solids, $3 \times (30 \times 30)$ sq. in. = 2700 sq. in.
For the smaller rectangular solids, $3 \times (30 \times 5)$ sq. in. = 450 sq. in.
For the small cube, (5×5) sq. in. = 25 sq. in.
3175 sq. in.

The number 3175 multiplied by the third dimension, 5, gives 5×3175 cu. in., or 15,875 cu. in., as found in § 353.

355. Cube Roots of Larger Numbers. The method of § 353 will apply to numbers of more than two periods, by considering *the part of the root already found as so many tens* with respect to the next figure of the root.

For example, required the cube root of 57,512,456.

		57 512 456 (386
		27
$3 \times 30^2 =$	2700	30 512
$3 \times (30 \times 8) =$	720	
$8^2 =$	64	
	3484	27 872
$3 \times 380^2 =$	433200	2 640 456
$3 \times (380 \times 6) =$	6840	
$6^2 =$	36	
	440076	2 640 456

356. Cube Roots of Decimals. If a cube root has decimal places, the cube will have *three times* as many.

Thus, if 0.11 is the cube root of a number, the number is $0.11 \times 0.11 \times 0.11 = 0.001331$. Hence, if a given number contains a decimal, we separate it into periods of three figures each, beginning at the decimal point and proceeding toward the left for the integral part, and toward the right for the decimal. The last period of the decimal must contain *three* figures, zeros being annexed when necessary.

Extract the cube root of 187.149248.

		187.149 248 (5.72
		125
$3 \times 50^2 =$	7500	62 149
$3 \times (50 \times 7) =$	1050	
$7^2 =$	49	
	8599	60 193
$3 \times 570^2 =$	974700	1 956 248
$3 \times (570 \times 2) =$	3420	
$2^2 =$	4	
	978124	1 956 248

Since there can be only one integral place, the decimal point is placed after the 5.

357. Approximate Cube Roots. If the given number is not a perfect cube, zeros may be annexed, and a value of the root may be found as near to the true value as we please.

Extract the cube root of 1250.6894.

$$
\begin{array}{rrl}
 & & 1\ 250.689\ 400\,(10.77 \\
 & & 1 \\
\hline
3 \times 10^2 = & 300 & 250 \\
\end{array}
$$

Since 300 is not contained in 250, the next figure of the root is 0.

$$
\begin{array}{rrl}
3 \times 100^2 = & 30000 & 250\ 689 \\
3 \times (100 \times 7) = & 2100 & \\
7^2 = & 49 & \\
\hline
 & 32149 & 225\ 043 \\
\hline
3 \times 1070^2 = & 3434700 & 25\ 646\ 400 \\
3 \times (1070 \times 7) = & 22470 & \\
7^2 = & 49 & \\
\hline
 & 3457219 & 24\ 200\ 533 \\
\hline
 & & 1\ 445\ 867
\end{array}
$$

358. Cube Roots of Common Fractions. The cube root of a common fraction is found by extracting the cube roots of the numerator and denominator; but, if the denominator is not a perfect cube, it is generally best to reduce the fraction to a decimal, and then extract the root.

EXERCISE 192

Extract the cube root, to two decimal places:

1. 71.296.	3. 7.1296.	5. 21.782.	7. 37.487.
2. 643.25.	4. 0.75475.	6. 0.1234.	8. 81.492.

Extract the cube root of the following fractions:

9. $\frac{8}{729}$. 10. $\frac{1331}{1728}$. 11. $\frac{4913}{6859}$. 12. $\frac{2197}{9261}$. 13. $\frac{3375}{4096}$.

Extract the cube root, to three decimal places:

14. 2. 15. 3. 16. 5. 17. $\frac{1}{4}$. 18. $\frac{2}{3}$.

359. Summary of Cube Root. We may summarize the process of cube root as follows:

Separate the number into periods of three figures each, beginning at the decimal point.

Find the greatest cube in the left-hand period and write its root for the first figure of the required root.

Cube this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

For a partial divisor, take three times the square of the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished slightly if necessary) will be the second figure of the root.

To this partial divisor add three times the product of the first figure of the root, considered as tens, by the second figure, and also the square of the second figure. This sum will be the complete divisor.

Multiply the complete divisor by the second figure of the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

Proceed in this manner until all the periods have been annexed. The result will be the cube root required.

If a given number contains a decimal, the last group on the right of the decimal point must be made to contain three figures, by annexing one zero or two zeros if necessary. In the examples of Exercise 193 carry the result to two decimal places in case the given number is not an exact cube.

360. Similar Solids. Solids that have exactly the same shape are called *similar solids.*

It is proved in geometry that

The volumes of similar solids are to each other as the cubes of corresponding dimensions.

The corresponding dimensions of similar solids are to each other as the cube roots of the volumes.

EXERCISE 193

Extract the cube root of:

1. 1,771,561.	4. 47,832,147.	7. 4826.809.
2. 1,295,029.	5. 11,390,625.	8. 0.000912673.
3. 2,048,383.	6. 87,528,384.	9. 0.114791256.

Find the edges of cubes of the following volumes:

10. 75 cu. in. 11. 830 cu. ft. 12. 92.5 cu. in. 13. $7\frac{1}{2}$ cu. in.

14. Find the diameter of an iron ball that will weigh 27 times as much as an iron ball 2 in. in diameter.

15. The weights of two iron cylinders of the same shape are as 2197 to 4913. Find the ratio of their heights.

16. Find the edge of a cube whose volume is equal to the volume of a rectangular solid $81'' \times 3'' \times 3''$.

17. Find the edge of a cubical cistern that holds as many gallons as a rectangular cistern $12' \times 8' \times 5\frac{1}{3}'$.

18. Find the edge of a cubical cistern that holds as many gallons as a rectangular cistern $18' \times 9' \times 4\frac{1}{2}'$.

19. The edge of a cube is 2 in., and the edge of another is 4 in. How many times the total surface of the smaller is the total surface of the larger? How many times the volume of the smaller is the volume of the larger?

20. The volume of a rectangular solid is 2197 cu. in., and the volume of a similar solid is 17,576 cu. in. How many times a dimension of the smaller solid is the corresponding dimension of the larger?

21. How many spheres 1 inch in diameter are equivalent in volume to a sphere 2 inches in diameter? to a sphere 5 inches in diameter? to a sphere 10 inches in diameter? to a sphere $15\frac{1}{2}$ inches in diameter?

EXERCISE 194

Problems without Numbers

1. How do you find the square of a number? the cube?

2. If you know the side of a square, how do you find the area?

3. If you know the area of a square, how do you find the side?

4. How do you find the square root of a number by factoring? also the cube root by factoring?

5. To what is the square of the sum of two numbers equal? How would you show it?

6. What is the object of separating a number into periods in extracting square root?

7. How do you proceed to extract the square root of an integer that is a perfect square?

8. How do you proceed to extract the square root of a number that contains decimal places?

9. How do you proceed to extract the square root of a common fraction?

10. How do you find the hypotenuse of a right triangle when the other sides are given?

11. How do you find the base of a right triangle, given the hypotenuse and the perpendicular?

12. How do you find the area of a triangle when the three sides are given?

13. What is the relation between the corresponding dimensions of similar figures?

14. Given the area of a circle, how do you find the radius?

15. How do you extract the cube root of a number?

CHAPTER XVI

METRIC MEASURES

361. The Metric System. The metric system is now used in several European countries, and in scientific work elsewhere. Since it is on the decimal scale it is much simpler than our system.

362. Meter. The unit of length is the *meter*.

363. Liter. The unit of capacity is the *liter* (pronounced lee'ter), a cube 0.1 of a meter on an edge.

364. Gram. The unit of weight is the *gram*. It is the weight of a cube of water 0.01 of a meter on an edge at the greatest density of water, a little above freezing.

365. The Prefixes. The system is understood as soon as the prefixes are known.

Just as 1 mill = $\frac{1}{1000}$ of a dollar,
so 1 millimeter = 0.001 of a meter.
Just as 1 cent = $\frac{1}{100}$ of a dollar,
so 1 centimeter = 0.01 of a meter.
Just as decimal means tenths,
so 1 decimeter = 0.1 of a meter.

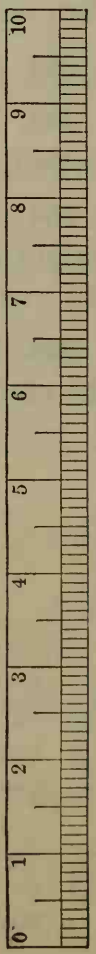
	THE PREFIX	MEANS	AS IN	WHICH MEANS	
From the Greek	myria-	10,000	myriameter	10,000	meters.
	kilo-	1,000	kilogram	1,000	grams.
	hekto-	100	hektoliter	100	liters.
	deka-	10	dekameter	10	meters.
		1		1	
From the Latin	deci-	0.1	decimeter	0.1	of a meter.
	centi-	0.01	centigram	0.01	of a gram.
	milli-	0.001	millimeter	0.001	of a meter.

366. Important Names. As in United States money we seldom speak of anything except dollars and cents, so in metric measures only those measures printed in black letters in the tables of this chapter are in common use.

All the units of the system are derived from the meter. Every compound name is accented on the first syllable ; thus, *mil'limeter.* The teacher should be supplied with a meter stick, a liter, and a cubic centimeter, and these can easily be made in school if necessary. The work should be made as practical as possible by actual measurements.

367. Measures of Length. The table of measures of length is as follows:

A myriameter	= 10,000	meters
A kilometer (km.)	= 1,000	meters
A hektometer	= 100	meters
A dekameter	= 10	meters
Meter (m.)		
A decimeter (dm.)	= 0.1	of a meter
A centimeter (cm.)	= 0.01	of a meter
A millimeter (mm.)	= 0.001	of a meter



10 centimeters = 1 decimeter.

The meter is about 39.37 in., or $3\frac{1}{4}$ ft., or a little over a yard; the kilometer is about 0.62 of a mile. When the metric system was invented in France over a century ago, the meter was intended to be one ten-millionth of the distance on the surface of the earth from the equator to the pole, but it varies slightly from this standard.

The abbreviations in this book are in common use. Some, however, use Km., Dm., dm., for kilometer, dekameter, and decimeter. We do not need abbreviations for the units not commonly used, such as the myriameter.

368. Reduction. Any one of these measures may be expressed in terms of another measure by *simply moving the decimal point to the right or left.*

Thus, as 245¢ = 24.5 d. = $2.45,
so 2475 mm. = 247.5 cm. = 24.75 dm. = 2.475 m.,
and 35.25 km. = 35,250 m. = 35,250,000 mm.

EXERCISE 195

Express as meters and decimals:

1. 25 km.
2. 3.7 km.
3. 0.7 km.
4. $62\frac{1}{2}$ km.
5. $57\frac{3}{4}$ km.
6. 237 dm.
7. 47.5 dm.
8. 2.75 dm.
9. 725 cm.
10. 68.5 cm.
11. 2965 cm.
12. 48,750 cm.
13. 29,375 mm.
14. 96,400 mm.
15. 250,000 mm.

Allowing 39.37 in. to a meter, express as meters:

16. 19.685 in.
17. 78.74 in.
18. 118.11 in.
19. 25 ft.
20. 4.8 ft.
21. 0.6 ft.
22. 3 ft. 3 in.
23. 3 ft. 3.37 in.
24. 29 ft. 6.33 in.

Express as feet:

25. 75 m.
26. $48\frac{1}{2}$ m.
27. $2.7\frac{1}{2}$ m.
28. 35 dm.
29. 7.5 dm.
30. 685 cm.
31. 4500 cm.
32. 6875 mm.
33. 21,475 mm.

Allowing 0.62 mi. to a kilometer, express as miles:

34. 5 km.
35. 75 km.
36. $2\frac{1}{2}$ km.
37. 4.75 km.
38. 272 km.
39. 48.9 km.

Express as kilometers:

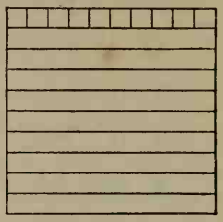
40. 325 mi.
41. 42.7 mi.
42. 0.75 mi.

43. If sound travels 340.5 m. a second, how many kilometers will it travel in 17 sec.?

44. If the distance from Paris to Brussels is 313 km., how many miles is it?

45. If the distance from Lyons to Marseilles is 218 mi., how many kilometers is it?

369. Square Measure. In square measure the scale is by hundreds instead of by tens. If the square here shown represents a square meter, it is evident that each of the ten equal horizontal bands is one tenth of a square meter, and each of the ten small squares therefore represents $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$, of a square meter. That is, 100 square decimeters = 1 square meter.



The table is as follows:

A square myriameter	=	100,000,000	square meters
A square kilometer (sq. km.)	=	1,000,000	square meters
A square hektometer	=	10,000	square meters
A square dekameter	=	100	square meters
Square meter (sq. m.)			
A square decimeter	=	0.01	of a square meter
A square centimeter (sq. cm.)	=	0.0001	of a square meter
A square millimeter (sq. mm.)	=	0.000001	of a square meter

370. Land Measure. In the measurement of land the square dekameter is called an *are* (pronounced är); the square hektometer is called a hektare (ha.). The hektare equals 2.47 acres, or nearly $2\frac{1}{2}$ acres.

EXERCISE 196

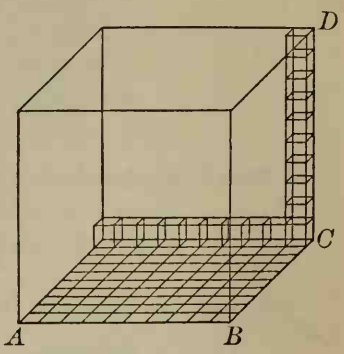
Express as square meters:

1. 275 sq. km.
2. 3.45 sq. km.
3. 0.75 sq. km.
4. 4875 sq. cm.
5. 9235 sq. cm.
6. 64,275 sq. cm.
7. 123,450 sq. mm.
8. 2,250,000 sq. mm.
9. 37,375,000 sq. mm.

Express as hektares:

10. 10,000 sq. m.
11. 45,000 sq. m.
12. 175,000 sq. m.
13. 325,500 sq. m.
14. 24.7 acres.
15. 1482 acres.
16. How long is a square field that contains 0.81 ha.?

371. Cubic Measure. In cubic measure the scale is by thousands. If the cube here shown represents a cubic meter, it is evident that each of the ten equal layers represents one tenth of a cubic meter; that each of the rectangular solids, DC, represents $\frac{1}{10}$ of $\frac{1}{10}$ of a cubic meter, and that each of the small cubes represents $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{1000}$, of a cubic meter. That is, 1000 cubic decimeters = 1 cubic meter.



The table is as follows:

A cubic hektometer	=	1,000,000	cubic meters
A cubic dekameter	=	1,000	cubic meters
Cubic meter (cu. m.)			
A cubic decimeter (cu. dm.)	=	0.001	of a cubic meter
A cubic centimeter (cu. cm.)	=	0.000001	of a cubic meter
A cubic millimeter (cu. mm.)	=	0.000000001	of a cubic meter

We may also have cubic kilometers and cubic myriameters. The terms above the cubic meter are seldom used.

372. Wood Measure. In measuring wood the cubic meter is called a *stere* (st.). Stere is pronounced stair.

EXERCISE 197

Express as cubic meters:

1. 725.25 st.
2. 2,750,000 cu. dm.
3. 4,625,750 cu. dm.
4. 9,125,000 cu. dm.
5. 8,125,000,000 cu. mm.
6. 9,275,000,000 cu. mm.
7. 25,325,000,000 cu. mm.
8. 37,250,000,000 cu. mm.

9. A pile of wood is 2.5 m. high, 1 m. wide, and 28.7 m. long. How many steres are there?

373. Measures of Capacity. The table of capacity is as follows:

A **hektoliter** (hl.)	=	100 liters
A dekaliter	=	10 liters
Liter (l.)		
A deciliter (dl.)	=	0.1 of a liter
A centiliter (cl.)	=	0.01 of a liter
A milliliter (ml.)	=	0.001 of a liter

A liter is a cubic decimeter. It is practically the same as a quart.

EXERCISE 198

Express as liters:

1. 750 hl.	**3.** 92.75 hl.	**5.** 750 dl.	**7.** 7500 ml.
2. 275 hl.	**4.** 9.375 hl.	**6.** 975 cl.	**8.** 6250 ml.

Express as hektoliters:

9. 3750 l.	**11.** 978.5 l.	**13.** $97\frac{1}{2}$ qt.	**15.** 65 gal.
10. 4875 l.	**12.** 25.75 l.	**14.** $38\frac{3}{4}$ qt.	**16.** $31\frac{1}{2}$ gal.

Express as quarts:

17. 37 hl.	**18.** 9.5 hl.	**19.** 750 dl.	**20.** 8250 cl.

21. How many liters are there in a cubic meter?

22. How many hektoliters of air are there in a room 7 m. long, 6 m. wide, and 3 m. high?

23. A cubical tank in a zoölogical garden is 2.25 m. on an edge. How many liters will it hold?

24. A bin is 11.25 m. long, 4.8 m. wide, and 1.6 m. deep. How many hektoliters will it hold? How many liters?

25. The average depth of a pond is 1.75 m., and its area is 3.5 ha. How many hektoliters of water does it contain?

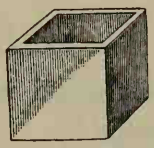
26. The height of a cylindrical tank is 12 m., and the diameter of the base is 9 m. How many liters will it hold?

374. Measures of Weight. The table of weight is as follows:

A metric ton (t.)	=	1,000,000 grams
A quintal (q.)	=	100,000 grams
A myriagram	=	10,000 grams
A kilogram (kg.)	=	1,000 grams
A hektogram	=	100 grams
A dekagram	=	10 grams
Gram (g.)		
A decigram	=	0.1 of a gram
A centigram (cg.)	=	0.01 of a gram
A milligram (mg.)	=	0.001 of a gram

375. Relation of Weight to Capacity.

1 cu. cm. of water weighs 1 g.
1 l. of water weighs 1 kg.
1 cu. m. of water weighs 1 t.



Cubic Centimeter

Gram Weight

The standard units vary slightly from these theoretical units.

A kilogram is about 2.2 lb.; and a metric ton is about 2204.6 lb.

EXERCISE 199

Express as grams:

1. 1.7 kg.
2. 0.9 kg.
3. 0.25 kg.
4. 3.75 kg.
5. 700 cg.
6. 950 cg.
7. 7500 mg.
8. 8125 mg.

Express as milligrams:

9. 7 g.
10. 8.7 g.
11. 0.27 g.
12. 4.25 g.
13. 15 kg.
14. 2.7 kg.
15. 75 cg.
16. 9.8 cg.

Express as kilograms:

17. 27 t.
18. 0.5 t.
19. 250 g.
20. 4400 lb.

Express as pounds:

21. 3 kg.
22. 75 kg.
23. 300 g.
24. 9000 mg.

376. Specific Gravity. The ratio of the weight of a given substance to the weight of an equal volume of water is called the *specific gravity* of the substance.

Thus, if a cubic decimeter of copper weighs 8.9 kg., the specific gravity of copper is 8.9, because 1 cu. dm. = 1 l., and 1 l. of water weighs 1 kg. Therefore copper is 8.9 times as heavy as water.

It is usually easy to find the volume of any substance by immersing it in a vessel full of water and measuring the amount that runs over.

By means of specific gravity the purity of metals and other substances may be determined.

In the case of gases the weight of the gas is usually compared with that of air or hydrogen instead of water.

EXERCISE 200

1. If 2 l. of alcohol weigh 1.58 kg., what is the specific gravity of alcohol?

2. If 5 l. of petroleum weigh $3\frac{1}{2}$ kg., what is the specific gravity of petroleum?

3. If 7 cu. cm. of cork weigh 1.68 g., what is the specific gravity of cork?

4. If 6.5 l. of milk weigh $6.69\frac{1}{2}$ kg., what is the specific gravity of milk?

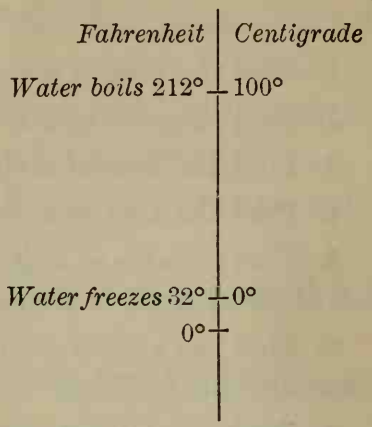
5. The specific gravity of silver is 10.5. Some spoons weighing 371.7 g., thought to be silver, are immersed in a jar full of water and 35.4 cu. cm. of water overflow. Do the spoons stand the specific-gravity test?

6. A body immersed in a liquid always weighs less than it does in air by the weight of the liquid that it displaces. If a piece of gold weighs 9.65 g. in air and 9.15 g. in water, what is its specific gravity?

7. If the specific gravity of steel is 7.8, what is the weight of a cylindrical steel rod 1 m. long and 5 cm. in diameter?

377. Measures of Temperature. The thermometer commonly used in the United States is known as the Fahrenheit. By this thermometer water freezes at 32° and boils at 212°.

The metric system uses the *Centigrade* thermometer. By this thermometer water freezes at 0° and boils at 100°.



378. Reduction from Fahrenheit to Centigrade. If we indicate Fahrenheit by Fah., and Centigrade by C., we have

$212° - 32°$ Fah. $= 180°$ Fah. above freezing $= 100°$ C.;

$1°$ Fah. above freezing $= \frac{100}{180}°$ C. $= \frac{5}{9}°$ C.

Therefore $98°$ Fah. $= 98° - 32° = 66°$ Fah. above freezing, and $66 \times \frac{5}{9}°$ C. $= 36\frac{2}{3}°$ C. That is, $98°$ Fah. $= 36\frac{2}{3}°$ C.

379. Reduction from Centigrade to Fahrenheit. Since

$100°$ C. $= 180°$ Fah. above freezing,

$1°$ C. $= 1.8°$ Fah. above freezing.

Hence $36\frac{2}{3}°$ C. $= 36\frac{2}{3} \times 1.8°$ Fah. above freezing $= 66°$ Fah. above freezing $= 66° + 32°$, or $98°$, Fah. above zero.

EXERCISE 201

Express in Centigrade these temperatures Fahrenheit:

1. 59°.	3. 75°.	5. 108°.	7. 150°.	9. 5°.
2. 42°.	4. 86°	6. 112°.	8. 200°.	10. 10°.

11. 10° below zero. 12. 15° below zero. 13. 25° below zero.

Express in Fahrenheit these temperatures Centigrade:

14. 40°. 15. 75°. 16. 10°. 17. 150°. 18. 200°.

19. 10° below zero. 20. 15° below zero. 21. 25° below zero.

EXERCISE 202

Review Problems

1. Find the area of a square 1.37 m. on a side.

2. Find the area of a rectangle 3.95 m. by 1.36 m.

3. Find the area of a circle whose radius is 75 cm.

4. Find the area of a circle whose diameter is 324 mm.

5. Find the area of a parallelogram whose base is 2.75 m. and whose altitude is 1.65 m.

6. Find the area of a triangle whose base is 325 cm. and whose altitude is 76 cm.

7. Find the altitude of a parallelogram whose area is 3.75 sq. m. and whose base is 1.875 m.

8. Find the base of a triangle whose area is 9.106 sq. m., and whose altitude is 2.9 m.

9. Find the weight of the water required to fill a tank 2.3 m. long, 0.98 m. wide, and 0.56 m. deep.

10. A liter of air weighs 1.292 g. Find the weight of air in a schoolroom 6.8 m. long, 5.2 m. wide, and 3 m. high.

11. How many liters of water will a tank hold that is 2.7 m. long, 1.4 m. wide, and 0.75 m. deep?

12. If a barrel full of water weighs 123 kg., and when empty weighs 10 kg., how many liters will it hold?

13. What part of a hektare is there in a piece of land 125 m. long and 65 m. wide?

14. If a 40-liter keg of olive oil weighs 40.5 kg., and the empty keg weighs 3.9 kg., what does a liter of oil weigh?

15. In a shop where machines are made for foreign trade an order is received to make some wheels 1.34 m. in diameter. How would the workmen express the diameter in inches?

16. A wheel is 0.85 m. in diameter. Express its circumference in inches.

17. What part of a hektare is there in a rectangular field 75 m. long and 65 m. wide?

18. An importer bought a quantity of goods weighing 347.5 kg. How many pounds did they weigh?

19. Taking the distance from Buffalo to Cleveland as 183 miles, how many kilometers is it?

20. Find the side of a square whose area is 2 ha. (2 decimal places).

21. Find the radius of a circle whose area is 62.832 sq. cm. (3 decimal places).

22. Find the diameter of a circle whose area is 125.664 sq. cm. (3 decimal places).

23. A lady shopping in Paris finds that some cloth costs her 7 francs a meter. Taking a franc as 19.3 cents, how much (to the nearest cent) does it cost a yard?

24. A lady bought some cloth in Paris at $8\frac{1}{2}$ francs a meter. How many dollars did it cost per yard?

25. Find the volume of a cylinder whose altitude is 3.5 m., and whose diameter is 2.2 m.

26. How many metric tons of water will it take to fill a cylindrical tank 13 m. in diameter to a depth of 5 m.?

27. A workman finds that the capacity of a cylinder on an imported engine is given as 11,200 cu. cm. He wishes to find what it is in cubic inches. Taking 1 m. as 39.37 in., express the capacity to 2 decimal places.

28. A mechanic wishes to find the weight of a steel bar that he cannot easily remove from a machine. The bar is 1.3 m. long and has a diameter of 3 cm. The specific gravity of steel being 7.8, what is the weight of the bar?

EXERCISE 203

Problems without Numbers

1. Give the metric table of length.

2. How do you reduce millimeters to meters? kilometers to meters? meters to kilometers?

3. How do you reduce inches to meters? meters to inches? kilometers to miles?

4. Give the metric table of square measure.

5. How do you reduce square millimeters to square meters? square meters to square kilometers?

6. How do you reduce hektares to acres?

7. Give the metric table of cubic measure.

8. How do you reduce cubic meters to cubic centimeters? cubic millimeters to cubic centimeters?

9. If you know the metric dimensions of a pile of wood, how do you find the number of steres?

10. Give the metric table of capacity.

11. How do you reduce liters to hektoliters? hektoliters to liters? liters to quarts?

12. If you know the dimensions of a rectangular tank, how do you find its contents in liters?

13. Give the metric table of weight.

14. How do you reduce milligrams to grams? centigrams to kilograms? grams to kilograms?

15. How do you reduce kilograms to grams? to pounds?

16. How would you find the specific gravity of a rectangular block of stone?

17. How would you find the specific gravity of an irregular piece of copper?

CHAPTER XVII

STOCKS AND BONDS

380. Corporations. A company composed of a number of persons authorized by law to do business of a certain nature is called a *corporation.*

A *municipal corporation* is a corporation organized for local government, such as a state, a county, a city, or a town.

A corporation organized to transact private business is called a *stock company.*

This is an age of stock companies. Almost all extensive business operations are carried on by such companies. Money is invested more frequently through or in stock companies than in any other way, and it is one of the necessities of business training that we know the general features of such corporations.

381. Capital Stock. The amount contributed by the members of a corporation to carry on the business is called the *capital stock, capital,* or simply *stock* of the company.

One of the equal parts into which the stock of a company is divided is called a *share* of stock.

The most common amount for a share of stock is $100, although shares of $10, $25, $50, and other amounts are issued.

382. Stock Certificates. A statement issued by a corporation specifying the number of shares of stock owned by the holder is called a *stock certificate.*

This certificate is usually signed by the president and either the secretary or the treasurer. Certificates of bank stock are usually signed by the president and the cashier.

A person who owns one or more shares of stock is called a *stockholder.*

383. Organization of Corporations. The stockholders of a company select a few of their number to manage the business. These members are called *directors.* The directors elect the president, secretary, treasurer, and other officers.

384. Dividends. A sum paid to the stockholders out of the earnings of a company is called a *dividend.*

The directors pay the expenses of running a company from all the earnings (the *gross earnings*), and from the remainder (*net earnings*) they usually lay aside a portion (*undivided profits*) for emergencies, and divide the rest among the stockholders as dividends.

Dividends are a certain per cent of the face value of the stock. Thus a 5% dividend on 80 shares of stock of $100 each is $400.

If a company is losing instead of earning money, stockholders are often required to pay *assessments* on their stock to make up the loss, although the company usually borrows money to help in a temporary emergency.

385. Par Value. The face value of stocks is called the *par value.*

The par value of one share (§ 381) is commonly $100, and should be so taken in all examples in this chapter.

386. Market Value. The price at which stocks sell is called their *market value.* If stocks sell for more than their face value, they are said to be *above par,* or at a *premium;* if for less than their face value, they are said to be *below par,* or at a *discount.*

If a company is prosperous and pays higher dividends, say 10%, than the money would earn in other ways, the stock will be sought by investors, and therefore a share of $100 may sell for a larger sum than the par value, perhaps $150 or more. This is because $10 (10% of $100) is $6\frac{2}{3}\%$ of $150, so that if a man pays $150 for the stock he is still getting $6\frac{2}{3}\%$ on his investment.

On the other hand, if a company is not prosperous and pays low dividends or none at all, then the stock will be below par.

The market value of stocks varies from day to day and often from minute to minute, depending upon the demand and supply.

387. Kinds of Stock. There are two leading kinds of stock, *preferred* and *common.*

Preferred stock is stock that has a fixed rate of dividend that must be paid before any other dividends can be paid.

The dividend on preferred stock is usually 5% to 7%. Investors who wish good security and a definite rate of income from year to year are inclined to buy preferred stock.

Common stock is stock that is entitled to share in the net earnings of a company after the dividends on the preferred stock have been paid.

If the net earnings of a company are very large, common stock may be worth more than preferred stock. For example, if the preferred stock of a company is entitled to 5%, and if the earnings are such as steadily to pay the common stockholders 10%, the common stock will be worth about twice as much as the preferred.

388. Buying Stock. A purchaser usually buys stock through a *broker,* and the broker obtains it through some other broker who has it for sale. The transaction between the brokers is usually carried on in a place called a *stock exchange.*

A stock exchange is merely a large auction room where brokers having stocks to sell may dispose of them to the highest bidder among the brokers who have orders to buy.

Brokers usually charge *brokerage* of $\frac{1}{8}\%$ for buying and $\frac{1}{8}\%$ for selling stock, computed on the par value. If stock is quoted in the newspaper at $96\frac{1}{2}$, it means that a \$100 share will cost the purchaser $\$96\frac{1}{2} + \$\frac{1}{8}$, or $\$96\frac{5}{8}$. It also means that the man who sells the stock receives $\$96\frac{1}{2} - \$\frac{1}{8}$, or $\$96\frac{3}{8}$, since he also has to pay brokerage.

The only fractions used in stock quotations are halves, quarters, and eighths. Fractional parts of a share are not sold. Stock purchased by speculators is usually in 100-share lots, although investors often purchase smaller amounts.

Stocks form a legitimate and common investment of earnings, in spite of the fact that many persons buy them on credit, hoping to make a profit by some sudden rise in value.

EXERCISE 204

Find the cost of the following stocks, adding $\frac{1}{8}\%$ brokerage in each case:

1. 100 shares @ $106\frac{3}{8}$.
2. 125 shares @ $112\frac{5}{8}$.
3. 140 shares @ $115\frac{1}{2}$.
4. 275 shares @ $127\frac{1}{4}$.
5. 350 shares @ $142\frac{3}{4}$.
6. 75 shares @ 62.
7. 80 shares @ $75\frac{1}{4}$.
8. 65 shares @ $98\frac{1}{2}$.
9. 125 shares @ $99\frac{1}{8}$.
10. 25 shares @ $127\frac{1}{2}$.

Find the amount received from the sale of the following stocks, deducting $\frac{1}{8}\%$ brokerage in each case:

11. 75 shares @ 82.
12. 25 shares @ $79\frac{1}{8}$.
13. 50 shares @ $96\frac{7}{8}$.
14. 120 shares @ $98\frac{1}{2}$.
15. 250 shares @ 125.
16. 125 shares @ 110.
17. 150 shares @ $112\frac{1}{2}$.
18. 175 shares @ $121\frac{1}{4}$.
19. 250 shares @ $142\frac{7}{8}$.
20. 275 shares @ par.

21. A man bought 50 shares of stock when it was quoted at $96\frac{7}{8}$, and sold it when quoted at par. Allowing, as usual, $\frac{1}{8}\%$ brokerage on each transaction, how much did he gain?

22. A man bought 75 shares of stock when the market price was 120, and sold it when the market price was 115. Allowing the usual brokerage, how much did he lose?

23. A man bought 125 shares of stock when the market price was $102\frac{7}{8}$, and sold it when the market price was still the same. Allowing the usual brokerage, how much did he lose on the transaction?

24. A man has \$3875 to invest. He decides that he will buy a certain stock quoted at $96\frac{3}{8}$. Allowing the usual brokerage, how many shares can he buy, and how much money will he have left?

389. Newspaper Quotations. In the daily newspapers may be found quotations showing the prices at which leading stocks were sold in the last session of the stock exchange.

The great center for trading in railway stocks is the New York Stock Exchange, on Wall Street. Many large cities have stock exchanges in which local stocks are bought and sold. Stocks of manufacturing and industrial corporations are known as *industrial stocks.*

The following are specimens of newspaper quotations:

American Express Co.	$208\frac{7}{8}$	Iowa Central	$31\frac{5}{8}$
Atchison, Topeka, & S. F.	$101\frac{7}{8}$	Louisville & Nashville	$127\frac{3}{8}$
Baltimore & Ohio	$111\frac{3}{4}$	Missouri Pacific	$68\frac{3}{8}$
Central of N.J.	231	N.Y. Central	$131\frac{1}{8}$
Chicago & Alton	$68\frac{1}{4}$	Northern Pacific	$132\frac{7}{8}$
Illinois Central	$149\frac{7}{8}$	Union Pacific	$184\frac{1}{8}$

In all cases remember to take into account the brokerage of $\frac{1}{8}\%$.

EXERCISE 205

Using the above quotations, find the cost of:

1. 150 shares of Iowa Central.
2. 125 shares of Illinois Central.
3. 75 shares of Baltimore & Ohio.
4. 110 shares of Chicago & Alton.
5. 25 shares of American Express Co.
6. 175 shares of Louisville & Nashville.
7. 180 shares of Central of New Jersey.
8. 50 shares of Atchison, Topeka, & Santa Fe.

Find the amount received from the sale of:

9. 25 shares of Missouri Pacific.
10. 35 shares of New York Central.
11. 75 shares of Northern Pacific.

Find the gain or loss in buying 25 shares of the following stocks as quoted on page 363, and selling at the price given below:

12. American Exp., 210.
13. Atchison, $103\frac{1}{2}$.
14. Balt. & Ohio, $109\frac{7}{8}$.
15. Central of N.J., $225\frac{1}{2}$.
16. Chic. & Alton, $68\frac{3}{4}$.
17. Ill. Cent., $151\frac{1}{8}$.
18. Iowa Cent., $33\frac{1}{2}$.
19. Louisv. & N., 129.
20. Missouri Pac., 71.
21. N.Y. Cent., $137\frac{1}{2}$.
22. Northern Pac., 135.
23. Union Pacific, $180\frac{1}{2}$.

24. A man bought 250 shares of American Express at the price quoted on page 363, and sold 100 shares at 207 and the rest at $209\frac{1}{2}$. Did he gain or lose on the lot, and how much?

25. A man bought 250 shares of Baltimore & Ohio at the quotation in Example 14. He kept it a year, receiving meantime a dividend of 6%, and sold it at 112. Money being worth 5%, did he gain or lose, and how much?

26. A man bought 25 shares of New York Central at 138, 125 shares of Iowa Central at 29, and 50 shares of Illinois Central at 150. He sold them at the quotations given on page 363. Did he gain or lose, and how much?

27. A man bought 150 shares of Atchison at $101\frac{7}{8}$. He borrowed the money to pay for it, at 5% interest. He kept the stock 6 months and sold it at the quotation given on page 363, having meantime received a dividend of 3%. Did he gain or lose, and how much?

28. A man bought 75 shares of American Express, 25 shares of Iowa Central, 50 shares of Chicago & Alton, and 150 shares of New York Central, at the quotations given on page 363, and sold them at the quotations given at the top of this page. Did he gain or lose, and how much?

390. Declaring Dividends. If the capital of a company is \$150,000, and the net earnings for six months amount to \$4875, the directors might declare a dividend of $3\frac{1}{4}\%$, since \$4875 is $3\frac{1}{4}\%$ of \$150,000. They will probably, however, declare a dividend not exceeding 3% (\$4500) and carry the balance as undivided profits to the surplus fund to meet an emergency.

EXERCISE 206

1. If the capital is \$125,000, and \$3125 is distributed as dividends, what is the rate of dividend?

2. If the capital is \$75,000, and a dividend of $3\frac{1}{2}\%$ is declared, what is the amount of the dividend?

3. If a company declares a dividend of $4\frac{1}{2}\%$, and this amounts to \$4050, what is the capital?

4. If a company declares a dividend of $5\frac{1}{2}\%$, and this amounts to \$4,152,500, what is the capital?

5. If a company declares a dividend of 4% on its capital stock, how much dividend will a man receive who owns 75 shares?

6. A company having a capital of \$75,000 loses \$1875. If an assessment is made to pay this loss, what per cent of his stock must each stockholder be assessed?

7. The net earnings of a company having a capital of \$150,000 amounted to \$7677.56 in 6 months. The company carried \$927.56 to its surplus fund and divided the remainder in dividends. What was the rate of dividend?

8. A railroad company having a capital of \$178,632,000 earns \$98,369,059 in one year. Its operating expenses are \$75,803,333, and it pays interest on its debt and carries to surplus a total of \$11,847,806, dividing the remainder in dividends. What is the rate of dividend for the year?

391. Rate of Income. If stock paying a 6% dividend is bought at $149\frac{7}{8}$, or 150 including the brokerage, the owner receives \$6 on every \$150 invested. He therefore receives \$6 ÷ \$150, or 4% on his investment.

Always add the brokerage to the market price.

EXERCISE 207

Given the rate of dividend and the market price, find the rate of income on the investment:

1. 5%, $124\frac{7}{8}$.
2. 8%, $199\frac{7}{8}$.
3. 7%, $174\frac{7}{8}$.
4. 7%, $139\frac{7}{8}$.
5. 9%, $149\frac{7}{8}$.
6. 10%, $124\frac{7}{8}$.
7. $4\frac{1}{2}$%, $112\frac{3}{8}$.
8. 8%, $159\frac{7}{8}$.
9. 7%, $87\frac{3}{8}$.
10. $4\frac{1}{2}$%, $74\frac{7}{8}$.
11. 4%, $94\frac{7}{8}$.
12. $4\frac{1}{2}$%, $82\frac{3}{8}$.
13. 5%, $109\frac{7}{8}$.
14. 4%, $98\frac{7}{8}$.
15. 6%, $120\frac{3}{8}$.
16. 7%, $130\frac{1}{8}$.
17. 10%, 190.
18. 15%, $299\frac{7}{8}$.

19. Which will bring the better income, a 5% promissory note or a 6% stock quoted @ $119\frac{7}{8}$?

20. Which will bring the better income, a 5% promissory note or a 7% stock quoted @ 150? How much better?

21. Which will bring the better income, a 5% promissory note or an 8% stock quoted @ $149\frac{7}{8}$? How much better?

22. Which will bring the better income, a 6% stock quoted @ $139\frac{7}{8}$ or a 5% stock quoted @ $119\frac{7}{8}$? How much better?

23. If an investor receives \$6 on each share of stock he owns, and this is 5% on what he paid for the stock, how much did it cost him? What was the quoted market price?

24. If an investor receives 6% on his investment in a stock that pays 9% dividends, how much did the stock cost him? What was the quoted market price?

392. Bonds. A written or printed promise to pay a specified sum at a given time, signed by the maker and often bearing his seal, is called a *bond*.

Bonds are usually secured by *mortgages* on the property of the maker, these being agreements by which the holders of the bonds may sell the property if either the interest or principal is unpaid.

393. Kinds of Bonds. Bonds are called *coupon bonds* if they have small promissory notes (coupons) annexed, which are cut off and presented for payment when the interest is due. They are called *registered bonds* if the owner's name is registered on the books of the company.

394. Illustrative Problems. How much is the income on \$7000 worth of 4% bonds?

The income is 4% of \$7000, or \$280.

Three-year bonds, paying 5%, are quoted at $102\frac{7}{8}$. What rate of income would an investor receive on the cost?

They cost $\$102\frac{7}{8} + \$\frac{1}{8}$, or \$103, for every \$100 face value.

They pay \$5 for every \$100 face value, but at the end of 3 years the investor receives only \$100 instead of \$103, or an average loss of \$1 a year (not counting interest on this amount). Therefore the net income is only \$5 − \$1, or \$4, a year on the \$103 invested.

Then \$4 ÷ \$103 = 0.03883+, the approximate rate of income.

EXERCISE 208

Find the income on the following bonds:

1. 4%, \$9000.
2. 5%, \$7000.
3. 6%, \$6000.
4. 3%, \$8500.
5. 4%, \$6500.
6. $3\frac{1}{2}$%, \$13,500.
7. $4\frac{1}{2}$%, \$17,000.
8. $5\frac{1}{2}$%, \$12,000.
9. $6\frac{1}{2}$%, \$17,000.
10. 6%, due in 2 years, quoted at $101\frac{7}{8}$.
11. 7%, due in 4 years, quoted at $101\frac{7}{8}$.
12. 5%, due in 10 years, quoted at $112\frac{3}{8}$.

EXERCISE 209

Problems without Numbers

1. If you know the newspaper quotation on a certain stock, how do you find the cost of a share?

2. If a certain stock is quoted at a specified amount above par, how do you find the cost of a share?

3. About what would have to be the rate of dividend on a railroad stock to have it at par? to have it below par?

4. If you own some railroad stock and should sell it through a broker and buy it back at the same quoted price, how would you compute your loss?

5. If you own some stock and wish to sell it, and know the market price, how much will you receive for the stock?

6. Knowing the capital and the net earnings of a company, and the amount to be carried to surplus, how do you find the rate of dividend?

7. Knowing the quotation on a certain stock, and the rate of dividend, how do you find the rate of income on the amount invested?

8. How do you find the income on a certain number of bonds of a given par value, the rate being known?

9. How do you find the income on a certain number of bonds of a given par value, the time they have to run and the rate being known?

10. If the rate of interest on some bonds is known, and also the amount of income, how do you find the par value of the bonds?

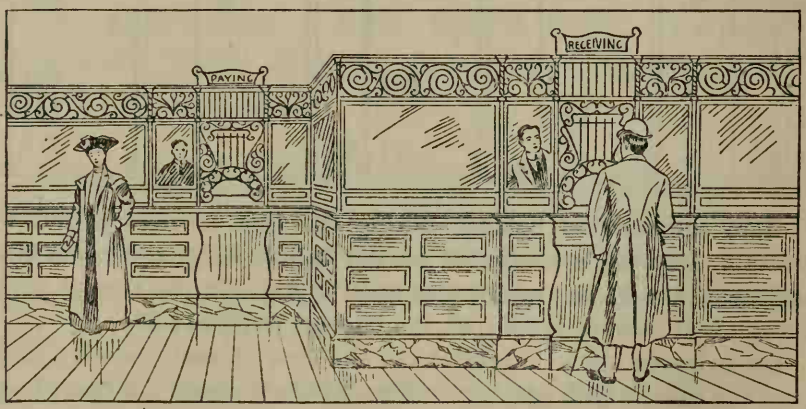
11. What is the difference between preferred stock and common stock? between common stock and a bond? How do you decide which is best for an investment?

CHAPTER XVIII

BANKING

395. Banks. A corporation authorized by law to receive and loan money and to perform certain other financial operations is called a *bank.*

Formerly there were many private banks, but now banks are usually corporations.



This picture represents the interior of a city bank. The clerk who receives money is called the *receiving teller;* the clerk who pays out money is called the *paying teller.*

396. Savings Banks. The banks where people keep their savings, receiving a low rate of interest, are carefully guarded by the laws of most of the states. These banks are called *savings banks.*

Nearly 9,000,000 persons in the United States have deposits in savings banks. These deposits amount in all to nearly $4,000,000,000.

397. Interest in Savings Banks. Usually savings banks pay interest semiannually, adding this to the principal on deposit. This is, therefore, a case of compound interest. The following is a specimen account at a bank paying 2% every six months (4% a year, payable semiannually), on the smallest balance on deposit during the period.

Date		Deposits		Interest		Payments		Balance	
1910									
July	1	$600	50					$600	50
July	20	75						675	50
Sept.	6					$120		555	50
Dec.	7	60						615	50
Dec.	20					65		550	50
1911									
Jan.	1			$11				561	50
May	9	200						761	50
July	1			11	22			772	72

Here the smallest balance during the first interest period was $550.50. Interest is computed on the dollars only, the cents being neglected. At 2% the interest on $550 is $11. In the second period the smallest balance is $561.50, the interest being $11.22.

Some banks allow interest from the first of each month; others from the first of each quarter; others, as above, from the first of each half year. The interest is computed on the smallest balance on hand between this day and the next interest day, and is usually added to the principal every half year.

In the above example, if the interest period had been 3 months and the interest had been credited every 6 months, then the interest would have been:

First quarter, 1% of $555 = $5.55.
Second quarter, 1% of $550 = 5.50. Credit $11.05.
Third quarter, 1% of $561 = 5.61.
Fourth quarter, 1% of $561 = 5.61. Credit $11.22.

In this case, therefore, the difference in interest would have been only five cents.

EXERCISE 210

In the following, find the balance due on the last date, interest 2% every six months on the smallest balance on deposit during six months preceding Jan. 1 and July 1:

1. Deposits: Jan. 1, 1910, $750; Feb. 3, $425; June 1, $37.50. Payments: Mar. 7, $230; May 6, $26.75. Balance, July 1, 1910.

2. Deposits: Jan. 1, 1910, $675.50; Mar. 2, $923.75; Sept. 5, $327.60. Payments: Feb. 7, $327.40; Oct. 10, $750. Balance, Jan. 1, 1911.

3. Deposits: Dec. 31, 1910, $500; Feb. 2, 1911, $300; Mar. 16, $250; Aug. 1, $50; Oct. 5, $125. Payments: Mar. 6, 1911, $325; May 11, $275; Sept. 12, $25; Nov. 15, $100. Balance, Jan. 1, 1912.

In the following, find the balance due on the last date, interest being 1% every three months on the smallest balance on deposit during the three months preceding Jan. 1, Apr. 1, July 1, and Oct. 1, and all interest being payable on Jan. 1 and July 1:

4. Deposits: Mar. 1, 1910, $200; June 3, $675; July 1, $350. Payments: Mar. 8, $50; May 17, $60; July 8, $50. Balance, Jan. 1, 1911.

5. Deposits: Apr. 1, 1911, $375; May 25, $460; June 14, $380; Aug. 8, $750. Payments: May 3, $75; Sept. 6, $50; Nov. 8, $130; Feb. 1, 1912, $200. Balance, July 1, 1912.

6. A man on July 1 deposited $2500 in a savings bank and left it for 3 years. The rate of interest was 4% a year and the interest was added to the principal semiannually. How much less interest did he receive than he would have received had he loaned the money at 5% simple interest?

398. Commercial Banks. Most banks receive money on deposit and pay it out on written orders, allowing no interest. Such banks are called *commercial banks.*

They are also known as *banks of deposit.* They afford a safe place for keeping money. They make their profit by loaning money. Savings banks are usually allowed to loan only on real estate or certain specified bonds of high grade as security, but commercial banks loan on promissory notes. Commercial banks also transmit money from one place to another, and make collections, as described later.

399. Depositing Money. When a reliable person wishes to open an account with a bank he begins by depositing some money. In depositing money a *deposit slip* is filled out each time, as here shown.

SECOND NATIONAL BANK
SPRINGFIELD, ILL.

Deposited for the account of
J. P. Roberts

Date *Feb. 3* 19*12*

Bills	*75*	
Gold	*10*	
Silver and small coin	*20*	
Check on *1st Nat.* Bank	*126*	*75*
Check on *Chicago Nat.* Bank	*37*	*60*
Total	*269*	*35*

The money and deposit slip are then presented to the *receiving teller* or to the *cashier*, together with a bank book in which the teller or cashier enters the amount deposited.

Many banks do not insist on having inserted in a deposit slip the name of the bank on which a check is drawn.

EXERCISE 211

Make out deposit slips for the following deposits, naming some bank in your vicinity:

1. Bills, $478; gold, $20; silver, $12; check on First National Bank, $93.75.

2. Bills, $295; silver, $27; checks on Farmers Bank, $275.50; Second National, $127.62.

3. Bills, $427; gold, $25; silver, $65; checks on Union Trust, $92.50; Manufacturers Bank, $326.75.

4. Bills, $396; gold, $75; silver, etc., $73.75; checks, $72, $40, $127.50, $345.75.

5. Bills, $426; silver, etc., $63.30; checks, $32.75, $16.50, $48.90, $123.45, $63.50.

6. Bills, $275; silver, etc., $50.75; checks, $27.62, $48.35, $62.90, $91.75, $126.30.

7. Bills, $300; silver, etc., $40; checks, $37.75, $52.50, $46.75, $75, $31.90, $43.

8. Bills, $250; silver, etc., $20; checks, $48, $70, $13.62, $19.83, $76.75, $49.32.

9. Bills, $420; gold, $75; silver, etc., $40; checks, $39.50, $42.75, $61.80, $120.35.

10. Bills, $37; checks, $14.75, $35.20, $48, $29.30, $62.75, $81.50, $8.15, $7.80.

11. Checks, $48.50, $16.90, $17.30, $21.25, $48.02, $37.70, $42.72, $30.96, $48.70, $9.20.

12. Checks, $148.62, $7.90, $4.08, $39.70, $62.80, $47, $106.05, $32.75, $81.22, $62.75, $38.40.

13. Bills, $428; gold, $80; silver, etc., $45.75; checks, $272.75, $121.62, $396.80, $92.75, $38.75, $326.42.

400. Drawing Money. When a person opens an account at a bank he is given a *check book*. Each page has one or more *checks* and *stubs*. On the presentation of a check properly filled out, signed, and indorsed the bank will pay the amount specified, if the drawer's account is sufficient, and the holder is properly identified.

Stub	*Check*
No. *575*	No. *575* MADISON, WIS., *Feb. 1*, 19*12*
Date *Feb. 1, 1912*	FIRST NATIONAL BANK OF MADISON
To *B. H. Roberts*	Pay to the order of
For *Bal. acct.*	*B. H. Roberts* $*75.*$\frac{75}{100}$
Amt. $*75.*$\frac{75}{100}$	*Seventy-five and* $\frac{75}{100}$ ~~~~ Dollars
	Rufus J. Perkins

In this case Rufus J. Perkins is the *drawer* and B. H. Roberts is the *payee*. The stub is kept by the drawer as a memorandum.

A check is usually payable to "self", in which case only the drawer can collect it; or to the order of the payee, in which case he must write his name across the back (*indorse* it); or to the payee or bearer, in which case any one can collect it; or to "cash", in which case any one can collect it. Of these forms, the one to the order of the payee, as above shown, is the most common.

In the above case, if B. H. Roberts draws the money he writes simply his name across the back. This is called a *blank indorsement*, or *indorsement in blank*. If he wishes to use it in paying some other person, for instance, Jacob C. Sturgess, he would write:

Pay to the order of
Jacob C. Sturgess
B. H. Roberts

This is called a *full indorsement*, and only Jacob C. Sturgess could collect it, unless he should indorse it in favor of some one else.

EXERCISE 212

1. Make out a check for $75.60, payable to the order of J. P. Flint. Indorse the check in blank.

2. Make out a check for $62.75, payable to the order of M. G. Thompson. Write the stub. Indorse the check to the order of R. P. Hull.

3. If a man had a balance of $273.75 at the beginning of last month, and has since given checks for $15.63, $28.96, $14.60, $5.75, $12.30, and $38.09, and made no deposits, what is his balance now?

4. If a merchant had a balance of $496.75 on the first of last month, and has since deposited $39.80, $126.70, and $98.90, and given checks for $72.83, $48.65, and $271.96, what is his balance now?

5. If a merchant had a balance of $592.80 on the fifteenth of last month, and has since deposited $98.60, $48.90, $5.70, $19.68, $72.75, $37.63, and $100, and has given checks for $38.40, $125.75, $30.42, and $75, what is his balance now?

6. A merchant's balance on the first of last week was $575.72. He has since deposited $162.75, $380.96, $401.75, $25.60, $75, and $129.80. He has given checks for $272.70, $92.35, $178.62, $325.25, and $68.70. What is his balance now?

7. A merchant's balance on the first of last week was $298.72. He has since deposited $42.80, $121.62, $148.90, $275.26, $72.50, $69.75, $14.02, and $9.70. He has given checks for $38.72, $129.40, $42.60, $25.80, $206.07, and $42.96. He has also paid by check a bill of $82.50 less 10% and 5%, and one of $48.75 less 3%. What is his balance now?

401. Borrowing from Banks. Banks make a business of lending money on short terms to reliable borrowers, taking promissory notes as evidence of the debt. Such a note is made more secure in one of three ways:

1. It is made payable to some responsible person who *indorses* it; that is, he signs his name across the *back*. This makes him responsible to the bank if the maker does not pay it, and if he is promptly notified.

2. It is signed by two or more makers, and is made payable to the bank. Such notes usually read, "We jointly and severally promise to pay," so that the bank may hold all the makers together responsible, or any one of them separately.

3. It is made payable to the order of the bank and is secured by the deposit of stocks, bonds, or other securities. These are called *collateral securities.*

402. Form of a Bank Note. The first of the above forms is as follows:

INDIANAPOLIS, IND., *May 15,* 19*13*

Sixty days after date, *I* promise to pay to the order of

S. M. Stockwell ~~~~~~~~~~ $*2000.*^{00}

Two Thousand ~~~~~~~~~~ Dollars

at **The First National Bank, Indianapolis, Ind.**

Value received *John Wilson*

Due *July 14, 1913*

This note is due, or it *matures*, on July 14. Formerly three *days of grace* were allowed on a note, so that the above note would mature on July 17, but this custom has been abolished in most states and is not considered in this book.

Bank notes are usually made for 30, 60, or 90 days.

When the time of a note is expressed in months, calendar months are used in computing the date of maturity; when the time is expressed in days, the exact number of days is used.

403. Bank Discount. The note given in § 402 does not mention any interest. This is because banks generally require the interest to be paid in advance.

Interest paid on a note in advance is called *bank discount*, and such a note is said to be *discounted*.

The rate of discount is understood to be by the year, as in the case of interest, unless there is a statement to the contrary.

404. Proceeds. The balance after deducting the discount from the face of a note is called the *proceeds*.

For example, what are the discount and the proceeds on the note given in § 402, the rate being 6%?

Face of note	$2000
Discount on $2000, 60 da., @ 6% .	20
Proceeds	$1980

EXERCISE 213

Find the discount and the proceeds on the following:

1. $250, 30 days, 6%.
2. $575, 30 days, 6%.
3. $625, 60 days, 6%.
4. $1250, 60 days, 5%.
5. $2250, 60 days, 5%.
6. $175, 30 days, 5%.
7. $1275, 60 days, $5\frac{1}{2}$%.
8. $2350, 60 days, $5\frac{1}{2}$%.
9. $275.50, 90 days, 6%.
10. $527.50, 90 days, 6%.

11. Make out a 60-day note for $1250, dated to-day, payable to R. H. Dodge, or order, at some bank in your vicinity. Discount it at 6% and find the proceeds.

12. Make out a 90-day note for $725, dated to-day, payable to P. M. Milton, or order, at some bank in your vicinity. Discount it at 6% and find the proceeds.

13. A cattle dealer needs $1700 for 30 days. He puts up collateral at a bank and borrows this amount. How much discount must he pay, the rate being 5%?

405. Discounting Interest-bearing Notes. It frequently happens that a manufacturer or dealer holds notes of his customers and wishes to get the money on them before they are due. In this case he may indorse the notes and sell them to a bank for the sum due at maturity, less the discount.

For example, a manufacturer has a note of one of his customers for $350, dated October 15 and due 3 months after date, with interest at 5%. He discounts this note at a bank at 6% on Nov. 16. What are the proceeds?

Face of the note	$350.00
Interest for 3 mo., 5%	4.38
Amount due at maturity . . .	$354.38
Discount, 60 da., 6%	3.54
Proceeds	$350.84

Here the time from November 16 to January 15, 60 days, is the *term of discount.*

Bankers compute the discount for the exact number of days in the term of discount. It should be noticed that *interest is computed on the face of the note at the beginning of the term, discount on the amount of the note at maturity.*

EXERCISE 214

Find the discount and the proceeds:

	Face	*Date*	*Due*	*Interest*	*Date of Discount*	*Rate of Discount*
1.	$250	Jan. 1	July 1	5%	Jan. 15	6%
2.	$375	Oct. 7	Jan. 7	5%	Oct. 7	6%
3.	$425	Oct. 18	Feb. 18	6%	Oct. 18	6%
4.	$550	Sept. 20	Jan. 20	5%	Nov. 1	6%
5.	$675	Nov. 16	May 16	6%	Dec. 1	5%
6.	$1250	Aug. 7	Feb. 7	5%	Nov. 15	5%
7.	$375.50	May 16	Sept. 16	6%	July 1	6%
8.	$827.25	July 1	Feb. 1	6%	Oct. 1	6%

9. A 60-day note for $1225, without interest, dated June 20, was discounted the day of its date at 6%. Find the discount.

10. A 60-day note for $1225, with interest at 5%, dated June 20, was discounted the day of its date at 6%. Find the discount.

11. A 60-day note for $1225, with interest at 6%, dated June 20, was discounted the day of its date at 5%. Find the discount.

12. A 60-day note for $1225, with interest at 6%, dated June 20, was discounted on July 20 at 6%. Find the discount.

13. A 30-day note for $2500, without interest, dated April 1, was discounted 15 days later at 6%. Find the discount and the proceeds.

14. A 90-day note for $1650, with interest at 5%, is discounted 30 days after date at 6%. Find the proceeds.

15. A 90-day note for $1650, with interest at 6%, is discounted 30 days after date at 5%. Find the proceeds.

16. A 90-day note for $1650, with interest at 6%, is discounted the day of its date at 6%. Find the proceeds.

17. A 90-day note for $1650, without interest, is discounted the day of its date at 6%. Find the proceeds.

18. How much greater are the proceeds on a note for $675 due 30 days hence, than on one for the same amount due 90 days hence, there being no interest in either case, and the rate of discount being 6% ?

19. On which can a man realize the more at a bank where the discount rate is 6%, a 6-months note for $2550 with interest at 6%, or a 4-months note for $2600 without interest? How much more?

406. Drafts. A written order by which one person directs another to pay a certain sum of money is called a *draft.*

A check is therefore a kind of draft. Usually the word *check* is used for an order on a bank, signed by a private individual; *draft* is used for an order other than a check, on one private individual or corporation signed by another. A draft may therefore be made by an individual on a company, by one bank on another, by a company on a private individual or firm, and so on.

407. Commercial Drafts. A draft made by one business house on another to secure the payment of a debt is called a *commercial draft.*

The following is a common form of commercial draft:

Cleveland, Ohio, *Aug. 8,* 19*12*

At *thirty* days sight pay to the order of

myself

$*975.*^{00}

Nine Hundred Seventy-five and $\frac{no}{100}$ Dollars

To *Robt J. Clements*

Louisville, Ky. *J. M. Simons*

Accepted Aug. 10, 1912
Robert J. Clements

In this case J. M. Simons has sold Robert J. Clements $975 worth of goods on 30 days' credit. When he sends the bill he also sends this draft. Mr. Clements writes "Accepted, Aug. 10, 1912, Robert J. Clements" across the face, and returns it to Mr. Simons. This becomes thereby a promise to pay $975 "at 30 days sight," that is, on Sept. 9, or 30 days after the acceptance on Aug. 10. Mr. Simons may now discount this at a bank, just as he would a promissory note.

This is coming to be a common way of borrowing money at a bank, replacing the older method by a promissory note. A general name for such evidences of debt as can be sold to banks is *negotiable paper.*

In this draft Mr. Simons is the *drawer*, Mr. Clements the *drawee*, and as the draft is payable to him, Mr. Simons is also the *payee.*

EXERCISE 215

1. A draft dated Aug. 8, at 30 days sight, for $975, is accepted on Aug. 10, and is discounted on Aug. 16 at 6%. What are the proceeds?

2. A draft dated Sept. 1, at 30 days sight, for $350, is accepted on Sept. 4, and is discounted on Sept. 7 at 6%. What are the proceeds?

3. A draft dated April 2, at 60 days sight, for $725, is accepted on April 7, and is discounted on April 12 at 6%. What are the proceeds?

4. A draft dated Oct. 15, at 60 days sight, for $1225.75, is accepted on Oct. 19, and is discounted on Oct. 25 at 6%. What are the proceeds?

5. A draft dated Feb. 7, at 90 days sight, for $1675.50, is accepted on Feb. 14, and is discounted on Feb. 22 at 5% What are the proceeds?

6. A draft dated Aug. 6, at 90 days sight, for $2170.60, is accepted and discounted on the day of its date. The rate of discount being $5\frac{1}{2}$%, find the proceeds.

7. A manufacturer sells goods listed at $725.50, discounted at 10% and 5%, and draws on the buyer at 60 days sight. The buyer accepts the draft and it is discounted 10 days later at 6%. Write the draft and find the proceeds.

8. A manufacturer sells goods listed at $1672.25, discounted at 8% and 3%, and draws on the buyer at 60 days sight. The buyer accepts the draft and it is discounted 5 days later at 5%. Write the draft and find the proceeds.

9. A wholesale house sells goods listed at $2737.50, discounted at 15%, 8%, and 3%, and draws on the buyer at 90 days sight. The buyer accepts the draft and it is discounted 30 days later at 5%. Write the draft and find the proceeds.

408. Finding the Face. If it is desired to borrow exactly \$1000 on a 60-day note that is to be discounted at a bank at 6%, the face of the note must evidently be such a sum as discounted will leave \$1000 as the proceeds.

First consider the face of the note as	\$1.00
Then the discount, @ 6% for 60 da., is . . .	0.01
Therefore the proceeds are	\$0.99

Hence \$1000 must be the proceeds of as many times \$1 as \$1000 ÷ \$0.99, or \$1010.10.

That is, the face of the note must be \$1010.10.

EXERCISE 216

1. What must be the face of a 60-day note in order that when discounted the day of its date at 6% the proceeds shall be \$497.50?

2. What must be the face of a 90-day note in order that when discounted the day of its date at 5% the proceeds shall be \$1777.50?

3. What must be the face of a 30-day note in order that when discounted the day of its date at 6% the proceeds shall be \$500?

4. A manufacturer sold some goods for \$1950 less 10% discount. He took the purchaser's note for 60 days for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

5. A manufacturer sold some goods for \$2400 less 10% and 5%. He took the purchaser's note for 30 days for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

6. A wholesale dealer sold some goods for \$3200 less 10% and 3%, due in 90 days. He took the purchaser's note for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

EXERCISE 217

Review Problems

1. Find the amount of $500, compounded semiannually, at the rate of 4% a year, for 5 years.

2. A man deposits $750 in a savings bank on July 1, 1910. His further deposits are as follows: Aug. 1, 1910, $300; Sept. 15, 1910, $250; Dec. 31, 1910, $1000; June 15, 1911, $500. He draws out as follows: Oct. 12, 1910, $400; June 20, 1911, $375; Aug. 2, 1911, $200. The savings bank pays 1% every three months on the smallest balance for the preceding quarter, on Jan. 1, Apr. 1, July 1, and Oct. 1. Find the balance on Jan. 1, 1912.

3. Make out a deposit slip for the following deposits, naming some bank in your vicinity: bills, $575; gold, $35; silver, $72.25; checks, $48.25, $72.50, $61.30, $227.52, and $86.95. If your balance was $326.42 before, what was it after the deposit?

4. A merchant's balance on the first of the month was $1725.60. He has since deposited $37.50, $296.75, $21.30, $572.32, and $217.65. He has given checks for $128.92, $27.36, $18.92, $78.76, and $273.96. How much is his present balance?

5. Make out a 90-day note for $1250, dated to-day, payable to R. P. Foster, or order, at some bank in your vicinity. Discount it at 6%. If with the proceeds you buy 10 shares of stock when quoted at $108\frac{3}{8}$, how much money will you have left?

6. Which is the greater, the proceeds of a note for $750 for one year, drawing interest at 6%, and discounted on the day of its date at 5%, or the amount of a note for $700 for one year at 6%? How much greater?

EXERCISE 218

Problems without Numbers

1. How is the interest due on the first of the year on a savings-bank account ascertained?

2. Which pays the better interest, if the money is left undisturbed for five years, a 4% promissory note or a savings-bank deposit at 4%? Why?

3. How do you fill out a deposit slip? After entering the items what operation do you perform? How do you make sure of the result?

4. If you know a man's balance in a bank a week ago and his deposits and checks since, how do you proceed to find his balance now?

5. How do you find the discount on a promissory note? How do you find the proceeds?

6. If a note drawing a certain rate of interest is discounted on the day it is made, at the same rate, are the proceeds greater than the face, or equal to it, or less? Why is this?

7. How can a manufacturer discount a claim against a purchaser, the claim not being yet due? How is the discount found?

8. If you know the proceeds and rate of discount, how do you find the face of a note?

9. If you know the face of a note and the proceeds, how do you find the discount? the rate of discount?

10. If a dealer buys some produce and agrees to pay for it in 90 days, with interest at 6%, and the seller takes a note and at once discounts it at 5%, will he receive more than the selling price, or the same amount, or less? Why?

CHAPTER XIX

EXCHANGE

409. Paying Bills at a Distance. If a man owes money at a distance, he may pay it in any one of several ways :

(1) He may send or carry the money.
(2) He may send a money order.
(3) He may send his check.
(4) He may send a bank draft.
(5) He may wait until the one whom he owes draws upon him, as mentioned in § 407.

410. Exchange. The payment of money by means of checks, money orders, or drafts is called *exchange.*

411. Paying by Money Order. A man who owes money at a distance may purchase a postal money order at a post office, an express money order at an express office, or a telegraphic money order at a telegraph office.

The cost of postal money orders is as follows: For sums not exceeding $2.50, 3 cents; over $2.50 to $5, 5 cents; over $5 to $10, 8 cents; over $10 to $20, 10 cents; over $20 to $30, 12 cents; over $30 to $40, 15 cents; over $40 to $50, 18 cents; over $50 to $60, 20 cents; over $60 to $75, 25 cents; over $75 to $100, 30 cents. Express money orders cost about the same. Postal money order rates to foreign countries vary with the country and the amount, — from 8 cents for sums not exceeding $10 to some countries, to $1 for sums over $90, but not exceeding $100, to other countries.

The rate for telegraphic money orders is double the cost of a ten-word message, plus 25 cents if the sum does not exceed $25, and plus 1% of the amount of the order for larger sums.

412. Paying by Check. Usually local bills are paid by check. A check sent to pay a bill at a distance is deposited by the payee in the bank where he keeps his account, and this bank sends it to the debtor's bank for collection, often making a slight charge for the trouble.

In some cities the banks agree to charge for collecting all out-of-town checks. This costs the payee of the check from ten cents upward, according to the amount involved.

A bank may guarantee that a check is good. This is done by the cashier writing the word "Certified" across the face, with the date and his signature.

EXERCISE 219

Find the cost of postal money orders for:

1. \$1.75.	4. \$9.92.	7. \$29.88.	10. \$47.50.
2. \$2.75.	5. \$19.20.	8. \$31.75.	11. \$65.90.
3. \$7.50.	6. \$25.50.	9. \$39.95.	12. \$96.75.

13. A man owing \$250 sent his check in payment. The creditor deposited the check and the bank charged him 0.1% for collecting. What was the net amount received by the creditor?

14. A manufacturer received checks for \$1200, \$650, \$2250, and \$1750. His bank charged him 0.1% for collecting. How much in all did he pay for the collecting of the four checks?

15. A man wishes to send a check to a friend for an amount such that after the bank has deducted 0.1% for collecting there will remain exactly \$3246.75. What shall he make the face of the check?

16. A man deposited a check for \$1500 in a bank, and the net proceeds after the bank had deducted its charge for collecting were \$1497. What was the rate that the bank charged for collecting?

413. Bank Drafts. A check drawn by one bank on another is called a *bank draft*, or simply a *draft*.

Banks usually charge from 0.1% to $\frac{1}{4}$% on the face of a draft to pay for their trouble and expense.

No. *2796*

FIRST NATIONAL BANK OF SALEM

SALEM, OREGON, *April 16,* 19*12*

Pay to the order of.....*Robert Stevens*.....$*95.*$\frac{50}{}$

Ninety-five and $\frac{50}{100}$ ~~~~~~~~~~~~~~~~~~~~ Dollars

To the Second National Bank,
New York City

D. B. Roberts
Cashier

If Robert Stevens of Salem, Oregon, owed Tiffany & Co. of New York $95.50, he might purchase a draft like the above and indorse it:

Pay to the order of Tiffany & Co., New York
Robert Stevens

He would then send the draft to Tiffany & Co., who would indorse it and deposit it in their bank for collection.

If Robert Stevens had an account at the First National Bank of Salem, he might not be charged any exchange. Otherwise he would probably be charged about 0.1% *premium*, or ten cents in this case.

Sometimes the premiums on drafts are quoted at a number of cents per thousand dollars.

EXERCISE 220

Find the cost of drafts for the following amounts at the premiums stated:

1. $2500, 0.1%.
2. $3750, 0.1%.
3. $4850, 0.1%.
4. $4800, 0.2%.
5. $5600, 0.2%
6. $250, $\frac{1}{8}$%.
7. $750, $\frac{1}{5}$%.
8. $640, $\frac{1}{4}$%.
9. $1280, $\frac{1}{4}$%.

414. Commercial Drafts. The fifth method mentioned in § 409 is that in which a commercial draft is used. These drafts have already been described (§ 407) and their use in connection with borrowing money has been explained.

For purposes of exchange, if J. M. Simons of Cleveland has sold to Robert J. Clements of Louisville $975 worth of goods on 30 days' credit, he may *draw upon* Mr. Clements by such a draft as that described in § 407. Instead, however, of sending the draft by mail and having it accepted, he may deposit it at once in his bank at Cleveland. This bank will then send it to some bank in Louisville, and that bank will present it to Mr. Clements for acceptance. If accepted it will hold the draft for 30 days, and then collect it and remit the money to Cleveland, charging the Cleveland bank a slight amount for collection, which will be charged finally to Mr. Simons.

EXERCISE 221

1. R. J. Lewis draws on J. P. Greene for $1750. The bank charges 0.1% for collection. What are the proceeds?

2. P. F. Brooks draws on M. R. Cook for $2150. The bank charges $\frac{1}{5}\%$ for collection. What are the proceeds?

3. M. J. Lange draws on F. P. Dow for $3600. The bank charges $\frac{1}{8}\%$ for collection. What are the proceeds? Write the draft.

4. F. P. Devoe collects a debt of $1950 against R. J. Stone by drawing upon him for the amount. The proceeds are $1948.05. What is the bank's rate for collection?

5. M. M. Miller draws upon R. D. St. Clair for $2150 and the bank charges $4.30 for making the collection. What rate does the bank charge? Write the draft.

6. R. R. Stewart sells F. M. Nixon 6 dressers @ $15.50, 4 bedsteads @ $12.50, and 4 doz. chairs @ $30 a dozen, all less 10% and 5%. He draws upon him for the amount and the bank charges 25¢ for collecting. What are the proceeds?

415. Variation in Rates. A postal or express money order always costs more than its face, the extra cost constituting a fixed rate of exchange for various amounts. This is not so with drafts, however, for they often can be bought at their face value, and occasionally even below the face value.

If the cost of a draft is more than the face, exchange is said to be *at a premium;* if the cost equals the face, exchange is said to be *at par;* if the cost is less than the face, exchange is said to be *at a discount.*

For small sums, say less than $500, drafts on New York, Chicago, and other commercial centers usually sell at a premium of about 0.1%, and they are usually cashed at par in any place for a customer of a bank; that is, a depositor receives their face value anywhere.

For large sums, like $500,000, the premium is about what it would cost to send the money by express, unless the bank that draws the draft already has such an amount in the bank on which it draws. For example, if a company in St. Louis wishes a draft on New York for $500,000, and if New York banks owe St. Louis banks a large amount just at that time, then the St. Louis bank might be glad to give such a draft, and might even give it for $100 less than the face in order to save the expense and loss of time in shipping the money that New York was just then owing it.

EXERCISE 222

Find the cost of drafts for the following amounts, the premiums per $1000 being given:

1. $27,500, $1.80.
2. $36,500, $2.20.
3. $37,250, $2.40.
4. $48,125, $1.60.
5. $26,500, $1.75.
6. $125,000, $1.85.

Find the cost of drafts for the following amounts at the discounts stated:

7. $48,250, $\frac{1}{10}\%$.
8. $75,000, $\frac{1}{5}\%$.
9. $64,800, $\frac{1}{4}\%$.
10. $327,500, $\frac{1}{20}\%$.
11. $775,250, $\frac{1}{20}\%$.
12. $1,750,275, $\frac{1}{10}\%$.

EXERCISE 223

Review Problems

Find the cost of the following drafts:

1. \$1250, at par.
2. \$1750, 0.2% premium.
3. \$2760, 0.1% premium.
4. \$4500, $\frac{1}{4}$% discount.
5. \$2700, 0.2% discount.
6. \$25,750, 0.2% premium.

7. A draft for \$27,500 was bought for \$27,486.25. Was exchange at a premium or a discount? Find the rate.

8. The Lion Construction Co. of Bloomington, Ind., draws at sight on R. B. James of Memphis, Tenn., for \$1760.75 through the First National Bank of Bloomington. Write the draft and find the charge for collection at 0.1%, to the nearest 5 cents.

9. The Acme Manufacturing Co. of Guthrie, Okla., draws at 30 days sight on J. B. Martin of Kansas City, Mo., for \$275.60 through the First National Bank of Guthrie. Write the draft and acceptance and find the charge for collection at 0.1%, to the nearest 5 cents.

10. Roberts Bros. of Louisville, Ky., sell to M. R. Freeman of Ashland, Ky., goods to the value of \$1257.60, subject to discounts of 10% and 6% if paid within 10 days. Mr. Freeman pays at once by check. Write the check and find the cost of collection at 0.1%, to the nearest 5 cents.

11. Ashland & Co. of Mobile, Ala., sell to F. P. Prentice of Bessemer, Ala., goods to the value of \$1425.50, subject to a discount of 6%. They draw for this amount on June 3, at 90 days sight. The draft is accepted on June 5 and discounted at 6% on June 7. If the cost of the collection is 0.1%, to the nearest 5 cents, find the net proceeds realized by Ashland & Co.

416. Foreign Exchange. Exchange carried on with foreign countries is called *foreign exchange.*

417. English Money. The table is as follows:

12 pence (d.) = 1 shilling (s. or /) = $0.243 +
20 shillings = 1 pound (£) = $4.8665

We usually think of the penny as about 2¢, the shilling as about 25¢, and the pound as about $5.

418. French Money. The table is as follows:

100 centimes (c.) = 1 franc (fr.) = $0.193

We usually think of the franc as about 20¢, and the 5-franc piece as about $1. This system is used in several countries.

419. German Money. The table is as follows:

100 pfennigs (pf.) = 1 mark (M.) = $0.238

We usually think of 4 pf. as 1¢, and 1 M. as about 25¢.

EXERCISE 224

Express as pence:

1. £7. 2. £24. 3. £5 6s. 4. £7 2s. 4d.

Express as shillings:

5. £27. 6. £32. 7. 120d. 8. 720d.

Express as pounds and shillings:

9. 95s. 10. 72s. 11. 127s. 12. 342s.

Express as francs and decimals:

13. 750 c. 14. 275 c. 15. 680 c. 16. 1250 c.

Express as marks and decimals:

17. 450 pf. 18. 275 pf. 19. 1680 pf. 20. 9275 pf.

420. Bills of Exchange. Foreign drafts are often called *bills of exchange.*

Bills of exchange, like domestic drafts, may be above par (when exchange is at a premium), at par, or below par (when exchange is at a discount).

English exchange, often called Sterling exchange, is quoted at dollars to the pound (4.88 meaning that $4.88 will buy a bill of exchange for £1). If English exchange is above 4.8665, exchange is at a premium; if below, at a discount.

French exchange is quoted either at francs to a dollar (5.15 meaning that $1 will buy a bill of exchange for 5.15 fr.), or at cents to the franc (19.7 meaning that 19.7¢ will buy a draft for 1 fr.).

German exchange is quoted either at cents to 4 marks (98 meaning that a bill of exchange for 4 M. will cost 98¢), or at cents to the mark (24.3 meaning that 24.3¢ will buy a draft for 1 M.).

When necessary to distinguish it from foreign exchange, exchange that is carried on within our own country is called *domestic exchange.*

421. Illustrative Problems. 1. How much will a bill of exchange for £75 6s. cost @ 4.90?

£75 6s. $= £75\frac{6}{20} = £75.3.$

Since £1 costs $4.90,

therefore £75.3 cost $75.3 \times \$4.90$, or $368.97.

2. How much will a bill of exchange for 26,000 fr. cost @ 5.20?

Since 5.20 fr. cost $1,

therefore 1 fr. costs $\frac{\$1}{5.20}$,

and 26,000 fr. cost $26000 \times \frac{\$1}{5.20}$, or $5000.

3. How much will a bill of exchange for 1600 M. cost @ $94\frac{1}{4}$?

Since 4 M. cost $\$0.94\frac{1}{4}$,

therefore 1 M. costs $\frac{1}{4}$ of $\$0.94\frac{1}{4}$,

and 1600 M. cost $1600 \times \frac{1}{4}$ of $\$0.94\frac{1}{4}$, or $377.

EXERCISE 225

Find the cost of the following bills of exchange:

1. £7 @ 4.89.
2. £9 @ 4.90.
3. £12 @ 4.87.
4. £25 @ 4.86.
5. £47 @ 4.85.
6. £78 @ 4.92.
7. £2 6s. @ 4.90.
8. £3 8s. @ 4.88.
9. £7 4s. @ 4.86.
10. £12 15s. @ 4.85.
11. 350 fr. @ 5.20.
12. 480 fr. @ 5.23.
13. 575 fr. @ 19.4.
14. 680 fr. @ 19.6.
15. 2975 fr. @ 19.2.
16. 4270 M. @ 98.
17. 2680 M. @ 97.
18. 3275 M. @ 24.1.
19. 4680 M. @ 23.9.
20. 5975 M. @ 23.6.

21. How large a draft on London can be bought for $3359.75 when exchange is 4.91?

22. How large a draft on Paris can be bought for $38.80, when exchange is 19.4?

23. How large a draft on Berlin can be bought for $160.70, when exchange is 95.2?

24. A merchant buys a shipment of Sheffield cutlery amounting to £48 6s. What will a bill of exchange for this amount cost @ 4.91?

25. An importer buys a lot of Lyons silk to the value of 27,375 fr. What will a bill of exchange for this amount cost @ 5.20?

26. A Chicago dealer imports some Nürnberg toys to the value of 2650 M. What do they cost him, exclusive of freight and duty, exchange being 96?

27. Which is better for a man who wishes to pay 1000 fr. in Paris, to buy his bill of exchange from a dealer at 5.20, or from one at 19.2? How much better?

EXERCISE 226

Problems without Numbers

1. What method will you take to pay a debt at a distance? Why? Is it fair to the one you owe?

2. If you purchase a bank draft when exchange is at a premium, how will you find the cost of the draft?

3. If you purchase a bank draft when exchange is at a discount, how will you find the cost of the draft?

4. If a man in another place owes you, state all the steps to be taken in drawing on him for the amount.

5. How will you find the actual value in English money of a sum in our money?

6. How will you find the actual value in our money of a sum in English money?

7. How will you find the actual value in French money of a sum in our money?

8. How will you find the actual value in our money of a sum in German money?

9. How will you find the cost of a draft for a certain number of pounds, when the rate of exchange is given?

10. How will you find the cost of a draft for a certain number of francs, when the rate of exchange is given?

11. How will you find the cost of a draft for a certain number of marks, when the rate of exchange is given?

12. How will you find the face of a draft on London, given the cost and the rate of exchange?

13. How will you find the face of a draft on Paris, given the cost and the quotation in cents?

14. How will you find the face of a draft on Munich, given the cost and the number of cents to a mark?

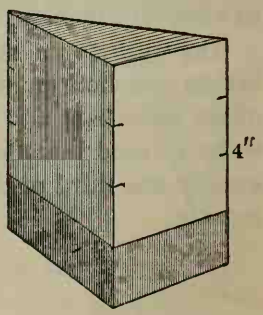
CHAPTER XX

MEASUREMENT OF SOLIDS

422. Prism. A solid bounded by two equal and parallel polygons and by parallelograms is called a *prism.*

The two equal and parallel polygons are called the *bases* of the prism, and the distance between them is called the *altitude.*

423. Volume of the Prism. In this figure it is evident that upon every square inch of the base there can be placed 1 cu. in., reaching 1 in. high. Therefore, if the prism is 4 in. high, the volume is 4 times the number of cubic inches in the shaded part.



Therefore, *the volume of a prism in cubic units equals the number of square units in the base, multiplied by the number of units in the altitude.*

424. Illustrative Problem. Find the volume of a prism with base 8 sq. in. and altitude 3 in.

3×8 cu. in. $= 24$ cu. in.

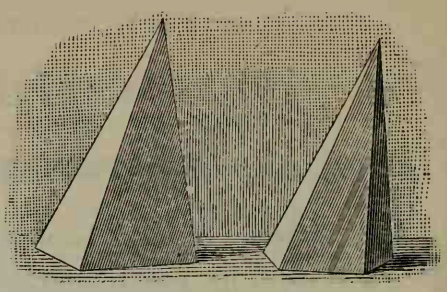
EXERCISE 227

Find the volumes of prisms with the following bases and altitudes:

1. 27 sq. in., 8 in.
2. 38 sq. in., $6\frac{1}{2}$ in.
3. 48 sq. in., $7\frac{3}{8}$ in.
4. 21.6 sq. in., 2.7 in.
5. 42.8 sq. in., 8.9 in.
6. 62.75 sq. in., 12.25 in.

425. Pyramid. A solid bounded by a polygon and by triangles that all meet in a point is called a *pyramid.*

The polygon is called the *base*, the triangles taken together are called the *lateral surface*, the point in which they meet is called the *vertex* of the pyramid, and the distance from the vertex to the base is called the *altitude.*

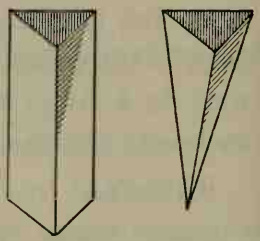


Pyramids

426. Volume of a Pyramid. *The volume of a pyramid equals one third the product of its base and altitude.*

The base and altitude are to be understood as abstract numbers, as in § 423.

The truth of the statement may be seen by taking a hollow pyramid and a hollow prism of the same base and altitude. The amount of water or sand that the pyramid will contain is exactly one third of the amount the prism will contain.



EXERCISE 228

Find the volumes of pyramids with bases and altitudes as follows:

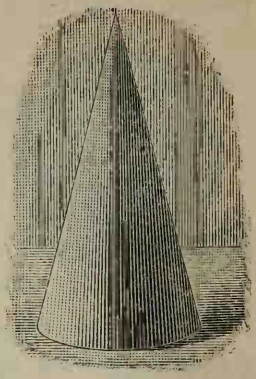
1. 32 sq. in., 9 in.
2. 48 sq. in., 13 in.
3. 64 sq. in., 15 in.
4. 96 sq. in., 22 in.
5. 132 sq. in., 26 in.
6. $8\frac{1}{4}$ sq. in., $2\frac{1}{2}$ in.
7. $9\frac{3}{8}$ sq. in., $3\frac{5}{8}$ in.
8. 8.7 sq. in., 3.1 in.
9. 12.3 sq. in., 4.7 in.
10. 15.75 sq. in., 8.1 in.

11. The Great Pyramid of Egypt has a square base 764 ft. on a side. It is 480 ft. 9 in. high. Assuming its faces smooth, what is its volume?

427. Cone. A solid bounded by a circle and a curved surface ending in a point is called a *cone.*

The circle is called the *base*, the curved surface is called the *lateral surface*, the point in which the lateral surface ends is called the *vertex*, and the distance from the vertex to the base is called the *altitude* of the cone.

In the cones here treated the vertex is directly over the center of the base.

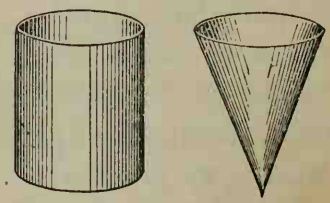


Cone

428. Volume of a Cone. *The volume of a cone equals one third the product of its base and altitude.*

For if made hollow, as in § 426, the contents of the cone will fill exactly one third of the cylinder of the same base and altitude.

429. Illustrative Problem. Find the volume of a cone whose base has a radius of 2 in., and whose altitude is 3 in.



Area of base $= 3.1416 \times 2^2$ sq. in.

Volume $= \frac{1}{3} \times 3 \times 3.1416 \times 2^2$ cu. in. $= 12.5664$ cu. in.

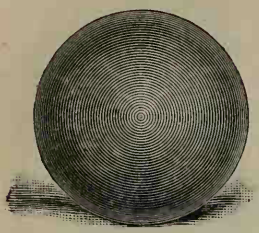
EXERCISE 229

Find the volumes of cones with bases and altitudes as follows:

1. 6 sq. in., 4 in.	6. $6\frac{3}{4}$ sq. in., $2\frac{1}{2}$ in.
2. 15 sq. in., 7 in.	7. $9\frac{7}{8}$ sq. in., $3\frac{3}{4}$ in.
3. 20 sq. in., 9 in.	8. $15\frac{3}{4}$ sq. in., $5\frac{3}{8}$ in.
4. 34 sq. in., 15 in.	9. 22.7 sq. in., 8.4 in
5. 68 sq. in., 21 in.	10. 35.75 sq. in., 12.3 in.

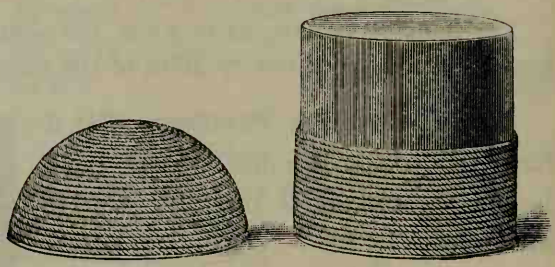
11. What is the volume of a cone whose base is a circle with a radius of 3 in., and whose altitude is 8 in.?

430. Sphere. A solid bounded by a curved surface, all points of which are equally distant from a point within, is called a *sphere.*



The point within from which all points on the surface are equally distant is called the *center.* A straight line from the center to the surface is called a *radius.* A straight line through the center and terminated at each end by the surface is called a *diameter.*

431. Surface of a Sphere. If we wind half of the surface of a sphere with a cord or tape as here shown, and then wind with exactly the same length of the cord or tape the surface of a cylinder having just the same radius, the height of the cylinder equaling the diameter, we find that the cord covers half the curved surface of the cylinder.



Therefore, *the surface of a sphere equals the curved surface of a cylinder of the same radius and height.*

The surface of a sphere also equals 3.1416 times the square of the diameter.

For we have seen (§ 250) that the curved surface of a cylinder equals the height times the circumference of the base, and since the height here equals the diameter, therefore

$$\text{height} \times \text{circumference} = \text{diameter} \times 3.1416 \times \text{diameter}$$
$$= 3.1416 \times \text{the square of the diameter.}$$

For practical purposes it is usually sufficient to take $3\frac{1}{7}$ instead of 3.1416 as the multiplier.

If we have given the surface of a sphere, we may find the diameter by dividing the surface by 3.1416, or $3\frac{1}{7}$, and extracting the square root of the quotient.

432. Illustrative Problem. What is the surface of a sphere with a radius of 3 in.?

Diameter $= 2 \times 3$ in. $= 6$ in.
Area $= 3.1416 \times 6^2$ sq. in.
$= 3.1416 \times 36$ sq. in.
$= 113.0976$ sq. in.

EXERCISE 230

Using $3\frac{1}{7}$ as the multiplier, find the areas of the surfaces of spheres with the following diameters:

1. 14 in.	4. 2.1 in.	7. $\frac{7}{8}$ in.	10. 224 in.
2. 35 in.	5. 2.8 in.	8. 0.7 in.	11. 371 in.
3. 42 in.	6. 4.9 in.	9. 0.56 in.	12. 434 in.

Find the same, using 3.1416 as the multiplier:

13. 7 in.	15. 9 in.	17. $7\frac{1}{2}$ in.	19. 2.2 in.
14. 6 in.	16. 4 in.	18. $6\frac{1}{4}$ in.	20. 3.5 in.

Find the same, using 3.1416, the following being the radii:

21. 1 in.	23. $\frac{1}{2}$ in.	25. 7 in.	27. 2.5 in.
22. 5 in.	24. 3 ft.	26. 9 in.	28. 3.7 in.

29. If the earth is considered a sphere of 4000 mi. radius, what is its surface? (Use $3\frac{1}{7}$.)

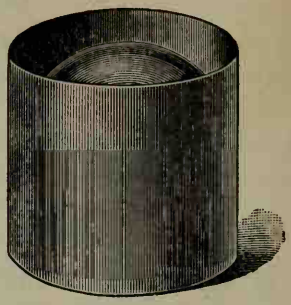
30. How many square inches of flannel will it take to cover a tennis ball 3 in. in diameter? (Use $3\frac{1}{7}$.)

31. If the sun is considered a sphere of 866,500 mi. diameter, what is its surface? (Use $3\frac{1}{7}$.)

32. How many square inches in the surface of a ball 44 in. in diameter? (Use $3\frac{1}{7}$.)

33. What is the diameter of a sphere whose surface is 154 sq. in.? (Use $3\frac{1}{7}$.)

433. Volume of a Sphere. If a sphere is placed in a cylinder of the same diameter and with an altitude equal to the diameter, and the vacant spaces are filled with sand, the sand will be found to fill exactly one third of the cylinder after the sphere is removed.



Therefore, *a sphere is equal to two thirds of a cylinder of the same diameter and altitude.*

The volume of a sphere is $\frac{4}{3} \times$ *3.1416* $\times$ *the cube of the radius.*

For the volume of the cylinder is the product of the altitude and the base, considered as abstract numbers (§ 249).

The altitude $= 2 \times$ radius,
and the base $= 3.1416 \times$ square of radius (§ 231).
Hence volume of cylinder $= 2 \times 3.1416 \times$ cube of radius,
and volume of sphere $= \frac{2}{3}$ of $2 \times 3.1416 \times$ cube of radius
$= \frac{4}{3} \times 3.1416 \times$ cube of radius.

434. Illustrative Problem. Find the volume of a sphere with a radius of 3 in.

$$\frac{4}{3} \times 3.1416 \times 3^3 \text{ cu. in.} = \frac{4 \times 3.1416 \times 3 \times 3 \times \cancel{3}}{\cancel{3}} \text{ cu. in.}$$

$$= 113.0976 \text{ cu. in.}$$

EXERCISE 231

Find the volumes of spheres with radii as follows:

1. 2 in.	4. 15 in.	7. 2.4 ft.	10. $2\frac{1}{2}$ in.
2. 5 in.	5. 23 in.	8. 4.1 ft.	11. $3\frac{3}{4}$ in.
3. 7 in.	6. 42 in.	9. 3.5 ft.	12. $4\frac{2}{3}$ in.

13. Find the weight of a lead ball 2 in. in diameter, lead being 11.35 times as heavy as water, and a cubic foot of water weighing 1000 oz.

EXERCISE 232

Review Problems

1. Compute the surface of a dome that is half of a sphere, the radius being 20 ft.

2. Find the volume of a sphere 6 in. in diameter and of one 12 in. in diameter.

3. If the moon is considered a sphere of radius 1080 mi., what is its volume?

4. A cornucopia has a circular top $2\frac{7}{8}$ in. across. It is $9\frac{1}{4}$ in. deep. How many cubic inches does it contain?

5. A ball 8 in. in diameter is placed in a cubical box 8 in. on an edge, inside measure. How many cubic inches of the box is not occupied by the ball?

6. A steel rod 4 ft. 8 in. long has a triangular cross section 2 sq. in. in area. If a cubic foot of water weighs 62.5 lb., and steel is 7.83 times as heavy as water, how much does the rod weigh?

7. A funnel in the shape of a cone is closed at the bottom. If the circular top is 3 in. across, and the funnel is 4 in. deep (to the vertex), how many cubic inches of water will it hold?

8. A prism has a base of 9.4248 sq. in. and an altitude of 15 in. How does its volume compare with that of a cone whose base has a radius 3 in., and whose altitude is 15 in.?

9. Find the volume of a sphere 1 ft. in diameter, of a cylinder of the same diameter and altitude, and of a cone of the same diameter and altitude. The volume of the sphere is how many times the volume of the cone? The volume of the cylinder is how many times the volume of the cone?

EXERCISE 233

PROBLEMS WITHOUT NUMBERS

1. Given the base and altitude of a prism, how will you find the volume ?

2. Given the volume and base of a prism, how will you find the altitude ?

3. Given the base and altitude of a pyramid, how will you find the volume ?

4. Given the base and altitude of a cone, how will you find the volume ?

5. Given the volume and altitude of a cone, how will you find the base ?

6. Given the diameter of a sphere, how will you find the surface ?

7. Given the radius of a sphere, how will you find the surface ?

8. Given the radius of a sphere, how will you find the volume ?

9. Given the diameter of a sphere, how will you find the volume ?

10. Given the edge of a cube, how will you find the volume ? how will you find the surface ?

11. Given the radius of a sphere, how will you find the circumference ?

12. Given the length of the earth's equator, how will you find the radius of the earth ? Then how will you find the surface of the earth ? the volume of the earth ?

13. Given the area of the entire surface of a cube, how will you find the area of one face ? the length of an edge ? the volume of the cube ?

CHAPTER XXI

VOCATIONAL PROBLEMS

435. Nature of the Problems. You are now nearing the end of your work in arithmetic, and it is necessary to consider some of the miscellaneous problems of business, — the problems that we may all meet in the practical American life of to-day.

EXERCISE 234

Problems of the Merchant

1. Supply the proper numbers in the spaces marked with an asterisk (*) in the following bill:

Houston, Texas, March 7, 1913

Mr. Robert J. Spencer, Tyler, Texas

Bought of Henderson Bros.

Terms: 6% 30 da., net 60 da.

March	7	50	gro. Pens	@ 78¢	$39		
		15	doz. Notebooks	@ $1.70	*		
		4	doz. Rulers	@ 1.32	*		
		30	gro. Lead Pencils	@ 3.35	*		
					*		
			Less 6%		*		*

Write bills similar to that of Ex. 1, inserting places, dates, and names, and completing each item:

2. 50 M envelopes @ \$2.10, 3 doz. fountain pens @ \$19.50, 8 doz. bottles ink @ 34¢ 5 gross erasers @ \$3.38. Terms: 8% 30 da., net 60 da.

3. 75 bbl. flour @ \$5.23, 63 gal. molasses @ 39¢, 75 gal. fancy sirup @ 58¢, 3 doz. cartons flour @ \$2.64, 260 lb. coffee @ 9.65¢. Discounted @ 6%.

4. 8 doz. hatchets @ \$10.75, 6 doz. pr. hinges @ \$4.35, 5 doz. carpenter's squares @ \$34.50, ½ gross locks @ \$42.50, 8 doz. files @ \$6.25. Discounted @ 40%, 10%, and 5%.

5. 575 yd. cotton flannel @ 38¼¢, 684 yd. cotton @ 7⅓¢, 364 yd. buckram @ 9½¢, 648 yd. sheeting @ 26¾¢, 327 yd. Java canvas @ 62½¢. Discounted @ 12%.

Make out a bill in each of the following examples, and write out a sight draft upon the buyer for the amount. Then find the charge for collection at 0.1%, to the nearest 5 cents.

6. L. P. Hall purchased from Myers Bros. 84 gross bone buttons @ 18½¢, 72 doz. pearl buttons @ 9¾¢, 694 yd. cambric @ 17½¢, 364 yd. cashmere @ 82¼¢, 408 yd. chambray @ 21¾¢. Discounted @ 12%.

7. F. J. Peabody purchased from Thompson & Co. 4 gross toothbrushes @ \$9.35, 3 doz. atomizers @ \$19.25, 8 doz. combs @ \$1.95, 4 doz. brushes @ \$18.37, ½ gross nail brushes @ \$27.50. Discounted @ 15% and 5%.

8. M. S. Robbins purchased from the Seacord Carpet Co. 480 yd. matting @ 18¼¢, 375 yd. matting @ 19⅔¢, 284 yd. carpet @ 48½¢, 8 rugs @ \$7.33, 4 doz. door mats @ \$4.75. Discounted at 3% and 1%.

EXERCISE 235

Problems of the Manufacturer

1. How much will a manufacturer receive for 6 rockers @ \$6.75, 8 dressers @ \$12.50, and 12 bedsteads @ \$15.75, discounted at 8%?

2. A manufacturer sells the following bill of goods: 4 doz. plates @ \$1.38, 8 doz. cups @ \$2.27, $\frac{1}{2}$ doz. covered dishes @ \$9.50, 12 doz. plates @ \$1.45, $\frac{1}{4}$ doz. pitchers @ \$6.40. The bill is discounted at 3% and he gains 20% on the goods. Required the cost of manufacture.

3. Iron expands $\frac{3}{16}$ in. to the foot when heated to a red heat. How much longer will a casting be when heated to a red heat, if it is 3 ft. 7 in. long before it is heated?

4. A manufacturing company has a capital of \$275,000, its annual business is $3\frac{1}{4}$ times its capital, and its annual profits are $7\frac{1}{2}$% of its annual business. How much are its annual profits?

5. A manufacturer sells brass lamps to a dealer at such a price that the dealer can sell them at \$6.60 and make a profit of 32%. The manufacturer himself makes a profit of 25%. What is the cost of manufacture per lamp?

6. On which will a manufacturer make the greater per cent of profit, — an article which it costs \$2.90 to make, and which he sells for \$3.33$\frac{1}{3}$ less 3%, or one costing \$8.70 to make, which he sells for \$12 less $\frac{1}{6}$? How much greater?

7. A manufacturer sold to a retail dealer the following bill of goods: 9 dining sets @ \$62.50, 16 rockers @ \$5.35, 8 sideboards @ \$32.50, 6 card tables @ \$8.75, 4 china closets @ \$17.50. He discounted this bill at 8% and 2%. How much did he receive for the goods?

8. In casting brass hinges the brass shrinks in cooling 1.5625% of its length when melted. How long must be the mold for a hinge that is to be $5\frac{7}{8}$ in. long?

9. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}\%$ as much as the iron. If the pattern weighs $32\frac{1}{2}$ lb., how much does the casting weigh?

10. A manufacturer has a water tank that is 8 ft. 3 in. wide, 12 ft. 6 in. long, and 9 ft. 8 in. deep. If a cubic foot of water weighs $62\frac{1}{2}$ lb., what is the weight of the water required to fill the tank?

11. A manufacturer pays a salesman a salary of $1250 a year and a commission of $1\frac{1}{2}\%$ on all sales, and allows him $1000 a year for expenses. If the salesman sells $172,500 worth of goods, what does the manufacturer pay him, including salary, commissions, and expenses?

12. An iron founder knows that an iron casting will weigh 17 times as much as the wooden pattern. If he has 5 patterns for the parts of a given piece that he wishes to cast, and these weigh 6 lb. 8 oz., 7 lb. 4 oz., 18 lb. 12 oz., 9 lb. 8 oz., and 12 lb. 4 oz., what will the casting weigh?

13. A manufacturer is building a chimney with a circular flue for his boiler room. He has two horizontal boilers, each containing 96 tubes 2 in. in diameter. His chimney must have the same cross-section area as the sum of all the cross-section areas of the tubes of the boilers. Required the diameter of the flue.

14. In a large dyeing establishment there is a tank 8 ft. long, 7 ft. wide, and 6 ft. deep. If the pressure on the end, when the tank is full of water, is equal to half the weight of the water in a rectangular tank whose base is the area of the end of the tank and whose height is the depth of the tank, how much is this pressure?

15. The following is the pay roll of a manufacturer for a week :

PAY ROLL *For the week ending* * **19** *

No.	Name	No. of hours per day						Total time	Wages per hour	Total wages	
		M.	T.	W.	T.	F.	S.				
1	J. P. Driscoll	9	9	8	$8\frac{1}{2}$	9	9	$52\frac{1}{2}$	25¢	$13	12
2	R. L. Bennett	9	$8\frac{1}{2}$	9	9	8	9	*	25¢	*	*
3	P. F. Purvis	9	$8\frac{1}{4}$	8	$8\frac{1}{2}$	9	9	*	30¢	*	*
4	B. J. Mead	8	9	9	9	$8\frac{1}{2}$	8	*	$33\frac{1}{3}$¢	*	*
5	R. K. King	8	8	9	9	9	9	*	35¢	*	*
6	M. L. Drake	9	9	8	0	9	9	*	$27\frac{1}{2}$¢	*	*
		*	*	*	*	*	*	*		*	*

Fill out the spaces marked with an asterisk (*).

Make out pay rolls (inserting names), the numbers, the hours per day, and wages per hour being as follows:

16. No. 1 : 9, 9, 9, 8, $8\frac{1}{2}$, 9, 30¢ ; No. 2 : 8, 8, 8, 9, 9, 8, $27\frac{1}{2}$¢ ; No. 3 : 8, $8\frac{1}{2}$, 9, 9, 9, 9, $28\frac{1}{4}$¢ ; No. 4 : 8, 8, 8, 8, $8\frac{1}{2}$, 8, $33\frac{1}{3}$¢ ; No. 5 : 8, 9, 8, 8, 8, $8\frac{1}{2}$, 32¢.

17. No. 1 : 9, 9, 9, 9, 9, 9, $37\frac{1}{2}$¢ ; No. 2 : 9, 9, 9, $8\frac{3}{4}$, $8\frac{3}{4}$, $8\frac{3}{4}$, 30¢ ; No. 3 : 9, 9, 9, 9, 9, $8\frac{3}{4}$, 32¢ ; No. 4 : 9, 8, $8\frac{3}{4}$, $8\frac{1}{2}$, 9, 9, 27¢ ; No. 5 : 9, 9, 9, $8\frac{1}{2}$, $8\frac{1}{4}$, 8, 28¢.

18. No. 1 : 8, 8, 8, 8, 8, 8, $32\frac{1}{4}$¢ ; No. 2 : 8, 8, $7\frac{3}{4}$, $7\frac{1}{2}$, $7\frac{3}{4}$, $7\frac{3}{4}$, 30¢ ; No. 3 : 8, 8, 8, $7\frac{3}{4}$, $7\frac{3}{4}$, $7\frac{3}{4}$, 28¢ ; No. 4 : $7\frac{1}{2}$, $7\frac{1}{4}$, $7\frac{1}{2}$, 8, $7\frac{3}{4}$, $7\frac{3}{4}$, 32¢ ; No. 5 : 8, 8, 8, 8, 8, 8, $33\frac{1}{3}$¢.

19. No. 1 : 8, $7\frac{3}{4}$, $7\frac{3}{4}$, $7\frac{3}{4}$, $7\frac{3}{4}$, $7\frac{1}{2}$, 28¢ ; No. 2 : 8, 8, 7, 0, 8, 8, 30¢ ; No. 3 : 8, 8, 8, $7\frac{3}{4}$, 8, 8, 35¢ ; No. 4 : 8, 8, 8, $7\frac{1}{2}$, 8, $7\frac{3}{4}$, $32\frac{3}{8}$¢ ; No. 5 : 8, 8, 8, 8, $7\frac{1}{2}$, $7\frac{1}{2}$, $32\frac{5}{8}$¢.

20. No. 1 : 8, 8, $7\frac{1}{2}$, 7, $6\frac{3}{4}$, 8, 31¢ ; No. 2 : 8, 7, 8, 8, $7\frac{3}{4}$, 8, $31\frac{1}{2}$¢ ; No. 3 : $7\frac{3}{4}$, $7\frac{1}{2}$, $7\frac{1}{4}$, 7, $6\frac{3}{4}$, 0, 28¢ ; No. 4 : 7, 7, $7\frac{1}{2}$, $7\frac{3}{4}$, 0, 7, 26¢ ; No. 5 : 8, 8, 8, 8, 8, 8, 36¢.

EXERCISE 236

Problems of the Contractor

1. These men have contracted to paper this house. The rooms are $16' \times 14'$, $12' \times 8'$, $18' \times 15'$, $12' \times 10'$, $15' \times 13'$, $16' \times 16'$, $10' \times 6'$, and $12' \times 12'$. They deduct $50'$ from the combined perimeters of the rooms and allow one roll of paper for every $3\frac{1}{2}$ ft. How many rolls do they need?

2. A contractor builds a brick wall 68 ft. 8 in. long, 5 ft. 8 in. high, and 2 bricks thick. Allowing 14 bricks per square foot of outside surface for a 2-brick wall, how much will the bricks cost at \$8.50 per M?

3. A man has a contract to put up the four walls of a building 80 ft. long, 35 ft. wide, and 78 ft. high. The walls are to be 18 in. thick, and 750 cu. ft. is to be allowed for openings. Allowing 22 bricks to a cubic foot, how much will the bricks cost at \$9.25 per M?

4. A contractor wishes to build a concrete wall $66' \times 8'$, and $18''$ thick. Concrete is $\frac{1}{9}$ Portland cement, which costs \$1.90 a barrel of 400 lb., and the rest is gravel costing delivered $87\frac{1}{2}¢$ a load (cubic yard). A cubic foot of dry cement weighs 150 lb. What will the materials cost?

EXERCISE 237

Problems of the Farm

1. A man had 8 acres planted to asparagus. The expense per acre for fertilizers was \$40, for labor before cutting \$20, for cutting and bunching \$40. From each acre he cut 1850 bunches and sold them at 10¢ a bunch, allowing a commission of 3% for selling. What was his profit from the 8 acres?

2. A farmer, building a barn, orders part of the lumber as follows: 16 pieces of timber $8'' \times 8''$, 12 ft. long; 4 pieces $6'' \times 8''$, 20 ft. long; 8 pieces $6'' \times 8''$, 18 ft. long; 4 pieces $6'' \times 6''$, 20 ft. long; 8 pieces $6'' \times 6''$, 18 ft. long; and 9 pieces $4'' \times 6''$, 12 ft. long. How much will this part of the lumber cost at \$26 per M?

3. A poultry raiser paid out for food for 350 hens during the first four weeks of December \$30.05. He sold 732 eggs @ 29¢ a dozen during the first two weeks, 564 eggs @ 36¢ a dozen during the third week, and 648 eggs @ 31¢ a dozen the fourth week. His receipts were what per cent greater than the cost of the feed?

4. A farmer spent on a 10-acre field the following sums: plowing, \$25; harrowing and rolling, \$18.50; seed, \$15.50; drilling, \$2.75; cutting and threshing, \$26.50. His income from the field was 220 bu. of wheat @ 75¢ a bushel, and \$3.25 for pasturage. Find the balance in his favor.

5. A farmer is planning to build a poultry house for 350 fowls. He estimates the average weight per fowl to be 6 lb., and he allows 10 cu. ft. of space to each pound of fowl. The house is to be 210 ft. long, 16 ft. wide, and 5 ft. high to the eaves. How high must be the ridgepole in order to give the required capacity?

EXERCISE 238

Workshop Problems

1. How much will 275 cu. in. of steel weigh, if 1 cu. ft. weighs 490 lb. ?

2. What is the weight of a circular piece of steel shafting 3 in. in diameter and 20 ft. long, 1 cu. ft. of steel weighing 490 lb. ?

3. If the pressure of steam in a boiler is 140 lb. to the square inch, find the pressure on a valve $2\frac{7}{8}$ in. in diameter.

4. A boiler contains 480 smoke tubes, each $2\frac{3}{8}$ in. in internal diameter. Find the total area for draft through them.

5. If a pressure of 120 lb. to the square inch is applied to the circular plunger of a cylinder 2 ft. in diameter, what is the total pressure ?

6. The rim of a fly wheel contains 1.56 cu. ft. of cast iron when it is 8 in. wide. If the rim were made a foot wide, how much would it weigh, 1 cu. ft. of cast iron weighing 450 lb. ?

7. Out of a rectangular sheet of steel which is 14′ 3″ long, 3′ $3\frac{1}{2}$″ wide, and $\frac{1}{2}$″ thick, a circular plate 13″ in diameter is cut. If 1 cu. ft. of steel weighs 490 lb., what is the weight of the part of the sheet that is left ?

8. If the drive wheel of an engine makes 140 revolutions a minute, and the cylinder is 9 in. in diameter and 14 in. long, and is filled with steam twice in each revolution, how many cubic feet of steam are used in a minute ?

9. A hollow steel shaft has an external diameter of 9″ and an internal diameter of 5″. If 1 cu. ft. of steel weighs 490 lb., find the weight of 10 ft. of this shaft.

EXERCISE 239

Problems in Transportation

1. If a freight car is 36 ft. long, 8 ft. 3 in. wide, and 7 ft. 6 in. high, inside measure, how many cubic feet does it contain?

2. In shipping certain goods at the rate of $97\frac{1}{2}¢$ per hundred pounds, there is allowed a discount of 15%. What is the net rate, to the nearest half cent?

3. In shipping certain goods the net rate after deducting 15% from the usual rate for that class is 34¢ per hundred pounds. What is the usual rate?

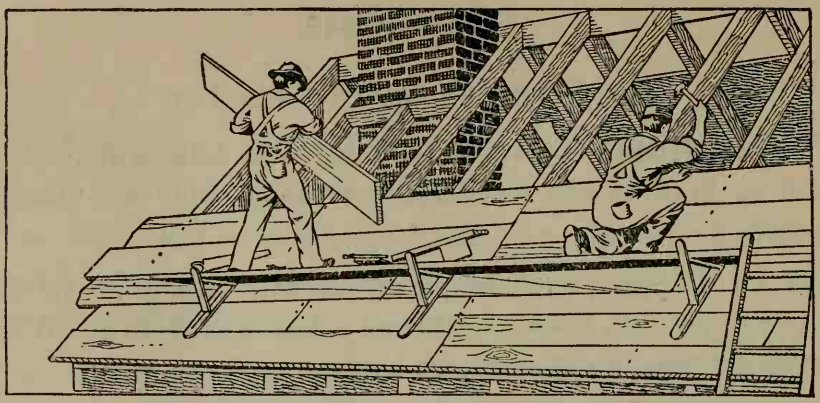
4. If a wooden coal car can carry 80,000 lb., and a steel car 100,000 lb., the steel car can carry what per cent more than the wooden car? The wooden car can carry what per cent less than the steel car?

5. A car 36 ft. long and 8 ft. 4 in. wide is loaded with wheat to a depth of 4 ft. 6 in. Reckoning a bushel as containing $1\frac{1}{4}$ cu. ft., and a bushel of wheat as weighing 60 lb., how many pounds does the wheat weigh?

6. If the car of Ex. 5 has a capacity of 80,000 lb., to what depth must it be filled with wheat to contain exactly this amount?

7. If a locomotive weighing $115\frac{1}{2}$ tons can exert a pull of $22\frac{1}{4}\%$ of its weight in starting a train, how many pounds of pull does it exert?

8. If a locomotive and coal tender together weigh 95 tons, and there are seven cars to the train, averaging 21 tons each, and a horizontal pull 0.5% of the total weight of the train is required to maintain the necessary speed on a level track, what is this pull in pounds?



EXERCISE 240

Problems of the Builder

1. From the eaves to the ridge of this roof is 13 ft., and the span is 24 ft. What is the rise of the roof?

2. The span of a common A-shaped roof is 26 ft., and the rise of the roof is 6 ft. What is the distance from the eaves to the ridge? (Two decimal places.)

3. A semicircular arch over a door has a span of 4′ 8″. Required the length of this semicircumference.

4. A builder is putting in a steam pipe 140 ft. long and 2 in. in diameter. What is the radiating surface (that is, the lateral surface that gives out heat)?

5. A builder is putting in a hemispherical dome over a tower that is 12 ft. 6 in. in diameter. What is the area of the surface of the dome?

6. The net excavation needed for a city apartment house is to be 85 ft. long, 50 ft. wide, and 7 ft. deep. In addition to this the excavation is to be extended 7 in. beyond each of the four sides, so as to allow space for working. How many loads (cubic yards) of earth must be removed?

EXERCISE 241

Problems of the Machinist

1. What is the circumference of a fly wheel that is 7′ 6″ in diameter?

2. An emery wheel 10 in. in diameter makes 2000 revolutions per minute. What is its grinding speed?

3. A fly wheel 6 ft. in diameter makes 100 revolutions per minute. At what rate per minute does a point on the rim travel?

4. How many revolutions per minute must be made by a circular saw 2 ft. in diameter, in order that the saw may have a cutting speed of 7539.84 ft. per minute?

5. A piston of an engine has an area of 175 sq. in. Required the diameter of the piston.

6. It is required to make a circular plate of the same material, thickness, and weight as a rectangular plate 2′ 4″ by 3′ 1″. What is its diameter?

7. If the diameter of a piston is 27 in., and the steam pressure is 120 lb. per square inch, what is the total pressure exerted on the piston?

8. What is the total external surface of the tubes in a boiler 18 ft. long, there being 112 tubes and the tubes being $3\frac{1}{2}$″ in diameter?

9. Two cogwheels are so geared that for every revolution of the larger the other revolves 12 times. The larger wheel has 204 cogs. How many cogs has the smaller?

10. The fly wheel of an engine 8 ft. in diameter makes 250 revolutions per minute. It is directly connected by a belt with a wheel 2 ft. 8 in. in diameter. How many revolutions per minute does the smaller wheel make?

EXERCISE 242

Problems in Farm Mechanics

1. Experiments show that, with a wagon having high wheels, it requires 84.48 lb. pull to move a ton on a dry, level, gravel road, and 30.4% more if the wagon has low wheels. How much power is required for the low-wheeled wagon?

2. Wind pressure increases as the square of the velocity of the wind. That is, when the wind blows 10 mi. an hour, the pressure is four times as much as when the wind blows 5 mi. an hour. When the wind blows 15 mi. an hour the pressure per square foot of area of a windmill is 1.135 lb. What is the pressure per square foot when the wind blows 30 mi. an hour? when the wind blows 40 mi. an hour?

3. The horse power developed by a windmill varies as the cube of the velocity of the wind. If a certain windmill develops $\frac{1}{40}$ of a horse power when the wind blows 5 mi. an hour, how much will it develop when the wind blows 35 mi. an hour?

4. If a man pumping water exerts a pressure of 18 lb. on the pump handle, and his hand is $6\frac{1}{4}$ times as far from the fulcrum as the rod that connects with the piston of the pump is distant from it, what is the lifting force exerted in raising the water?

5. If a 2-inch piston in a pump requires a lifting force of 54 lb., how much will a 3-inch piston require in the same well?

6. If a pump having a $2\frac{1}{2}$-inch cylinder, and working at a certain rate, delivers enough water per hour for 25 cows, a pump having a 3-inch cylinder, working at the same rate, would deliver enough for how many cows?

CHAPTER XXII

GENERAL REVIEW

436. Nature of the Problems. The problems in this chapter have been selected from recent official examination papers. They show some of the demands made upon the schools and may assist in preparing pupils for examinations.

EXERCISE 243

PROBLEMS IN COMPUTATION WITH INTEGERS

Add, checking the results:

1.	2.	3.	4.	5.
4368	2748	4227	8284	7856
5962	6299	3729	2643	9280
6857	3176	6248	1716	3442
7976	4387	5135	7357	2635
8795	8132	4273	2148	4331

Subtract, checking the results:

6.	7.	8.	9.	10.
8284	9307	9002	4871	3407
1796	2969	3763	2998	1879

Multiply, checking the results:

11.	12.	13.	14.	15.
4879	7963	3148	6779	8764
6273	4072	2909	9283	8297

Divide, checking the results:

16. 92,358 ÷ 233. 17. 123,540 ÷ 225. 18. 62,500 ÷ 250.

EXERCISE 244

Problems in Computation with Common Fractions

1. Simplify $\dfrac{2\frac{1}{6} + 2\frac{5}{8} \div \frac{9}{14}}{3 \times 1\frac{3}{4}} + \dfrac{1}{6} \times 6\frac{1}{7}$. (In all such cases perform first the multiplications and divisions as indicated by the symbols, and then the additions and subtractions in the order given.)

2. Simplify $\dfrac{(\frac{7}{20} + \frac{1}{5} \times 1\frac{5}{8}) \div (6 \times \frac{3}{4})}{\frac{3}{1000}}$. (Work first within the parentheses.)

3. Simplify $\dfrac{(1\frac{7}{24} - \frac{5}{18}) \div (1\frac{7}{18} \times 2\frac{25}{34})}{6\frac{3}{8} - \frac{9}{20} \times \frac{5}{6}}$.

4. Simplify $\dfrac{\frac{3}{8} + 1\frac{3}{16} \div 1\frac{19}{20}}{2 - 1\frac{13}{22} + \frac{8}{77} \times 1\frac{1}{20}} - \dfrac{19}{36}$.

5. Simplify $\dfrac{\frac{2}{5} + 3\frac{3}{5} \div 4\frac{1}{2}}{4\frac{4}{5} \times \frac{2}{3}}$, and express the result both as a common fraction and as a decimal.

6. Simplify $\dfrac{(\frac{4}{15} + 1\frac{1}{3}) \div (3\frac{1}{9} \times \frac{4}{7})}{1 - \frac{2}{3} \times \frac{3}{20}}$.

7. Simplify $\dfrac{1\frac{4}{5} \times 3\frac{3}{4} \div 6\frac{3}{4}}{\frac{2}{3} \div \frac{7}{12} - \frac{4}{7}}$.

8. Simplify $\dfrac{(\frac{2}{3} + 4\frac{1}{2}) \times (3\frac{1}{2} - \frac{1}{3})}{2\frac{1}{2} \times 3\frac{1}{2}}$.

9. Simplify $\dfrac{4\frac{1}{2} + \frac{2}{3} - (4\frac{1}{2} - \frac{2}{3})}{2 + (3\frac{1}{2} - \frac{1}{8})}$.

10. Simplify $\dfrac{2\frac{1}{2} \times (3 + 4\frac{1}{4})}{3\frac{1}{2} \div (2 + 3\frac{1}{8})}$.

11. Simplify $\dfrac{(\frac{1}{4} + \frac{3}{5}) \times (\frac{3}{4} - \frac{1}{5})}{\frac{3}{10} + \frac{1}{2}}$.

12. Simplify $\dfrac{\frac{1}{6} \times 1\frac{13}{16} + 1\frac{1}{6} \times 6\frac{1}{4} - 1\frac{1}{3} \times 5\frac{4}{9}}{\frac{1}{6} \times 2\frac{5}{6} \times 5\frac{2}{3}}$.

EXERCISE 245

Problems in Computation with Decimals

1. Simplify $\dfrac{3.63 \times 3.5}{8.47 \times 0.5}$.

2. Simplify $\dfrac{1.25 - 1.33\frac{1}{3} \div 1.66\frac{2}{3}}{240 \times .015}$ (See p. 416, Ex. 1.)

3. Simplify $\dfrac{1.75 - 0.27 \div 0.36}{2.64 - 2 \times 1.07}$.

Add the following:

4.	**5.**	**6.**	**7.**
2.074	7.265	4.527	5.2793
3.672	8.030	3.298	4.2606
9.42	4.723	1.111	1.1111
8.307	9.481	5.473	4.7207

Subtract:

8.	**9.**	**10.**	**11.**
9.6087	8.1306	21.4273	32.0049
4.7998	5.0987	8.6998	26.8297

Multiply:

12.	**13.**	**14.**	**15.**
5.2346	208.73	29.008	0.07289
3.1416	3.1416	42.606	0.02134

Divide:

16. 53.4072 ÷ 3.1416.

17. 0.062727 ÷ 0.02987.

18. 74.3094 ÷ 4.1283.

19. 0.00037356 ÷ 0.00283.

Divide to three decimal places:

20. 27 ÷ 24.8.

21. 35 ÷ 4.16.

22. 8.4 ÷ 2.96.

23. 5.3 ÷ 0.072.

24. 6.05 ÷ 3.07.

25. 7.92 ÷ 0.063.

EXERCISE 246

Problems in Computation with Compound Numbers

Add:

	ft.	in.		yd.	in.		lb.	oz.
1.	42	7	2.	27	27	3.	15	12
	33	9		32	18		26	9
	26	7		41	26		31	8
	52	11		62	15		42	15
	46	8		33	32		17	13
	75	10		48	12		36	7

Subtract:

	ft.	in.		yd.	in.		lb.	oz.
4.	27	2	5.	127	15	6.	37	6
	18	9		68	27		19	14

Multiply:

7. 26 ft. 4 in. × 35

8. 42 yd. 23 in. × 76

9. 86 lb. 14 oz. × $37\frac{1}{2}$

Divide:

10. 111 yd. ÷ 4.

11. 115 ft. 1 in. ÷ 7.

12. 156 lb. 15 oz. ÷ 9.

13. 1482 yd. 8 in. ÷ 40.

14. 222 bu. ÷ 27 bu. 3 pk.

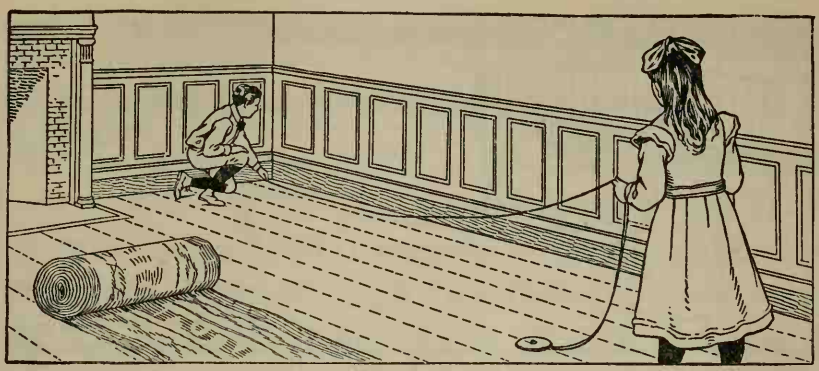
15. 117 ft. 3 in. ÷ 16 ft. 9 in.

16. Reduce 7 mi. 10 rd. 2 yd. 2 ft. 9 in. to inches.

17. Reduce 33 rd. 16 ft. 10 in. to inches; to feet and a decimal; to rods and a decimal.

18. Add 3 mo. 2 wk. 3 da., 5 mo. 1 wk. 6 da., 6 mo. 3 wk. 5 da., and 10 mo. 4 da., allowing 4 weeks to the month.

19. From 1915 yr. 8 mo. 2 da. subtract 1911 yr. 10 mo. 20 da.



EXERCISE 247

Problems in Painting and Carpeting

1. This room is $18' \times 15'$. How much will it cost at 65¢ a yard for matting 1 yd. wide to cover the floor, no allowances being made?

2. Find the cost at 15¢ a square yard of painting the sides, ends, and tops of 10 freight cars $34' \times 8' \times 7'$.

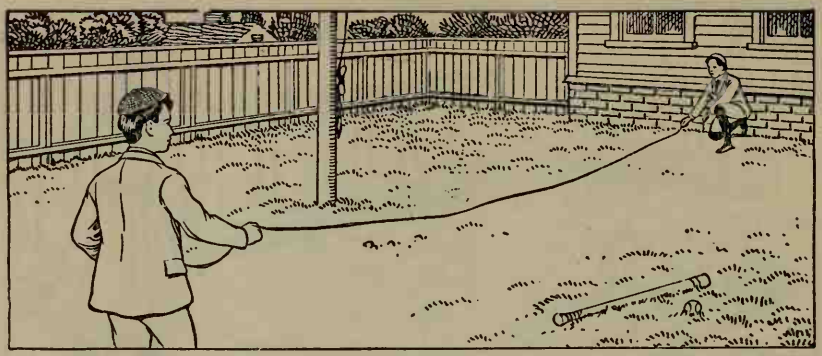
3. How much will it cost at \$1.25 a yard to carpet a room 15′ by 12′ with plain carpet $\frac{3}{4}$ yd. wide?

4. How much will it cost at \$1.25 a yard to cover the floor of a room $17'\,6'' \times 15'\,9''$ with 27-inch carpet, allowing 3 yd. for matching patterns?

5. Find the cost at 18¢ a square yard of painting the walls and ceiling of a room $16' \times 14' \times 10\frac{1}{2}'$, making no allowance for openings.

6. Find the cost of carpeting a room 30′ by 27′ with carpet $\frac{3}{4}$ yd. wide, 3 yd. being allowed for matching, and the carpet costing \$2.25 a yard.

7. A painter, estimating the cost of a job, finds there are 925 sq. yd. of surface to be covered. His bid is at the rate of 16¢ per square yard. What is the amount of his bid?



EXERCISE 248

Problems in Practical Measurement

1. These boys are laying out a baseball diamond in the form of a square 90 ft. between bases. How far is it from first base directly across to third base?

2. Find the cost of 20 joists, each $4'' \times 6''$ and 16′ long, at \$32 per M.

3. Find the cost of 40 joists 18′ long, 10″ wide, and 4″ thick @ \$27.50 per M.

4. What is the cost of fencing a $2\frac{1}{2}$-acre rectangular lot 25 rd. long @ 45¢ a rod?

5. How many cubic feet in a cistern that holds 54 bbl. of water, allowing 231 cu. in. to a gallon?

6. Find the cost of 20 joists $2'' \times 4''$ and 12′ long, and 24 boards $1\frac{1}{2}''$ thick, 8″ wide, and 10′ long @ \$28 per M.

7. Allowing 2150.4 cu. in. to a bushel, how many tons of coal can be placed in a bin containing 560 cu. ft., if the weight of 1 bu. of coal is 80 lb.?

8. A picket fence incloses a rectangular field $42' \times 30'$. The pickets are $2\frac{1}{2}''$ wide and $1\frac{1}{2}''$ apart. Find the cost of the pickets @ \$3.25 a hundred.

EXERCISE 249

Problems in Longitude and Time

1. The longitude of Cincinnati is 84° 26′ W. and that of Berlin is 13° 23′ 45″ E. Find the difference in local time.

2. The longitude of Boston is 71° 4′ W. and that of Rochester is 77° 51′ W. Find the difference in local time.

3. The longitude of St. Paul is 93° 5′ W. and that of St. Petersburg 30° 18′ E. Find the difference in local time.

4. New York is 74° 0′ 3″ west longitude and Chicago is 87° 38′ west; how much sooner does the sun rise at New York than at Chicago?

5. When it is noon at Greenwich it is 5 hr. 20 min. 59.4 sec. P.M., local time, at Madras, India. What is the longitude of Madras?

6. When it is noon at Portland, Me., local time, it is 4 hr. 41 min. 1.2 sec. P.M. at Greenwich. What is the longitude of Portland?

7. When it is noon at Greenwich it is 6 hr. 12 min. 48 sec. A.M., local time, at Nashville, Tenn. What is the longitude of Nashville?

8. When it is noon at the Cape Hatteras lighthouse, local time, it is 5 hr. 2 min. 5 sec. P.M. at Greenwich. What is the longitude of the Cape Hatteras lighthouse?

9. When it is noon at Greenwich it is 6 hr. 2 min. 22.2 sec. A.M., local time, at Madison, Wis. What is the difference between local and standard time at Madison?

10. On a certain day a sea captain found his Greenwich chronometer had lost 2 hr. 45 min. 30 sec., compared with the sun. Find his longitude at the time of the observation. Explain fully.

EXERCISE 250

Problems in Profit and Loss

1. Some goods were sold for \$1015.30 at a loss of $31\frac{1}{4}\%$. Find the loss.

2. Beans bought @ \$1.60 a bushel are sold @ 7¢ a quart. Find the per cent of gain or of loss.

3. A grocer sold tea for $87\frac{1}{2}$¢ a pound, thus gaining $12\frac{1}{2}$¢ a pound. What was his gain per cent?

4. A watch sold for \$228 at a loss of 5%. How much would have been gained by selling it at a gain of 5%?

5. Papers bought at the rate of 60 for 50¢ are sold for 1¢ each. Find the per cent of gain and the cost of 75 papers.

6. A box of 150 oranges is bought for \$1.40. The oranges are sold at 20¢ per dozen. Find the gain per cent.

7. By selling a farm for \$4228 a farmer lost $12\frac{1}{2}\%$. How much would he have gained by selling it for \$5040?

8. A boy bought oranges at 20¢ per dozen and sold them at the rate of 2 for 5 cents. Find his gain per cent.

9. A grocer buys 50 bushels of potatoes at 60¢ a bushel and retails them at 20 cents a peck. Find his whole gain and his gain per cent.

10. A grocer sold 4 chests of tea @ \$20.08 a chest; on 3 chests he gained $25\frac{1}{2}\%$ and on 1 chest he lost 20%. Find the total cost of the tea.

11. A merchant marked an article so as to gain 15%, but sold it for cash at such a price as to gain $3\frac{1}{2}\%$. What per cent of the marked price did he deduct?

12. A merchant marks goods $12\frac{1}{2}\%$ above cost and sells them 20% below the marked price. Find his per cent of loss. Write the analysis.

EXERCISE 251

Problems Involving Commission

1. An agent sold 325 bu. of wheat @ 80¢ a bushel, charging a commission of 5%. Find the net proceeds.

2. An agent charged \$6.63 for selling 52 bushels of tomatoes @ 85¢ a bushel. Find the rate of his commission.

3. A real estate agent charged \$295.50 for selling a piece of property for \$9850. What was the rate of his commission?

4. The net proceeds of the sale of 244 bbl. of flour is \$1124.23, the commission being 3%. Find the selling price of the flour per barrel.

5. An agent bought for a dealer 240 baskets of peaches @ 75¢ a basket, charging $3\frac{1}{2}\%$ commission. Find the total cost to the dealer.

6. An agent sold 44 wagons @ \$135 each and paid his principal \$5197.50 as the proceeds of the sale. Find the rate of the agent's commission.

7. A real estate agent charged \$562.50 for selling a piece of property. The rate of commission being $4\frac{1}{2}\%$, at what price did he sell the property?

8. A commission merchant sold 45 barrels of apples at \$2.80 a barrel, charging 6¢ a barrel for storage and a commission of $5\frac{1}{2}\%$. Find the net proceeds of the sale.

9. A commission merchant sold some produce, charging 3% for his services. He remitted to his principal \$906.95. At what price did he sell the produce?

10. A manufacturer sent his agent \$3177.81 to pay for some leather, and the commission of $\frac{1}{2}\%$. How much did the agent invest and how much was his commission?

EXERCISE 252

Problems in Insurance

1. What is the face of the policy when the premium at $\frac{3}{4}\%$ is \$21?

2. A house valued at \$3216 is insured for $\frac{5}{6}$ of its value at $\frac{3}{4}\%$. What premium is paid?

3. A house valued at \$2750 is insured for 80% of its value at $\frac{3}{4}\%$. What premium is paid?

4. A man pays \$105 once in 3 years for insuring his house at $\frac{1}{4}\%$ annually. Find the face of the policy.

5. A house is insured for 80% of its value at 1%, the premium being \$36.80. What is the value of the house?

6. A house valued at \$4400 is insured for 80% of its value, the premium being \$44. What is the rate of insurance?

7. A man has a life insurance policy of \$7500 on which he pays an annual premium of \$28.60 per thousand. How much does he pay in premiums in five years?

8. A man has a life insurance policy on which he pays \$201 a year. The premiums are at the rate of \$26.80 per thousand. What is the face of the policy?

9. A man who pays an annual premium of $2\frac{1}{2}\%$ on a life insurance policy for \$1500 dies after 18 payments. How much more did his heirs receive than he paid?

10. A man insures his stock of goods for \$6000, this being 80% of their value. The rate is \$1.15 per thousand. What is the value of the property and the annual premium?

11. A man pays an annual premium of \$29.60 a thousand on a life insurance policy. If the total annual premium is \$244.20, what is the face of the policy?

EXERCISE 253

Problems in Taxes

1. The assessed valuation of the property of a certain town is $1,872,500, and the tax levied is $14,043.75. Find the rate of tax on $1000.

2. A tax of $22,400 is raised in a certain school district to pay for a new school building. A resident who is assessed $5600 pays a tax of $62.72. What is the total valuation?

3. In a certain school district assessed at $80,000 a tax of $1200 is raised. Mr. B's assessed valuation is $6840. Find the rate of taxation and Mr. B's tax.

4. The assessed valuation of a city is $65,489,000; the amount to be raised by tax is $229,211.50. Find the tax on property assessed at $12,400.

5. The assessed valuation of a certain school district is $2,559,000; the amount to be raised by tax is $11,515.50. Find the tax on a business block assessed at $85,000.

6. The assessed valuation of a certain school district is $2,345,000; the amount to be raised by tax is $10,552.50. Find the tax on property assessed at $4500.

7. The assessed valuation of a certain city is $48,250,000; the amount to be raised by tax is $337,750. Find the tax on a piece of property assessed at $75,000.

8. If the tax rate is 11 mills, what must a man pay who has a farm assessed at $4750, a village residence assessed at $3500, and personal property assessed at $1500?

9. A tax of $16,500 is raised in a certain town as its share of the cost of a bridge. If the assessed valuation of the town is $2,750,000, what is the rate of taxation for this purpose?

EXERCISE 254

Problems in Trade Discount

1. Find the net amount of a bill for \$880 subject to discounts of $2\frac{1}{2}\%$, 5%, and 10%.

2. On a bill of \$620 commercial discounts of 5% and 5% are allowed. What is the net price?

3. Find the net cost of a piano marked \$450, with trade discounts of 3%, 8%, and 10%.

4. What is the list price of an article whose net cost is \$6.15 after discounts of 20% and 25% are deducted?

5. What per cent of profit will a man make by paying \$17.10 for an article, with discounts of 20%, 10%, and 5% from the list price, if he sells it at the list price?

6. Furniture is bought at trade discounts of 5% and 16% from the list price, and the net cost is \$518.70. Find the list price of the furniture.

7. Find the net amount of a bill of \$470, the trade discounts being 20% and 5%. What single rate of discount is equivalent to these discounts?

8. What single discount is equivalent to a discount series of 40%, 20%, 10%, and 5%?

9. Which is the better for the purchaser, and how much better: two successive trade discounts of 25% and 10% on a bill of \$500, or a single discount of $33\frac{1}{3}\%$?

10. Which is the better for the seller, and how much better: trade discounts of 20%, 12%, and 3%, on a bill of \$800, or a single discount of $31\frac{3}{4}\%$?

11. Which is the better for the purchaser, and how much better: trade discounts of 10%, 10%, and 10%, on a bill of \$625.75, or a single discount of $27\frac{1}{10}\%$?

EXERCISE 255

Problems in Interest and Discount

1. Find the simple interest on \$920 at $4\frac{1}{4}\%$ for 2 yr.

2. Find the simple interest on \$685.70 at $5\frac{1}{2}\%$ for 1 yr. 7 mo. 20 da.

3. Find the simple interest on \$653.25 @ $3\frac{1}{2}\%$ from Sept. 28, 1910, to June 18, 1913.

4. Find the simple interest on \$786 at $5\frac{1}{2}\%$ from Dec. 29, 1910, to Oct. 2, 1911.

5. How long must \$650.82 be on interest at 5% to amount to \$761.44?

6. Find the interest at $5\frac{1}{2}\%$ on a note of \$500 dated Oct. 22, 1910, and paid June 7, 1912.

7. A 60-day note, without interest, is discounted at a bank 13 days after date at 6%. The proceeds are \$734.70. What is the face of the note?

8. A 90-day note for \$376.85 with interest at 5% is due March 30. Find the amount due and the date of the note.

9. A 60-day note for \$630 without interest, dated Dec. 27, is discounted at a bank on Jan. 16, at 6%. Find the discount.

10. The proceeds of a 90-day note, without interest, discounted 32 days before it is due @ 6%, are \$574.42. Find the face of the note.

11. Find the proceeds of a 60-day note for \$420, without interest, dated Aug. 29, 1910, and discounted at a bank five days later at 6%.

12. A note of \$600 for 6 months without interest, dated Dec. 4, is discounted at a bank on Dec. 29 at 6%. Find the discount and the proceeds.

EXERCISE 256

Problems in Stocks and Bonds

1. Find the cost of 36 shares of N. Y. Central stock at $130\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

2. What will be the total cost to the purchaser of 96 shares of stock at $126\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

3. What per cent is realized from an investment in $4\frac{1}{2}\%$ stock at $124\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

4. Find the number of shares of stock that can be bought for \$5160.75 @ $122\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

5. What dividend is a stock paying when the rate on the investment is $4\frac{4}{9}\%$, if the stock was bought for $110\frac{1}{8}$?

6. A man bought stock at 97 and sold at 102, brokerage in each case $\frac{1}{8}\%$. Find the net gain per share.

7. What would be the cost of an investment in U.S. 4's (United States 4% bonds) @ $131\frac{1}{4}$, brokerage $\frac{1}{8}\%$, to secure an annual income of \$720?

8. A man sold through his broker 156 shares of stock @ $3\frac{1}{2}\%$ discount, paying a brokerage of $\frac{1}{8}\%$. Find the brokerage and the net proceeds of the sale.

9. How much better rate of income will be received from an investment in 6% stock at $137\frac{3}{8}$ than in 4% stock at $109\frac{7}{8}$, brokerage in each case being $\frac{1}{8}\%$?

10. A man paid \$2550 for some 5% railway stock at $127\frac{3}{8}$, brokerage $\frac{1}{8}\%$. How many shares did he buy, and what was his annual income from the stock?

11. A man bought 50 shares of stock at $120\frac{1}{4}$, and 6 months later a $3\frac{1}{2}\%$ dividend was declared; he then sold the stock at 118, brokerage in each case $\frac{1}{8}\%$. Find his net gain or loss, money being worth 5%.

EXERCISE 257

Problems in Ratio, Proportion, and Analysis

1. Find the fourth term of the proportion $6\frac{2}{5} : 3\frac{1}{3} :: 1\frac{3}{5}$; that is, $6\frac{2}{5} : 3\frac{1}{3} = 1\frac{3}{5} : (?)$. (The use of the symbol :: for = is nearly obsolete.)

2. The means of a proportion are $3\frac{1}{2}$ and $\frac{5}{21}$. One of the extremes is $\frac{5}{12}$. Write the proportion.

3. A prize of \$60 was divided between two persons in the ratio of $\frac{2}{3}$ to $\frac{5}{6}$. How much was the share of each?

4. If 18 men can do a piece of work in 24 days, in how many days should 27 men do the work? Solve by (*a*) analysis; (*b*) proportion.

5. X can do a piece of work in $3\frac{1}{2}$ days, Y in 3 days, and Z in $2\frac{3}{4}$ days. How many days will it take them to do it if they all work together?

6. In 20 days 36 men can do a piece of work. Find how many men it would take to do the work in 15 days. Solve by proportion.

7. A map is drawn so that 1 in. represents 0.8 mi. on the surface of the earth. How many acres of land are represented by a portion of the map 3 in. square?

8. A tank can be filled by a pipe in $3\frac{1}{2}$ hours and can be emptied by another pipe in 5 hours. If the tank is empty and both pipes are open, how long will it take to fill the tank, the flow in the pipes being constant?

9. Two stockmen, A and B, maintain a common drinking trough and share the annual cost of repairs in proportion to the number of cattle owned by each. A has 1253 head of cattle and B has 2747. If the annual cost of repairs is \$80, how much should each contribute?

EXERCISE 258

Problems in the Metric System

1. A meter is 39.37 in. long. How many rods in a kilometer ?

2. Find the weight, in kilograms, of the water in a tank 3 m. long, 2 m. wide, and 1.6 m. deep.

3. How many blocks 2 dm. long and 12 cm. wide will be required to pave a court 14 m. by 10.8 m. ?

4. A tank containing 5400 l. of water is 4 m. long and 1.8 m. wide. Find the depth of water in the tank.

5. A cistern 2.5 m. by 3.6 m. contains 14 kiloliters of water. How deep is the water ?

6. Find the cost of the milk in a vat 1.5 m. long, 8 dm. wide, and 4 dm. deep, @ 4¢ a liter.

7. A wood pile containing 13.86 steres is 5.5 m. long and 1.8 m. wide. Find the height of the pile.

8. A circular park contains 78.54 ha. Find the distance from the center to the circumference.

9. A tank is 5.4 dm. wide, 2.5 dm. deep, and 1.75 m. long. Find the weight, in kilograms, of the water it contains when full of water.

10. Sea water is 2.8% salt, and is 1.025 times as heavy as fresh water. How many kilograms of salt can be obtained from 100 cu. m. of sea water ?

11. Find the weight in kilograms of a bar of iron 2 m. long, having a rectangular cross section 8 cm. by 12 cm., iron being 7.8 times as heavy as water.

12. In a circular reservoir 18 m. in diameter rainfall increased the depth of the water 17 cm. How many liters of water were added to the reservoir ?

EXERCISE 259

Problems Involving Square Root

1. Find the square root of 119,716.

2. Find the square root of 170,569.

3. Find to three decimal places the square root of 6.

4. Find to three decimal places the square root of 30.

5. Determine the number of rods around a square field, the diagonal of which is 380 rd.

6. The diagonal of a square is 15 ft. Find to two decimal places the length of one side.

7. The side of a square is 15 ft. Find to two decimal places the length of the diagonal.

8. The area of a square is 37 sq. ft. Find to two decimal places the length of one side.

9. Separate 15,876 into prime factors, and from these factors determine the square root of the number.

10. The area of a circle is 113.0976 sq. in. Find the length of the radius.

11. The area of a circle is 153 9384 sq. ft. Find the length of the radius.

12. The foot of a 37-foot ladder is 12 ft. from the wall of a building against which the top rests. How high does the ladder reach on the wall?

13. How far from the wall of a house must the foot of a 50-foot ladder be placed in order that the top of the ladder may touch a window sill 48 ft. from the ground?

14. A rope stretched from the top of a 28-foot pole just reaches the ground 21 ft. from the foot of the pole. Assuming the rope to be straight, how long is it?

EXERCISE 260

Miscellaneous Problems

1. If 48 yd. of cloth costing 3s. 6d. a yard are sold for £10 4s., what is the per cent of gain?

2. What will it cost for rails for 5 mi. of double track, if the rails cost $28 a ton and weigh 100 lb. per yard?

3. A and B together have $153. If $\frac{3}{4}$ of A's money is equal to $\frac{2}{3}$ of B's, how much has each?

4. A horse tied to a stake can reach the grass 30 ft. in any direction from the stake. Over how many square feet of land can the horse graze?

5. Allowing 1000 shingles to 120 sq. ft., find how many thousand shingles should be bought for a pitched roof each side of which is 58 ft. long and 24 ft. wide.

6. A corner lot has 96 ft. front and is 180 ft. deep. Find the cost of laying a 4-foot stone walk on the front and side, at 18¢ per square foot.

7. Find in inches the depth of a cylindrical tank 5 ft. in diameter that has the same capacity as a rectangular cistern 8 ft. square and 6 ft. deep.

8. A boat travels 15 mi. down stream in $2\frac{1}{2}$ hr. The boat's rate of travel in still water is $4\frac{1}{2}$ mi. an hour. In what time can the boat return?

9. A man has $\frac{5}{8}$ of his property invested in a farm, $\frac{1}{6}$ in railway stock, and the remainder, $1500, is deposited in a bank. Find the value of the farm and of the railway stock.

10. Two men start from points 33 mi. apart and walk towards each other, the first at the rate of $4\frac{1}{2}$ mi. per hour, and the second at the rate of $3\frac{3}{4}$ mi. per hour. How far from where the second man started will they meet?

11. The amount of water that flows over Niagara Falls averages 264,000 cu. ft. a second. Find the average number of gallons carried over the falls in a second.

12. The diagonal of a square is 20 ft. Find to two decimal places the length of one side.

13. If $\frac{7}{8}$ in. on a map represents an actual distance of $35\frac{1}{2}$ mi., what distance on the map represents 28.4 mi.?

14. How many loads of gravel averaging 1 cu. yd. will be required to grade $2\frac{3}{4}$ mi. of road, the gravel to be laid 14 ft. wide and 6 in. deep?

15. If a dealer pays \$9737.50 for some wool, including the purchasing agent's commission of $2\frac{1}{2}\%$, how much is the amount of the commission?

16. The sugar contained in the sugar beet is $6\frac{1}{4}\%$ of the weight of the beet. How many pounds of beets will be required to produce $122\frac{1}{2}$ lb. of sugar?

17. How many boards 12 ft. long will be required to build a straight fence 4 boards high about a rectangular field 40 rd. wide containing 20 acres?

18. Find the circumference of a circle whose diameter is 22.6 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355 : 113.

19. If 15 burners, each consuming 6 cu. ft. of gas per hour, are used on an average 4 hr. per day for 365 days, what will be the gas bill at 80¢ per thousand?

20. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 150 lb. of flint glass.

21. A bell weighing 1250 lb. is composed of 77% copper, 21% tin, and the rest zinc. How many pounds of each metal does the bell contain?

22. Bronze contains 9 parts of copper by weight to 2 parts of tin. How many pounds of tin in a bronze statue that weighs 2662 lb.?

23. The time from one new moon to the next is 29 da. 12 hr. 44 min. 3 sec. Express this in days and the decimal fraction of a day.

24. Find the difference in the number of acres in two fields, one of which contains 75 sq. rd., and the other of which is 75 rd. square.

25. Find to the nearest pound the weight of the air in a schoolroom $30' \times 16' \times 12'$, if water is 770 times as heavy as air and weighs 1000 oz. per cubic foot.

26. If a certain gun metal is composed of 84% copper and the rest tin, how many pounds of tin will be needed with 58.8 lb. of copper to make the gun metal?

27. A man, computing the cost of 54 articles, multiplied by mistake the cost of each by 45 instead of 54, and obtained \$121.50. What was the cost of the 54 articles?

28. A company with a capital of \$350,000 earned net \$28,250 last year. It paid a 7% dividend and carried the balance to surplus. How much did it carry to surplus?

29. A gardener finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in a bin that is 25% longer, 25% narrower, and has the same depth?

30. Find the cost of a stair carpet at \$1.35 a yard for a straight flight of 19 steps 10 in. wide and 8 in. rise, allowing 15 in. extra for the bottom of the stairs and the same for the top.

31. Water is flowing into a rectangular tank whose base is 6 ft. 8 in. long and 4 ft. 6 in. wide, at the rate of 2 cu. ft. in 3 min. How long will it take the water to fill it to a depth of 6 ft.?

32. Find the cost of 325 boards 16 ft. long, 4 in. wide, and $\frac{7}{8}$ in. thick at \$27 per M.

33. Find the number of dry quarts a cylindrical measure $16\frac{1}{2}$ in. in diameter and 1 ft. high will hold.

34. At what rate will \$1250 yield \$112.50 interest in 2 yr.? at what rate will it yield \$206.25 interest in 3 yr.?

35. What fraction of the year 1912 is the time from January 20 to March 15, including both of these days?

36. On seven successive days at noon the thermometer indicated 66°, 72°, 61°, 68°, 73°, 62°, 65°. What was the average for the week?

37. On seven successive days the barometer indicated 29.22 in., 29.18 in., 29.65 in., 30.02 in., 30.12 in., 30.18 in., and 30.21 in. What was the average for the week?

38. A cubic inch of iron weighs $4\frac{4}{9}$ oz. Find the weight in pounds of an iron bar 1 in. square at the end and 1 yd. long; 1 in. square at the end and 15 ft. long.

39. The driving wheels of a locomotive are 6 ft. 8 in. in diameter. How many revolutions a minute does each wheel make when the locomotive is traveling 48 mi. an hour?

40. When water is heated from 32° to 212° it expands $\frac{1}{24}$ in volume. If a cubic foot of water at the freezing point weighs $62\frac{1}{2}$ lb., what does a cubic foot of boiling water weigh?

41. A man bought a bill of goods in Paris to the value of 2600 fr. He paid 40% ad valorem duty and \$17.65 freight, and sent a draft for the purchase price when exchange was 19.4. What was the total cost?

42. A pole is broken by the wind 12 ft. above the ground. The two parts hang together, but the upper part is bent over and touches the ground 12 ft. from the base. How long was the pole originally? (Two decimals.)

43. If it takes 1 bbl. of lime and $\frac{5}{8}$ cu. yd. of sand to make mortar for 1000 bricks, and if $22\frac{1}{2}$ bricks are allowed per cubic foot of wall, how much lime and how much sand are needed for a wall $28' \times 6' \times 1'\,6''$?

44. The diagonal of one face of a cube is $\sqrt{450}$ in. Find the number of square inches in the entire surface of the cube and the number of cubic inches in the volume.

45. If a horse is tethered by a 40-foot rope, the length of the largest circumference he can walk around is how many times what it would be if he were tethered by a 20-foot rope?

46. In Ex. 45 the horse can graze over how many times as much ground in the first case as in the second?

47. One man can do a piece of work in 8 days and another man can do it in 10 days. How many days will it take the two together to do the work?

48. If X can build a wall in 18 days, Y in 15 days, and Z in 12 days, how many days will it take all three working together to build the wall?

49. If X, Y, and Z working together can dig a ditch in 24 hr., and X and Z working together can dig it in 32 hr., how many hours will it take Y alone to dig the ditch?

50. A water tank can be filled by three pipes, running separately, in 4 hours, 5 hours, and 6 hours respectively. In how many hours will the empty tank be filled by all three running together?

51. If the eye is 8 in. above the surface of a lake it can see about 1 mi.; if it is $2^2 \times 8$ in. above the surface, 2 mi.; if it is $3^2 \times 8$ in. above the surface, 3 mi.; and so on, the height being approximately 8 in. multiplied by the square of the distance in miles. About how far can a man see in one direction, if his eye is 36 ft. above the surface of the sea? (Two decimals.)

APPENDIX

437. Subjects Treated. In this Appendix are included certain subjects now generally omitted from courses in arithmetic, but still demanded by schools that have not diminished the time formerly assigned to this study. There are also included several tables that will prove helpful to the student and teacher, and will be of value for reference after the school course is completed. Local conditions should determine what portion of this material should be used. It may all be omitted if desired, or such selections may be made as seem best.

Arithmetical and Geometric Progressions, for example, are best treated in algebra, but for pupils who take no algebra the subject has considerable interest and offers opportunity for exercise in computation. Compound Interest, used chiefly in computing the growth of funds the interest on which is reinvested when due, follows naturally from the study of Geometric Progression, and affords practice in the use of tables. Repeating Decimals, while of little practical value, are interesting applications of progressions, and are frequently met in official examinations. The Proof by Casting out Nines is so important that teachers are advised always to use it with their classes. A large amount of extra drill work has also been provided, from which teachers may select problems as needed. A few problems of a recreative nature have been added that can hardly fail to appeal to many pupils.

438. Arithmetical Progression. A series of numbers that increase or decrease by a common difference is called an *arithmetical progression.*

Thus the numbers 6, 9, 12, 15 form an arithmetical progression with the common difference 3. The numbers of a progression are called its *terms.*

439. Finding a Required Term. In the arithmetical progression

1st	2d	3d	4th	5th	6th
4,	7,	10,	13,	16,	19,

it is evident that any term, as for example the 6th, is found by adding to the first term the product of the common difference by a number 1 less than the number of the term. In this case, the 6th term is $4 + 3 \times 5$, or 19.

In an expression like $4 + 3 \times 5$ the multiplication is performed before the addition. If the addition were to be performed first, the expression would be written $(4 + 3) \times 5$.

If the series is a decreasing series, we subtract from the first term the product of the common difference by a number 1 less than the number of the term. Thus, the 12th term of the series 50, 47, 44, 41, etc., is $50 - 3 \times 11$, or 17.

Therefore, *to find any required term of an arithmetical progression, multiply the common difference by a number that is 1 less than the number of the required term. Add this product to the first term if the series is an increasing series; subtract it from the first term if the series is a decreasing series.*

EXERCISE 261

1. Find the 5th term of 4, 9, 14, etc.
2. Find the 10th term of 3, 8, 13, etc.
3. Find the 12th term of 5, 8, 11, etc.
4. Find the 6th term of 63, 58, 53, etc.
5. Find the 7th term of 100, 92, 84, etc.

440. Finding the Sum. Taking seven terms of the series, 1, 3, 5, etc., the sum is indicated by

$$1 + 3 + 5 + 7 + 9 + 11 + 13$$

In reverse order it is $13 + 11 + 9 + 7 + 5 + 3 + 1$

Hence, *twice the sum* is $14 + 14 + 14 + 14 + 14 + 14 + 14$

Hence, *once the sum* is $\frac{1}{2}$ of 7×14, or 49.

Therefore, *to find the sum of an arithmetical progression, take half the product of the number of terms by the sum of the first and last terms.*

Thus the sum of nine terms of an arithmetical progression whose first term is 3 and last term 41, is $\frac{1}{2}$ of $9 \times (3 + 41)$, or 198.

EXERCISE 262

Given the first and last terms, and the number of terms, find the sum:

	First	*Last*	*No.*		*First*	*Last*	*No.*
1.	3	30	10	**7.**	11	41	11
2.	5	45	11	**8.**	12	56	11
3.	7	40	12	**9.**	15	70	12
4.	4	39	8	**10.**	45	13	9
5.	6	30	13	**11.**	57	21	7
6.	7	35	15	**12.**	49	35	15

Given the first term, common difference, and number of terms, find the last term (§ *439*) *and then the sum:*

	First	*Diff.*	*No.*		*First*	*Diff.*	*No.*
13.	3	3	8	**17.**	5	7	6
14.	7	2	9	**18.**	7	8	10
15.	5	9	7	**19.**	2	3	20
16.	1	6	6	**20.**	1	4	15

Find the sum of:

21. 1, 4, 7, to 8 terms.
22. 2, 5, 8, to 9 terms.
23. 4, 6, 8, to 12 terms.
24. 5, 8, 11, to 10 terms.
25. 7, 8, 9, to 25 terms.
26. 11, 16, 21, to 7 terms.
27. 14, 19, 24, to 6 terms.
28. 23, 20, 17, to 7 terms.

29. What is the sum of the first 25 numbers, beginning with 2?

30. What is the sum of the first 50 odd numbers, beginning with 1?

31. What is the sum of the first 20 numbers that are divisible by 3, beginning with 3?

32. How many times in a day does a clock strike that strikes the hours only?

33. Supposing a body to fall through space $16\frac{1}{12}$ ft. in the first second of its fall, and in each succeeding second $32\frac{1}{6}$ ft. more than in the second just before, how far will it fall in the 6th second? how far in 6 seconds?

34. From the preceding example, how far will a body fall in the 10th second of its fall? in 10 seconds?

35. In a potato race 30 potatoes are placed in a straight line 3 ft. distant from one another. A boy starting from a basket in the line 3 ft. from the first potato is required to pick them up one by one, beginning with the nearest one, and carry them to the basket. To finish the race, how far must the boy run?

36. Solve the preceding example on the supposition that there are 100 potatoes.

37. Solve Example 35 on the supposition that the boy picks up the farthest potato first and the nearest one last, and compare the results.

441. Geometric Progression. A series in which each term after the first is obtained by multiplying the preceding term by a constant multiplier is called a *geometric progression.*

The constant multiplier is called the *ratio* of the progression.

Thus the numbers 2, 6, 18, 54, form a geometric progression, the ratio being 3. The numbers 27, 9, 3, 1, $\frac{1}{3}$, also form a geometric progression, the ratio being $\frac{1}{3}$.

442. Finding a Required Term. In the geometric progression

1st	2d	3d	4th	5th
2,	6,	18,	54,	162,

it is evident that any term, as for example the 5th, is the product of the first term by the power of the ratio that is 1 less than the number of the term.

That is, the 2d term is 2×3; the 3d term is 2×3^2; the 4th term is 2×3^3; the 5th term is 2×3^4; and so on.

Therefore, *to find any required term of a geometric progression, raise the ratio to a power 1 less than the number of the required term, and multiply this by the first term.*

EXERCISE 263

Given the first term and ratio to find the term whose number is indicated:

	First	*Ratio*	*No.*		*First*	*Ratio*	*No.*
1.	1	2	6	7.	64	$\frac{1}{2}$	6
2.	2	4	5	8.	81	$\frac{1}{3}$	4
3.	5	3	7	9.	128	$\frac{1}{2}$	4
4.	7	5	5	10.	243	$\frac{1}{3}$	4
5.	6	6	6	11.	256	$\frac{1}{4}$	3
6.	7	4	5	12.	729	$\frac{1}{9}$	3

13. Find the 8th term of 1, 4, 16, 64, 256, etc.

14. Find the 9th term of 2, 6, 18, 54, 162, etc.

443. Finding the Sum. Taking five terms of the series 2, 6, 18, etc., the sum is indicated by $2 + 6 + 18 + 54 + 162$. If we multiply this sum by the ratio 3, we have:

$3 \times$ the sum is $\quad 6 + 18 + 54 + 162 + 486$

$1 \times$ the sum is $\quad 2 + 6 + 18 + 54 + 162$

$2 \times$ the sum is $\quad 486 - 2$, by subtracting.

Hence the sum is $\quad \dfrac{486 - 2}{2}$.

The numerator of this fraction is the difference between the first term and the product of the last term by the ratio; the denominator is the difference between the ratio and 1.

Therefore, *to find the sum of a geometric progression, multiply the last term by the ratio and subtract the first term from the product. Then divide the remainder by the ratio minus 1.*

If the ratio is less than 1, simply reverse the progression. That is, to find the sum of the series 128, 64, 32, 16, where the ratio is $\frac{1}{2}$, find the sum of the series 16, 32, 64, 128, where the ratio is 2.

EXERCISE 264

Given the first and last terms and the ratio, find the sum:

	First	*Last*	*Ratio*		*First*	*Last*	*Ratio*
1.	2	128	2	6.	5	625	5
2.	1	243	3	7.	3	108	6
3.	1	256	2	8.	2	162	3
4.	3	192	4	9.	32	1	$\frac{1}{2}$
5.	4	108	3	10.	729	81	$\frac{1}{3}$

Find the sum of:

11. 3, 6, 12, to 8 terms.
12. 1, 5, 25, to 7 terms.
13. 4, 8, 16, to 6 terms.
14. 405, 135, 45, to 5 terms.

444. Geometric Progression and Compound Interest. To what sum will \$1000 amount, at 6% compound interest, in 4 yr. ?

Original principal	\$1000	
Amount at end of 1st year	\$1060,	or $1.06 \times \$1000$
Amount at end of 2d year	\$1123.60,	or $1.06 \times \$1060$
Amount at end of 3d year	\$1191.02,	or $1.06 \times \$1123.60$
Amount at end of 4th year	\$1262.48,	or $1.06 \times \$1191.02$

The first term is \$1000, the ratio 1.06, and the fifth term \$1262.48.

To what sum will \$250 amount, in 5 years, at 4% compound interest?

By § 442 the *sixth* term equals

$$1.04^5 \times \$250, \text{ or } 1.21665 \times \$250, \text{ or } \$304.16.$$

445. Compound Interest Table. The table on page 444 shows the amount of \$1 at compound interest at various rates. The compound interest on \$1 is found by subtracting 1 from the proper number shown in the table.

(1) What principal will in 10 yr., at 6% compound interest, yield \$1898.04 interest?

The interest on \$1 for 10 yr. at 6% is \$0.79085 (see the table).

Since \$0.79085 is the interest on \$1,

\$1898.04 is the interest on $\frac{\$1898.04}{0.79085}$, or \$2400.

(2) In what time will \$1600, at $4\frac{1}{2}$% compound interest, yield \$1000 interest?

Since \$1600 yields \$1000, \$1 will yield $\frac{1}{1600}$ of \$1000, or \$0.625, in the same time, and \$1 will amount to \$1.625. By the table, \$1 will, in 11 yr. at $4\frac{1}{2}$%, amount to \$1.62285, and in 12 yr. to \$1.69588. Hence the required time is a little more than 11 yr.

(3) At what rate, compound interest, will \$1500 yield \$1201.41 interest in 15 yr. ?

Since \$1500 yields \$1201.41 interest in 15 yr., \$1 in 15 yr. will yield $\frac{1}{1500}$ of \$1201.41, or \$0.80094, and \$1 will amount in 15 yr. to \$1.80094. In the table, opposite 15 yr., we find in the 4% column the amount of \$1 is \$1.80094. Therefore the rate is 4%.

Table showing the amount of $1 at compound interest for :

YEAR	2 PER CENT	2½ PER CENT	3 PER CENT	3½ PER CENT	4 PER CENT
1	1.02000	1.02500	1.03000	1.03500	1.04000
2	1.04040	1.05063	1.06090	1.07123	1.08160
3	1.06121	1.07689	1.09273	1.10872	1.12486
4	1.08243	1.10381	1.12551	1.14752	1.16986
5	1.10408	1.13141	1.15927	1.18769	1.21665
6	1.12616	1.15969	1.19405	1.22926	1.26532
7	1.14869	1.18869	1.22987	1.27228	1.31593
8	1.17166	1.21840	1.26677	1.31681	1.36857
9	1.19509	1.24886	1.30477	1.36290	1.42331
10	1.21899	1.28009	1.34392	1.41060	1.48024
11	1.24337	1.31209	1.38423	1.45997	1.53945
12	1.26824	1.34489	1.42576	1.51107	1.60103
13	1.29361	1.37851	1.46853	1.56396	1.66507
14	1.31948	1.41297	1.51259	1.61870	1.73168
15	1.34587	1.44830	1.55797	1.67535	1.80094
16	1.37279	1.48451	1.60471	1.73399	1.87298
17	1.40024	1.52162	1.65285	1.79468	1.94790
18	1.42825	1.55966	1.70243	1.85749	2.02582
19	1.45681	1.59865	1.75351	1.92250	2.10685
20	1.48595	1.63862	1.80611	1.98979	2.19112

YEAR	4½ PER CENT	5 PER CENT	5½ PER CENT	6 PER CENT	7 PER CENT
1	1.04500	1.05000	1.05500	1.06000	1.07000
2	1.09203	1.10250	1.11303	1.12360	1.14490
3	1.14117	1.15763	1.17424	1.19102	1.22504
4	1.19252	1.21551	1.23882	1.26248	1.31080
5	1.24618	1.27628	1.30696	1.33823	1.40255
6	1.30226	1.34010	1.37884	1.41852	1.50073
7	1.36086	1.40710	1.45468	1.50363	1.60578
8	1.42210	1.47746	1.53469	1.59385	1.71819
9	1.48610	1.55133	1.61909	1.68948	1.83846
10	1.55297	1.62889	1.70814	1.79085	1.96715
11	1.62285	1.71034	1.80209	1.89830	2.10485
12	1.69588	1.79586	1.90121	2.01220	2.25219
13	1.77220	1.88565	2.00577	2.13293	2.40985
14	1.85194	1.97993	2.11609	2.26090	2.57853
15	1.93528	2.07893	2.23248	2.39656	2.75903
16	2.02237	2.18287	2.35526	2.54035	2.95216
17	2.11338	2.29202	2.48480	2.69277	3.15882
18	2.20848	2.40662	2.62147	2.85434	3.37993
19	2.30786	2.52695	2.76565	3.02560	3.61653
20	2.41171	2.65330	2.91776	3.20714	3.86968

EXERCISE 265

1. Find the compound interest at 6% for 5 yr. on $287.

2. Find the compound interest at $3\frac{1}{2}\%$ for 12 yr. 6 mo. on $686.70.

3. Find the amount of $960 for 9 yr. at 5%, interest compounded semiannually.

4. Find the amount of $1216 for 5 yr. at 8%, interest compounded quarterly.

5. What principal will in 14 yr. at $5\frac{1}{2}\%$ compound interest amount to $1880.78?

6. At what rate compound interest will $2200 yield $4749.40 interest in 17 yr.?

7. In what time will $1800 at 5% compound interest amount to $3000?

8. In what time will $962.44 yield $1080.72 interest at 6% compound interest?

9. What principal will in 20 yr. at 4% compound interest yield $2500 interest?

10. At what rate compound interest will $462.50 yield $277.98 interest in 12 yr.?

11. What principal will in 10 yr. at 6% amount to $3612.22, interest being compounded semiannually?

12. In what time at 5% will $1250 amount to $2000, interest being compounded semiannually?

13. At what rate per annum will $500 amount to $779.83 in 9 yr., interest being compounded semiannually?

14. At what rate compound interest will $1500 amount to $2229.15 in 9 yr.?

15. At what rate compound interest will $2500 amount to $3620.75 in 15 yr.?

446. Repeating Decimals. A decimal that contains a constantly recurring figure or series of figures is called a *repeating decimal* or *circulating decimal.*

Thus the decimal 0.272727 . . . is a repeating decimal, the figures constantly recurring being 27.

The figure or series of figures that constantly recurs in a repeating decimal is called the *repetend.*

If the repetend begins in the first decimal place, the decimal is called a *pure* repeating decimal; otherwise the decimal is called a *mixed* repeating decimal. Thus 0.272727 . . . is a pure repeating decimal and 0.3272727 . . . is a mixed repeating decimal.

Repeating decimals are rarely used, the decimal being carried usually to as many places as desired and the remaining figures being omitted.

447. Symbol. In writing a repeating decimal we stop with the last figure of the repetend and place dots over the first and last figures.

Thus we write $0.\dot{2}\dot{7}$ for 0.272727 . . . and $0.\dot{3}$ for 0.333 . . .

The repetend may be made to begin at any place we please after the first repeating figure. Thus, 4.13548548 . . . may be written $4.13\dot{5}4\dot{8}$, or $4.135\dot{4}8\dot{5}$, or $4.1354\dot{8}5\dot{4}$, and so on.

448. Reducing a Repeating Decimal to a Common Fraction.

(1) Required to reduce $0.\dot{2}\dot{7}$ to a common fraction.

From	100 times the decimal,	or	27.2727 . . .
take	1 times the decimal,	or	0.2727 . . .
Then	99 times the decimal	=	27.
Therefore	the decimal	=	$\frac{27}{99}$, or $\frac{3}{11}$.

(2) Reduce $0.5\dot{2}4\dot{3}$ to a common fraction.

First, suppose we had the repeating decimal $0.\dot{2}4\dot{3}$.

From	1000 times the decimal,	or	243.243243 . . .
take	1 times the decimal,	or	.243243 . . .
Then	999 times the decimal	=	243.
Therefore	the decimal	=	$\frac{243}{999}$, or $\frac{9}{37}$.

Then $0.5\dot{2}4\dot{3}$ must equal $0.5\frac{9}{37}$, or $\frac{194}{370}$, or $\frac{97}{185}$.

449. Repeating Decimals and Geometric Progressions. If we consider the first example in § 448 we see that

$$0.\dot{2}\dot{7} = \tfrac{27}{100} + \tfrac{27}{10000} + \tfrac{27}{1000000} + \cdots,$$

a geometric progression that decreases towards zero, with a ratio $\frac{1}{100}$. If we reverse the order, the first term will be 0 and the ratio 100. Then, by § 443, the sum equals $\dfrac{100 \times \frac{27}{100}}{100 - 1}$, or $\frac{27}{99}$, or $\frac{3}{11}$, as found in § 448.

Therefore, *to reduce a repeating decimal to a common fraction, write the repetend for the numerator, and as many 9's for the denominator as there are figures in the repetend.*

EXERCISE 266

Reduce to repeating decimals :

1. $\frac{1}{3}$.	6. $\frac{2}{7}$.	11. $\frac{1}{11}$.	16. $\frac{8}{15}$.
2. $\frac{2}{3}$.	7. $\frac{5}{7}$.	12. $\frac{5}{11}$.	17. $\frac{1}{22}$.
3. $\frac{1}{6}$.	8. $\frac{1}{9}$.	13. $\frac{1}{12}$.	18. $\frac{5}{22}$.
4. $\frac{5}{6}$.	9. $\frac{5}{9}$.	14. $\frac{7}{12}$.	19. $\frac{1}{30}$.
5. $\frac{1}{7}$.	10. $\frac{7}{9}$.	15. $\frac{1}{15}$.	20. $\frac{1}{90}$.

Reduce to common fractions or mixed numbers :

21. $0.\dot{3}$.	26. $0.\dot{2}\dot{0}$.	31. $0.\dot{2}4\dot{2}$.	36. $0.7\dot{2}8\dot{3}$.
22. $0.\dot{6}$.	27. $0.\dot{2}\dot{1}$.	32. $0.\dot{1}2\dot{7}$.	37. $0.11\dot{3}\dot{6}$.
23. $0.\dot{5}$.	28. $0.\dot{3}\dot{0}$.	33. $0.\dot{1}6\dot{1}$.	38. $2.1\dot{3}3\dot{6}$.
24. $0.\dot{7}$.	29. $0.\dot{9}\dot{0}$.	34. $0.\dot{2}4\dot{0}$.	39. $4.3\dot{1}6\dot{2}$.
25. $0.\dot{9}$.	30. $0.\dot{9}\dot{9}$.	35. $0.\dot{8}6\dot{3}$.	40. $7.124\dot{3}$.

41. Add $0.\dot{7}$, $0.6\dot{3}$, and $0.\dot{2}\dot{1}$, reducing first to common fractions.

42. Subtract $0.\dot{1}\dot{3}$ from $2.\dot{3}$, and express the result as a repeating decimal.

450. Decimal Scale. The common mode of representing numbers is called the *decimal scale of notation.*

This is because the value of the places increases from right to left in a tenfold ratio, the word *decimal* being derived from a Latin word (*decem*) meaning *ten.* In this scale of notation 10 is called the *radix* or *base* of the system.

451. Other Scales. Instead of the radix 10, any other integral number might be used as the radix.

Thus, the number 543 stands for

$5 \times 10^2 + 4 \times 10 + 3$ in the scale of ten,
$5 \times 8^2 + 4 \times 8 + 3$ in the scale of eight,
and $5 \times 6^2 + 4 \times 6 + 3$ in the scale of six.

Therefore, in the scale of ten, 12 means 1 ten and 2 units, or twelve ; in the scale of eight, 12 means 1 eight and 2 units, or ten ; in the scale of five, 12 means 1 five and 2 units, or seven.

The civilized world now uses only the scale of ten.

452. Reducing from One Scale to Another. There is some interest in changing from one scale to another, as in the following examples:

(1) Express 25 in the scale of eight.

This means that we are to find how many eights and units it contains. There are evidently 3 eights and 1 unit besides. Therefore 25 in the scale of ten would be 31 in the scale of eight.

(2) Express 6532 in the scale of six.

This means that we are to find how many sixes, six squares, six cubes, and so on, it contains. Dividing, we have :

$$\begin{array}{rl} 6\overline{)6532} & \\ 6\overline{)1088} & \text{remainder } 4 \\ 6\overline{)181} & \text{remainder } 2 \\ 6\overline{)30} & \text{remainder } 1 \\ 5 & \text{remainder } 0 \end{array}$$

We have, therefore $5, \times 6^4 + 0 \times 6^3 + 1 \times 6^2 + 2 \times 6 + 4$, and the number in the scale of six is 50,124.

(3) Change 50,124 from the scale of six to the scale of eight.

$$
\begin{array}{rl}
8\overline{)50124} & \\
8\overline{)3440} & \text{remainder } 4 \\
8\overline{)250} & \text{remainder } 0 \\
8\overline{)20} & \text{remainder } 6 \\
1 & \text{remainder } 4
\end{array}
$$

Since 50,124 is in the scale of 6, each figure is *six* times the value it would have one place to the right. Hence, we divide $6 \times 5 + 0$ by 8, and we obtain 3 for the quotient and 6 for the remainder. The next partial dividend, $6 \times 6 + 1$, or 37, divided by 8 gives 4 for the quotient and 5 for the remainder. The next partial dividend, $6 \times 5 + 2$, or 32, divided by 8 gives 4 for the quotient and 0 for the remainder ; and so on. Therefore the required number is 14,604.

(4) Change 14,604 from the scale of eight to the scale of ten.

$$
\begin{array}{rl}
10\overline{)14604} & \\
10\overline{)1215} & \text{remainder } 2 \\
10\overline{)101} & \text{remainder } 3 \\
6 & \text{remainder } 5
\end{array}
$$

Therefore the required number is 6532.

EXERCISE 267

Change 3863 in the decimal scale to the scale of :

1. Seven. 2. Eight. 3. Three.

Change 1000 in the decimal scale to the scale of :

4. Four. 6. Three. 8. Eleven.
5. Seven. 7. Nine. 9. Twelve.

Change to the decimal scale :

10. 41,543 in the scale of seven.
11. 41,543 in the scale of six.
12. Change 1235 from the scale of six to the scale of eight.

453. Settlement of Accounts. Find the balance due on the following account, Sept. 10, 1912, computing interest at 6% on each item from its date to the day of settlement, and reckoning the time in days :

1912		Dr.	Int.	1912		Cr.	Int.
June 29.	To Mdse.	$250	$3.04	July 3.	By Cash.	$200	$2.30
July 13.	To Mdse.	400	3.93	July 17.	By Cash.	125	1.15
July 27.	To Mdse.	500	3.75	July 31.	By Cash.	350	2.39
				Sept. 10.	By bal. acct.	475	
Settled Sept. 10, '12.				Sept. 10.	By bal. int.		4.88
		$1150	$10.72			$1150	$10.72

Hence, the cash balance is $475 + $4.88, or $479.88.

When the balance of account and the balance of interest fall on *opposite* sides, the cash balance is their *difference*.

EXERCISE 268

Find the cash balance, Sept. 10, 1912, of the following accounts, reckoning interest at 6% :

1.

1912		Dr.	1912		Cr.
May 13.	To Mdse.	$250.00	May 27.	By Cash.	$200.00
May 28.	To Mdse.	610.00	June 22.	By Cash.	500.00
June 17.	To Mdse.	300.00	June 29.	By Cash.	400.00

2.

1912		Dr.	1912		Cr.
Mar. 7.	To Mdse.	$350.00	Apr. 3.	By Cash.	$150.00
Apr. 10.	To Mdse.	98.50	May 2.	By Cash.	150.00
May 25.	To Mdse.	300.00	June 4.	By Cash.	200.00

3.

1912		Dr.	1912		Cr.
May 8.	To Mdse.	$250.00	June 22.	By Cash.	$200.00
June 5.	To Mdse.	670.00	July 20.	By Cash.	500.00
July 3.	To Mdse.	200.00	Aug. 19.	By Cash.	300.00

454. Division by Factors. If it is required to divide one number by another which can be separated into factors, it is often easier and more convenient to divide the number by one of the factors of the divisor, the quotient by another factor, the second quotient by a third factor, and so on.

For example, required to divide 22,756 by 1155.

The divisor, 1155, equals $3 \times 5 \times 7 \times 11$.

$$\begin{array}{r|l} 3)22756 & \\ 5)7585 & \ldots 1 \\ 7)1517 & \\ 11)216 & \ldots 5 \\ 19 & \ldots 7 \end{array}$$

Here $22{,}756 \div 3 = 7585$ *threes* and 1 *unit* over; 7585 *threes* $\div 5 = 1517$ *fifteens;* 1517 *fifteens* $\div 7 =$ 216 *one hundred fives* and 5 *fifteens* over; 216 *one hundred fives* $\div 11 = 19$ *eleven hundred fifty-fives* and 7 *one hundred fives* over. Therefore the quotient is 19 and the remainder is $1 + 5 \times 15 + 7 \times 105$, or 811.

Therefore, to divide one number by another,

Divide the number by one of the factors of the divisor; divide the quotient by another factor of the divisor; and so on until all the factors have been used as divisors.

Multiply each remainder by all the divisors that precede the divisor that produced it. The sum of these products is the complete remainder.

It is best to use the largest factors possible, not exceeding 12.

EXERCISE 269

Divide by using factors:

1. 2875 by 48.
2. 4792 by 49.
3. 8463 by 75.
4. 9648 by 121.
5. 8762 by 81.
6. 7645 by 144.
7. 6589 by 99.
8. 8431 by 64.
9. 7865 by 128.
10. 8475 by 125.
11. 9175 by 88.
12. 37,560 by 616.
13. 87,542 by 495.
14. 92,653 by 504.
15. 88,475 by 378.
16. 75,555 by 1152.
17. 82,643 by 891.
18. 97,682 by 396.
19. 65,877 by 252.
20. 85,789 by 192.
21. 84,602 by 539.

455. Check by Casting out Nines. One of the most valuable checks upon computations, particularly in multiplication and division, is the one known as Casting out Nines, and this will now be explained.

456. Excess of Nines. The remainder after dividing any integer by 9 is called the *excess of nines* in the integer.

Thus the excess of nines in 11 is 2, because $11 \div 9 = 1$, with 2 remainder. The excess of nines in 48 is 3, the excess in 85 is 4, the excess in 108 is 0.

457. Finding the Excess of Nines. Instead of dividing by 9 to find the excess there is a much shorter method. Consider, for example, the number 2348.

We may separate this into $2000 + 300 + 40 + 8$.

$$\begin{aligned} 2000 &= 2 \times 999 + 2 \\ 300 &= 3 \times 99 + 3 \\ 40 &= 4 \times 9 + 4 \\ 8 &= 8 \\ \hline 2348 &= \text{a multiple of } 9, + 2 + 3 + 4 + 8 \end{aligned}$$

That is, 2348 contains 9 exactly except for the sum of its digits, 2, 3, 4, 8.

Therefore, *the excess of nines in any number is equal to the excess in the sum of the digits.*

458. Casting out Nines. In finding the remainder from dividing the sum of the digits by 9, that is, in casting out the nines, we may of course omit the nines, or any two or three digits which we see at a glance will make 9.

Thus in casting out the nines from 1,926,754, we see at once that $1 + 2 + 6 = 9$ and $5 + 4 = 9$, and the single 7 will be the remainder. So in 254,786, we reject 5, 4, and 2, 7, and add $8 + 6$; from the sum we reject 9 and have 5 left.

459. Check in Multiplication. *The excess of nines in a product equals the excess in the product of the excesses in the factors.*

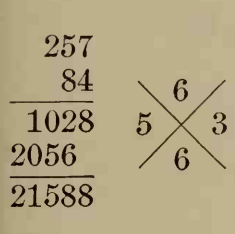
Check the product of 61 × 47 by casting out nines.

The remainder after the nines are cast out:

From 47 is 2
From 61 is 7 } Multiply
From 2867 is 5. From 14 is 5.

The product of the two *numbers* has an excess 5, and the product of the two *excesses* has 5 remaining after the nines are cast out. Therefore the work may be assumed to be correct. The check does not, however, discover errors that are independent of the sum of the digits.

A convenient arrangement of work is as follows:



In the two angles at the sides of the cross write 5 and 3, the excesses in the factors. At the top write the excess in the product of these excesses, 6 being the excess in 3 × 5, or 15. At the bottom write the excess in the product, 6 being the excess in 21,588. The top and bottom numbers in the cross should then agree.

460. Check in Division. *The excess of nines in a dividend equals the excess in the product of the excesses in the divisor and quotient, plus the excess in the remainder.*

For, if 1,348,708 ÷ 498 is 2708 with remainder 124, then 1,348,708 = 2708 × 498 + 124. Now 8, the excess in 2708, times 3, the excess in 498, is 24; and 6, the excess in 24, plus 7, the excess in 124, is 13, or simply 4. Since the excess in 1,348,708 is 4, the work may be assumed to be correct.

EXERCISE 270

Multiply, and check by casting out nines:

1. 75 × 628.
2. 34 × 872.
3. 27 × 691.
4. 36 × 987.
5. 246 × 489.
6. 492 × 572.

Divide, and check by casting out nines:

7. 144 ÷ 12.
8. 1524 ÷ 12.
9. 1331 ÷ 11.
10. 2714 ÷ 15.
11. 1342 ÷ 21.
12. 27,634 ÷ 128

461. Tables of Denominate Numbers. The following tables are here brought together for convenience in reference. Certain tables of equivalents have been added, and these will frequently be found useful in making reductions and in avoiding long multiplications.

Table of Length. (*Page 166*)

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

			ft.	in.
		yd.	1 =	12
	rd.	1 =	3 =	36
mi.	1 =	$5\frac{1}{2}$ =	$16\frac{1}{2}$ =	198
1 =	320 =	1760 =	5280 =	63,360

A *hand* (4 in.) is used in measuring the height of horses; a *fathom* (6 ft.) and *cable length* (120 fathoms) in measuring depths of water; a *knot* (nautical mile, 1.152 common or statute miles, or 6080.27 ft.) in measuring distances at sea. Carpenters, mechanics, and others usually write 2′ 6″ for 2 ft. 6 in.

Table of Square Measure. (*Page 168*)

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

				sq. ft.		sq. in.
			sq. yd.	1	=	144
		sq. rd.	1 =	9	=	1296
	A.	1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$	=	39,204
sq. mi.	1 =	160 =	4840 =	43,560	=	6,272,640
1 =	640 =	102,400 =	3,097,600 =	27,878,400	=	4,014,489,600

Table of Cubic Measure. (*Page 169*)

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

	cu. ft.	cu. in.
cu. yd.	1 =	1728
1 =	27 =	46,656

A perch of stone or masonry is usually 1 rd. long, 1 ft. high, and $1\frac{1}{2}$ ft. thick, and contains $24\frac{3}{4}$ cu. ft., but this varies in different parts of the country. A cubic yard of earth is considered a load.

Table of Avoirdupois Weight. (*Page 170*)

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

	lb.	oz.
T.	1 =	16
1 =	2000 =	32,000

100 lb. is sometimes called a *hundredweight* (cwt.). The ton of 2000 lb. is sometimes called the *short ton*, there being an old ton of 2240 lb., known as the *long ton*, which is used in the customhouse and in some wholesale transactions in mining products. There is also the *long hundredweight* of 112 lb., but it is rarely used except in English trade.

Table of Troy Weight. (*Page 170*)

24 grains (gr.) = 1 pennyweight (pwt. or dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

The avoirdupois pound contains 7000 gr., the troy pound 5760 gr. For the *carat*, see page 170.

Table of Apothecaries' Weight. (*Page 170*)

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb.)

Table of Liquid Measure. (*Page 172*)

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
63 gallons = 1 hogshead (hhd.)

			pt.	gi.
		qt.	1 =	4
	gal.	1 =	2 =	8
bbl.	1 =	4 =	8 =	32
1 =	31½ =	126 =	252 =	1008

A gallon contains 231 cubic inches.

Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads. Whenever barrels or hogsheads are used as *measures*, a barrel means 31½ gallons, and a hogshead means 63 gallons. A barrel contains about 4⅕ cu. ft.

Apothecaries and physicians use the following table of liquid measure, but it need not be learned:

60 minims (℥) = 1 dram (℥ lx. or f. ʒ)
8 fluid drams = 1 ounce (fl. drm. viij. or f. ℥)
16 ounces = 1 pint (fl. oz. xvj. or O)
8 pints = 1 gallon (cong.)

Table of Dry Measure. (*Page 175*)

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

		qt.	pt.
	pk.	1 =	2
bu.	1 =	8 =	16
1 =	4 =	32 =	64

A bushel contains 2150.42 cu. in., or about 1¼ cu. ft. A dry quart contains 67.2 cu.in., while the liquid quart contains only 57.75 cu. in. In measuring grain, seeds, and small fruits the measure must be *even* full. In measuring apples, potatoes, and other large articles the measure must be *heaping* full.

Table of Surveyors' Linear Measure. (*Page 177*)

7.92 inches (in.) = 1 link (l.)
25 links = 1 rod (rd.)
100 links = 1 chain (ch.)
80 chains = 1 mile (mi.)

			l.	in.
		rd.	1 =	7.92
	ch.	1 =	25 =	198
mi.	1 =	4 =	100 =	792
1 =	80 =	320 =	8000 =	63,360

Table of Surveyors' Square Measure. (*Page 177*)

16 square rods (sq. rd.) = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
1 square mile = 1 section (sec.)
36 sections = 1 township (T.)

		sq. ch.	sq. rd.
	A.	1 =	16
sq. mi.	1 =	10 =	160
1 =	640 =	6400 =	102,400

Table of Time. (*Page 178*)

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
366 days = 1 leap year

			min.	sec.
		hr.	1 =	60
	da.	1 =	60 =	3600
wk.	1 =	24 =	1440 =	86,400
1 =	7 =	168 =	10,080 =	604,800
yr. mo.				
1 = 12 =	365 =	8760 =	525,600 =	31,536,000
	366 =	8784 =	527,040 =	31,622,400

462. Metric System Equivalents.

Length

Meter	= 39.370 in.	Inch	= 0.0254 m.
	= 3.281 ft.	Foot	= 0.3048 m.
	= 1.093 yd.	Yard	= 0.9144 m.
	= about 1.1 yd.		= about 0.9 m.
Millimeter	= 0.039 in.	Inch	= 25.400 mm.
Centimeter	= 0.3937 in.		= 2.540 cm.
Kilometer	= 0.62137 mi.	Mile	= 1.60935 km.
	= about $\frac{3}{5}$ mi.		= about 1.6 km.

Surface

Sq. meter	= 10.764 sq. ft.	Sq. foot	= 0.0929 sq. m.
	= 1.1960 sq. yd.	Sq. yard	= 0.8361 sq. m.
Hektare	= 2.471 A.	Acre	= 0.4047 ha.
	= about $2\frac{1}{2}$ A.		= about $\frac{2}{5}$ ha.
Sq. kilometer	= 0.3861 sq. mi.	Sq. mile	= 2.59 sq. km.

Volume

Cu. centimeter	= 0.061 cu. in.	Cu. inch	= 16.3872 cu. cm.
Cu. meter	= 35.314 cu. ft.	Cu. foot	= 0.0283 cu. m.
	= 1.3079 cu. yd.	Cu. yard	= 0.7646 cu. m.
Stere	= 0.2759 cd.	Cord	= 3.6245 st.

Capacity

Liter	= 1.05668 liq. qt.	Liq. quart	= 0.94636 l.
	= 0.9081 dry qt.	Dry quart	= 1.1012 l.
	= 0.26417 gal.	Gallon	= 3.78543 l.
	= 0.11351 pk.	Peck	= 8.80982 l.
Hektoliter	= 2.83774 bu.	Bushel	= 0.35239 hl.

Weight

Gram	= 15.4324 gr.	Grain	= 0.0648 g.
	= about $15\frac{1}{2}$ gr.		= about $\frac{3}{50}$ g.
	= 0.03527 oz. av.	Ounce av.	= 28.3495 g.
Kilogram	= 2.20462 lb. av.	Pound av.	= 0.45359 kg.
	= about $2\frac{1}{5}$ lb.		= 0.00045 t.
Met. ton.	= 2204.62 lb. av.	Short ton	= 0.90718 t.

463. Spanish Land Measures. In those sections of the United States that formerly belonged to Mexico, the Spanish measures are still used in land surveying.

The *vara* is taken as the unit of measure.

In Mexico the length of the vara is 32.9927 in., in California 33 in., in Texas $33\frac{1}{3}$ in. In the problems below its length will be taken as $33\frac{1}{3}$ in., as in Texas. In California it should be taken as 33 in.

A *labor* is a square each side of which is 1000 varas.

A *square league* is a square each side of which is 5000 varas.

464. Table of Length.

1 vara = $33\frac{1}{3}$ inches
3 varas = 100 inches
36 varas = 100 feet
108 varas = 100 yards
1900.8 varas = 1 mile

465. Table of Area.

1,000,000 square varas = 1 labor = 177.136 acres
25 labors = 1 square league = 4428.4 acres
5645.376 square varas = 1 acre

EXERCISE 271

Change to yards:

1. 540 varas. 2. 936 varas. 3. 257 varas.

Change to feet:

4. 864 varas. 5. 324 varas. 6. 750 varas.

Change to miles and decimals of a mile:

7. 3801.6 varas. 8. 8553.6 varas. 9. 22,809.6 varas.

Change to labors or square varas:

10. 3542.72 acres. 11. 10,000 sq. ft. 12. 1 sq. mi.

466. Legal Weights in Pounds per Bushel.

	Apples	Barley	Beans	Beets	Buck-wheat	Shelled Corn	Flaxseed	Oats	Onions	Peas	Potatoes	Sweet Potatoes	Rye	Wheat
U.S. customs.		48			42		56	32		60	60		56	60
Ala.		47	60			56		32		60		55	56	60
Ariz.		45	55					32					56	60
Ark.	50	48	60		52	56	56	32	57	60	60	50	56	60
Cal.		50			40			32					54	60
Colo.		48	60		52			32	57		60		56	60
Conn.	48	48	60	60	48		55	32	52	60	60	54	56	60
Del.														60
D.C.											60			
Fla.	48	48	60			56		32	56			60	56	60
Ga.		47	60		52	56	56	32	57	60		55	56	60
Hawaii		48						32					56	60
Idaho	45	48			42		56	36			60		56	60
Ill.		48	60		52	56	56	32	57			50	56	60
Ind.		48	60		50	56		32	48		60	55	56	60
Iowa	48	48	60		52	56	56	32	57		60	46	56	60
Kans.	48	48	60		50		56	32	57		60	50	56	60
Ky.		47	60		56	56	56	32	57	60	60	55	56	60
La.		48											56	60
Me.	44	48	60	60	48			32	52	60	60		50	60
Md.								26			56			
Mass.	48	48	60		48	50	55	32	52	60	60	54	56	60
Mich.	48	48	60		48	56	56	32	54	60		56	56	60
Minn.	50	48	60	50	50	56		32	52	60		55	56	60
Miss.		48	60		48	56	56	32	57	60		60	56	60
Mo.	48	48	60		52	56	56	32	57	60		56	56	60

Legal Weights in Pounds per Bushel (*Continued*).

	Apples	Barley	Beans	Beets	Buck-wheat	Shelled Corn	Flaxseed	Oats	Onions	Peas	Potatoes	Sweet Potatoes	Rye	Wheat
Mont.	45	48	60	50	52	56	56	32	57	60	60		56	60
Nebr.		48	60		52	56	56	32	57	60		50	56	60
Nev.														
N.H.			62					32		60	60		56	60
N.J.	50	48	60		50		55	30	57	60		54	56	60
N.Mex.														
N.Y.	48	48	60		48		55	32	57	60		54	56	60
N.C.		48			50		55	32		60			56	60
N.Dak.	50	48	60	60	42	56	56	32	52	60		46	56	60
Ohio	50	48	60	56	50	56	56	32	55	60		50	56	60
Okla.		48	60	60	42	56	56	32	52	60		46	56	60
Oreg.	45	46			42			32			60		56	60
Pa.		47			48			32	50		56		56	60
R.I.	48	48	60	50	48	56	56	32	50	60		54	56	60
S.C.														
S.Dak.		48	60	60	42	56	56	32	52	60		46	56	60
Tenn.	50	48	60	50	50	56	56	32	56	60		50	56	60
Tex.	45	48	60		42	56	56	32	57			55	56	60
Utah														
Vt.	46	48	62	60	48			32	52	60	60		56	60
Va.		48	60		52	56	56	30	57	60		56	56	60
Wash.	45	48			42		56	32			60		56	60
W.Va.		48	60		52		56	32			60		56	60
Wis.	50	48	60	50	50		56	32	57	60		54	56	60
Wyo.														

467. Drill Problems. Teachers occasionally wish more drill problems than a text-book supplies. An opportunity is offered in the following exercise to select from 15,800 problems in the fundamental operations, but teachers are advised to omit the entire exercise unless there is need for special attention to some particular deficiency. The problems are arranged as follows:

1. Add each of the 10 numbers in the first column of I to each of the lettered numbers of the second column, thus giving 100 examples.

For example, (7) + (c) = 10,125 + 27 = 10,152.

2. In the same way, subtract, giving 100 examples.
3. In the same way, multiply, giving 100 examples.
4. In the same way, divide, giving 100 examples.
5. Proceed in the same way for II, giving 400 more examples, or 800 on this page. In the same way III affords 2500 different examples, and so on, making a total of 15,800 different examples.

EXERCISE 272

I		II	
1. 2025.	a. 9.	1. 1176.	a. 4.
2. 8100.	b. 15.	2. 4704.	b. 6.
3. 6075.	c. 27.	3. 2352.	c. 7.
4. 16,200.	d. 45.	4. 5880.	d. 8.
5. 12,150.	e. 25.	5. 3528.	e. 21.
6. 18,225.	f. 75.	6. 9408.	f. 49.
7. 10,125.	g. 81.	7. 8232.	g. 98.
8. 14,175.	h. 225.	8. 10,584.	h. 147.
9. 22,275.	i. 135.	9. 12,936.	i. 196.
10. 20,250.	j. 675.	10. 11,760.	j. 392.

Add, subtract, multiply, and divide, using each number of the first column with each lettered number of the second column:

III		IV	
1. 8640.	a. 2.	1. 10,368.	a. 16.
2. 5184.	b. 8.	2. 20,736.	b. 32.
3. 3456.	c. 4.	3. 51,840.	c. 27.
4. 6912.	d. 6.	4. 41,472.	d. 81.
5. 17,280.	e. 9.	5. 62,208.	e. 54.
6. 34,560.	f. 3.	6. 31,104.	f. 36.
7. 51,840.	g. 16.	7. 72,576.	g. 64.
8. 55,296.	h. 18.	8. 93,312.	h. 48.
9. 58,752.	i. 24.	9. 134,784.	i. 96.
10. 65,664.	j. 27.	10. 155,520.	j. 162.
11. 10,368.	k. 32.	11. 176,256.	k. 108.
12. 12,096.	l. 96.	12. 103,680.	l. 192.
13. 32,832.	m. 12.	13. 145,152.	m. 144.
14. 13,824.	n. 36.	14. 165,888.	n. 288.
15. 15,552.	o. 72.	15. 114,048.	o. 324.
16. 29,376.	p. 54.	16. 196,992.	p. 648.
17. 20,736.	q. 48.	17. 186,624.	q. 576.
18. 15,552.	r. 108.	18. 124,416.	r. 384.
19. 69,120.	s. 216.	19. 228,096.	s. 1728.
20. 25,920.	t. 288.	20. 207,360.	t. 1296.
21. 27,648.	u. 144.	21. 259,200.	u. 2592.
22. 414,720.	v. 864.	22. 238,464.	v. 1152.
23. 103,680.	w. 432.	23. 269,568.	w. 5184.
24. 829,440.	x. 192.	24. 248,832.	x. 3456.
25. 207,360.	y. 1728.	25. 217,728.	y. 10,368.

Add, subtract, multiply, and divide, using each number of the first column with each lettered number of the second column:

V		VI	
1. 64.68.	a. 0.4.	1. 189.728.	a. 0.4.
2. 582.12.	b. 0.8.	2. 3794.56.	b. 0.8.
3. 12.936.	c. 0.6.	3. 5.69184.	c. 1.6.
4. 258.72.	d. 2.4.	4. 75891.2.	d. 3.2.
5. 7.7616.	e. 20.	5. 94.864.	e. 1.4.
6. 905.52.	f. 0.07.	6. 113.8368.	f. 2.8.
7. 3880.8.	g. 1.10.	7. 13.28096.	g. 0.56.
8. 194.04.	h. 7.7.	8. 1517.824.	h. 0.11.
9. 106.424.	i. 5.5.	9. 17.07552.	i. 1.21.
10. 84.084.	j. 26.4.	10. 18.9728.	j. 0.77.
11. 3.234.	k. 4.9.	11. 2087.008.	k. 0.14.
12. 129.36.	l. 0.35.	12. 227.6736.	l. 0.49.
13. 13.5828.	m. 15.	13. 246.6464.	m. 8.8.
14. 4.5276.	n. 8.8.	14. 2656.192.	n. 3.52.
15. 12.2892.	o. 0.66.	15. 2845.92.	o. 1.76.
16. 0.135828.	p. 30.	16. 303.5648.	p. 6.16.
17. 51.744.	q. 4.4.	17. 474.32.	q. 5.39.
18. 97.020.	r. 1.65.	18. 3794.56.	r. 0.98.
19. 711.48.	s. 38.5.	19. 455.3472.	s. 24.2.
20. 0.122892.	t. 24.5.	20. 3415.104.	t. 8.47.
21. 1164.24.	u. 1.05.	21. 41740.16.	u. 11.2.
22. 5.8212.	v. 0.22.	22. 3225.376.	v. 6.05.
23. 103.488.	w. 5.39.	23. 36.04832.	w. 61.6.
24. 0.9702.	x. 0.49.	24. 3984.288.	x. 2.24.
25. 0.6468.	y. 0.12.	25. 4363.744.	y. 48.4.

Add, subtract, multiply, and divide, using each number of the first column with each lettered number of the second column:

VII

1. $\frac{5}{8}$.	a. $\frac{1}{2}$.
2. $\frac{7}{8}$.	b. $\frac{1}{3}$.
3. $\frac{5}{6}$.	c. $\frac{1}{4}$.
4. $\frac{3}{4}$.	d. $\frac{1}{5}$.
5. $\frac{3}{5}$.	e. $\frac{1}{6}$.
6. $\frac{4}{7}$.	f. $\frac{1}{8}$.
7. $\frac{5}{7}$.	g. $\frac{2}{5}$.
8. $\frac{6}{7}$.	h. $\frac{3}{8}$.
9. $\frac{5}{9}$.	i. $\frac{1}{9}$.
10. $\frac{7}{9}$.	j. $\frac{2}{9}$.
11. $\frac{8}{9}$.	k. $\frac{4}{9}$.
12. $\frac{7}{10}$.	l. $\frac{1}{12}$.
13. $\frac{9}{10}$.	m. $\frac{5}{12}$.
14. $\frac{6}{11}$.	n. $\frac{1}{16}$.
15. $\frac{7}{12}$.	o. $\frac{3}{16}$.
16. $\frac{7}{16}$.	p. $\frac{5}{16}$.
17. $\frac{9}{16}$.	q. $\frac{7}{16}$.
18. $\frac{13}{16}$.	r. $\frac{1}{24}$.
19. $\frac{15}{16}$.	s. $\frac{5}{24}$.
20. $\frac{17}{32}$.	t. $\frac{7}{24}$.
21. $\frac{23}{24}$.	u. $\frac{11}{24}$.
22. $\frac{19}{24}$.	v. $\frac{1}{32}$.
23. $\frac{17}{48}$.	w. $\frac{3}{32}$.
24. $\frac{29}{48}$.	x. $\frac{5}{32}$.
25. $\frac{51}{100}$.	y. $\frac{5}{64}$.

VIII

1. $4\frac{1}{2}$.	a. 2.
2. $3\frac{1}{4}$.	b. 3.
3. $3\frac{3}{4}$.	c. $1\frac{1}{8}$.
4. $4\frac{1}{8}$.	d. $1\frac{3}{8}$.
5. $5\frac{3}{8}$.	e. $2\frac{5}{8}$.
6. $3\frac{5}{8}$.	f. $2\frac{7}{8}$.
7. $4\frac{7}{8}$.	g. $1\frac{1}{5}$.
8. $3\frac{1}{16}$.	h. $2\frac{3}{5}$.
9. $4\frac{3}{16}$.	i. $1\frac{4}{5}$.
10. $3\frac{5}{16}$.	j. $2\frac{1}{6}$.
11. $3\frac{7}{16}$.	k. $1\frac{5}{6}$.
12. $4\frac{9}{16}$.	l. $1\frac{1}{10}$.
13. $3\frac{11}{16}$.	m. $2\frac{3}{10}$.
14. $4\frac{15}{16}$.	n. $1\frac{1}{16}$.
15. $3\frac{1}{24}$.	o. $1\frac{3}{16}$.
16. $3\frac{5}{24}$.	p. $2\frac{5}{16}$.
17. $4\frac{11}{24}$.	q. $1\frac{9}{16}$.
18. $3\frac{1}{32}$.	r. $1\frac{1}{24}$.
19. $3\frac{3}{32}$.	s. $2\frac{5}{24}$.
20. $4\frac{5}{32}$.	t. $2\frac{7}{24}$.
21. $3\frac{9}{32}$.	u. $1\frac{1}{32}$.
22. $3\frac{1}{48}$.	v. $1\frac{5}{32}$.
23. $4\frac{5}{48}$.	w. $2\frac{1}{48}$.
24. $3\frac{7}{48}$.	x. $1\frac{1}{64}$.
25. $3\frac{1}{10}$.	y. $2\frac{3}{100}$.

468. Arithmetical Recreations. Most of the following problems have had a long and interesting history. They are part of the inheritance of the school, and they show some of the amusing features of arithmetic. They claim to have no practical bearing, and they should be omitted unless taken in the spirit intended, as mere recreations showing the lighter side of the subject.

EXERCISE 273

1. Multiply 142,857 by 7.

2. Multiply 37 by 3; by 6; by 9; by 12; by 15; by 18; by 21.

3. How can you write one hundred with Arabic figures without using zeros?

4. How can you write an even number, using only the odd digits?

5. If a goose weighs 10 lb. and half of its own weight, what is the weight of the goose?

6. How many days would it take to cut a piece of cloth 50 yd. long into 1-yard lengths, one yard being cut off daily?

7. If a post in a pond stands $\frac{1}{3}$ in the ground, $\frac{1}{4}$ in the water, and 10 feet above the water, how long is it?

8. A bottle and its stopper cost \$1.10, and the bottle cost \$1 more than the stopper. What did the stopper cost?

9. A new steel coal car and its contents weigh 139,700 lb., and the contents weighs 60,300 lb. more than the car. What does the car weigh?

10. Two men purpose to cross the Rocky Mountains. X can walk 13 miles a day and Y only 10 miles a day. Therefore Y starts 9 days ahead. How long will it take X to overtake Y?

11. A snail climbing up a post 20 ft. high ascends 5 ft. every day and slips down 4 ft. every night. How long will it take the snail to reach the top of the post?

12. A visitor to a class said, "Good morning, you 100 pupils." One of them replied, "We are not 100, but our number, and the number again, and its half, and its fourth, and one more, make 100." How many were there?

13. A man borrowed as much money as he had and then spent $10. He again borrowed as much money as he then had and again spent $10. A third time he borrowed as much money as he then had and then spent $10. He then had nothing left. How much had he at first?

14. One man said to another, "If you give me $1, I shall have as much as you." The other replied, "If you give me $1, I shall have twice as much as you." How much did each have?

15. In several parts of the world twenty-four-hour time is used. Assuming the clock to strike the hours from 1 to 24, how many strokes would such a clock make in a day? How many, if it also struck each half hour?

16. Two Arabs met at meal time, one having 4 loaves and the other 3. Just as they were beginning to eat, a third Arab, without any loaves, joined them, and they all ate equally. The third Arab then produced 7 pence and said, "Divide this equitably." How much did each receive?

17. A Persian fable relates that a father left to his three sons 17 camels, the first son to have half, the second a third, and the youngest a ninth. They could not divide them, and so a wise man loaned them another camel, and they divided the 18, the first taking 9, the second 6, and the youngest 2. This made 17, and they returned the one that was borrowed. Was the division right?

18. It is said that the inventor of the game of chess was asked by the king to name his reward. He asked for a single grain of wheat to place on the first square, 2 on the second, 4 on the third, 8 on the fourth, and so on, doubling each time, until the 64 squares were completed. How many grains of wheat did the inventor ask?

19. A woman dropped a basket of eggs and broke them all. Some one asked her how many she had, and she said she did not know, but that when she put them in the basket by twos there remained one egg; when she took them out by threes there also remained one egg; and when she took them out again, five at a time, there remained none. What is the smallest number that she could have had?

20. A thief leaving an orchard met three other thieves, who threatened to expose him unless he gave them a share of his apples. To the first he gave half, who then returned 12 to him. To the second he gave half of the remainder, who then returned 7. To the third he gave half of what he then had, 4 being returned. He then had 20 apples. How many did he steal?

21. Two men are employed by a manufacturing company, one beginning with a salary of $1000 a year, with a yearly increase of $200, and the other at a salary of $1000 a year, with a half-yearly increase of $50. In each case the salary was paid every six months. Which had the larger income?

22. The first of two bottles is half full of water and the second is half full of vinegar. From the first bottle a tablespoonful of water is taken and poured into the second bottle. A tablespoonful of the mixture is then taken from the second bottle and poured into the first bottle. Is the quantity of water removed from the first bottle greater or less than the quantity of vinegar removed from the second bottle?

DEFINITIONS OF COMMON TERMS

Abstract Number. A number used without reference to any particular unit (§ 3).

Addends. Numbers added (§ 22).

Addition. The operation of finding a number equal to two or more numbers taken together (§ 18).

Agent. A person who transacts business for another (§ 270).

Aliquot Part. An exact divisor of another number (§ 157).

Amount. The sum of principal and interest (§ 290).

Analysis. Any process of reasoning by which we prove a mathematical truth.

Annual Interest. Periodic interest in which installments are due annually (§ 301).

Antecedent. The first term of a ratio (§ 310).

Area. The number of square units in a surface (§ 170).

Arithmetic. The science that treats of numbers and the methods of using them.

Arithmetical Progression. A series of numbers that increase or decrease by a common difference (§ 438).

Assets. All the property of an estate, individual, or corporation.

Bank. A corporation authorized by law to receive and loan money and to perform certain other financial operations (§ 395).

Bank Discount. Interest paid in advance on a note (§ 403).

Base. The number of which some per cent is to be taken (§ 259).

Bond. A written or printed promise to pay a specified sum at a given time, signed by the maker and often bearing his seal (§ 392).

Broker. An agent who makes sales or purchases for his principal without having actual possession of the goods (§ 271).

Brokerage. A sum paid a broker for transacting business (§ 269).

Cancellation. The process of shortening work in division by removing or canceling equal factors from both dividend and divisor (§ 90).

Capital Stock. The amount contributed by the members of a corporation to carry on the business (§ 381).

Check. Any method tending to verify an operation. A written order on a bank to pay money (§ 400).

Commercial Discount. A reduction from the list price of an article, from the amount of a bill of goods, or the amount of a debt (§ 264).

Commercial Draft. A draft made by one business house on another to secure the payment of a debt (§ 407).

Commission. A sum paid an agent for transacting business (§ 269).

Commission Merchant. An agent who has actual possession of goods to be sold for his principal (§ 271).

Common Denominator. A denominator common to two or more fractions (§ 106).

Common Divisor. The same as Common Measure (§ 80).

Common Fraction. A fraction expressed by two numbers, one under the other with a line between them (§ 92).

Common Measure. A number that exactly divides each of two or more numbers (§ 80).

Common Multiple. A multiple of two or more numbers (§ 86).

Complex Fraction. A fraction that has a fraction in one or both of its terms (§ 124).

Composite Number. A number that is not a prime number (§ 70).

Compound Interest. The total interest when interest is added to the principal and becomes a part of it at specified intervals (§ 302).

Compound Number. A denominate number involving two or more different units (§ 164).

Concrete Number. If the unit of a number is named, the number and the unit together is called a *concrete number* (§ 3).

Consequent. The second term of a ratio (§ 310).

Consignee. The person or firm to whom goods are sent.

Consignor. The person or firm who sends goods to another.

Corporation. A company composed of a number of persons authorized by law to do business of a certain nature (§ 380).

Creditor. A person or firm to whom money is due.

Cube. The third power of a number (§ 45, § 346).

Cube Root. One of the three equal factors of a number (§ 347).

Customs. Taxes imposed by law on merchandise imported (§ 285).

Debtor. A person who owes money to another.

Decimal Fraction. A fraction whose denominator is not written, but is some power of 10 (§ 128).

Decimal Point. The period written at the left of tenths (§ 129).

Decimal Scale. The common mode of representing numbers, each order having a value ten times that of the order to the right (§ 450).

Denominate Number. A concrete number expressing measure (§ 163).

Denominator. The number below the line in a common fraction (§ 93).

Difference. The result found in subtraction (§ 28).

Discount. Allowance made for the payment of money before it becomes due. Also, the difference between the market value and the face value when the market value is *below* the face value (§ 386). A reduction from the list price of goods (trade discount) (§ 264).

Discount Series. A series of trade discounts (§ 265).

Dividend. In division, the given number which is equal to the product of a given factor (called divisor) and a required factor (called quotient) (§ 51). In business, the sum paid on each share of stock from the profits of the business (§ 384).

Divisible Number. A number is *divisible* by another when it is exactly divisible, that is, divisible without a remainder (§ 72).

Division. The process by which, when a product and one of two factors are given, the other factor is found (§ 49).

Divisor. The number by which a given dividend is divided (§ 52).

Draft. A written order directing one person to pay a specified sum of money to another (§ 406).

Drawee of a Draft. The person to whose order the sum of money named in the draft is to be paid (§ 407).

Drawer of a Draft. The person who signs the draft (§ 407).

Duties. The same as customs (§ 285).

Even Number. A number that is exactly divisible by 2 (§ 73).

Exact Divisor. A number that divides another without a remainder (§ 71).

Exact Interest. Interest computed for exact days, and at 365 days to the year (§ 300).

Exchange. A system of paying debts, due to persons living at a distance, without actually sending the money (§ 410).

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor (§ 328).

Extremes. The first and last numbers of a proportion (§ 315).

Factors. Integral numbers whose product is a given number (§ 38, § 67).

Figures. Symbols used to represent numbers in the common system of notation. Also diagrams used to represent geometric forms.

Fraction. A number that shows what part or what number of parts of a unit is taken (§ 91).

Geometric Progression. A series of numbers in which each term after the first is obtained by multiplying the preceding term by a constant multiplier (§ 441).

Greatest Common Divisor. The same as Greatest Common Measure.

Greatest Common Measure. The greatest number that exactly divides each of two or more numbers (§ 81).

Improper Fraction. A fraction whose numerator is not less than the denominator (§ 96).

Indorse. To write across the back of a note, draft, or check (§ 400).

Insurance. An agreement by which, for a lawful consideration, a company agrees to pay an indemnity for loss by specified cause (§ 272).

Integers. Numbers applied to whole units (§ 4).

Interest. Money paid for the use of money (§ 288).

Inverse Ratio. The result of interchanging the terms of a ratio (§ 319).

Least Common Multiple. The smallest common multiple of two or more numbers (§ 87).

License. A permit to engage in some specified act or to enjoy some special privilege (§ 280).

Like Numbers. Numbers applied to the same unit (§ 21).

Long Division. The method of dividing in which the processes are written in full (§ 60).

Longitude. The distance in circular measure east or west of a certain meridian agreed upon (§ 206).

Lowest Terms. If the terms of a fraction are prime to each other, the fraction is expressed in *lowest terms* (§ 100).

Maker of a Note. The person who signs a note (§ 304).

Market Value. The price at which goods or stocks sell (§ 386).

Maturity of a Note. The date at which a note legally becomes due.

Means. The two middle numbers of a proportion (§ 315).

Meridian. The shortest line on the earth's surface passing from one pole to the other (§ 205).

Minuend. The number from which the subtrahend is taken (§ 28).

Mixed Number. A number composed of an integer and a fraction taken together (§ 103).

Multiple. The product of two or more integers is called a *multiple* of each (§ 37, § 85).

Multiplicand. A number to be multiplied (§ 36).

Multiplication. The process of taking a number as many times as there are units in another number (§ 35).

Multiplier. A number by which another is multiplied (§ 36).

Net Price. The amount of a bill after the discount is made (§ 264).

Net Proceeds. The money that remains of the money received for property after all expenses and discounts are paid.

Notation. A system of expressing numbers by symbols (§ 5).

Note. A written promise to pay a specified sum of money on demand or at a specified time (§ 303).

Number. That which shows how many times a unit is taken, or what part of a unit is taken (§ 2).

Numerals. The characters used in any system of notation (§ 7).

Numeration. The reading of numbers (§ 5).

Numerator. The number above the line in a common fraction (§ 93).

Odd Number. A number that is not an even number (§ 74).

Order of Units. Each successive place in a number that may be occupied by a figure (§ 9).

Par Value. The face value of stocks (§ 385).

Partial Payments. Payments of a part of a note (§ 306).

Payee. The person to whom a note, check, or draft is payable (§ 304, § 407).

Per Cent. Another name for hundredths (§ 252).

Percentage. The part of arithmetic treating of per cent (§ 252). The result found by taking a certain per cent of the base (§ 259).

Perimeter. The distance around a polygon (§ 222).

Period. A group of three figures (§ 11).

Policy. A written contract of insurance (§ 273).

Poll Tax. A tax levied upon a person (§ 278).

Power. The result of taking a number any number of times as a factor (§ 45, § 325).

Premium. Money paid for insurance (§ 273). Also the excess of market value above par value (§ 386).

Prime Factor. A factor that is a prime number (§ 69).

Prime Number. A number that has no factors (§ 68).

Prime to Each Other. Two integers that have no common measure, always excepting 1 (§ 82).

Principal. The person for whom another transacts business (§ 270). Money loaned on which interest is paid (§ 289).

Proceeds. The balance after deducting the discount from the face of a note (§ 404).

Product. The result found in multiplication (§ 36).

Promissory Note. A written promise to pay a specified sum of money on demand or at a specified time (§ 303).

Proper Fraction. A fraction whose numerator is less than the denominator (§ 95).

Property Tax. A tax levied upon property (§ 279).

Proportion. An expression of equality between two ratios (§ 314).

Quotient. The number found by division (§ 53).

Rate of Interest. The rate per cent of the principal paid in interest for one year (§ 290).

Rate per cent. Rate in hundredths (§ 259).

Ratio. The relative magnitude of two numbers, as expressed by the fraction which has the first number for the numerator and the second for the denominator (§ 309).

Reciprocal of a Fraction. The fraction inverted (§ 121).

Reduction. The process of changing the unit of a number without changing the value (§ 98, § 165).

Remainder. The part of the dividend that is left in division (§ 54).

Root. One of the equal factors of a number.

Security. Property used to guarantee the payment of any debt.

Short Division. The process of dividing in which the operations of multiplying and subtracting are performed mentally (§ 57).

Similar Decimals. Decimals that have the same number of decimal places (§ 134).

Similar Fractions. Fractions with the same denominator (§ 107).

Simple Fraction. A fraction whose terms are both integers (§ 124).

Specific Gravity. The ratio of the weight of a given substance to the weight of an equal volume of water (§ 376).

Square. The result of taking a number twice as a factor (§ 45, § 326).

Square Root. One of the two equal factors of a number (§ 327).

Stock. Capital invested in business (§ 381).

Subtraction. The process of taking one number from another, or of finding what number must be added to one of two numbers to make the other (§ 27).

Subtrahend. The number taken away in subtraction (§ 28).

Sum. The result obtained in addition (§ 19).

Taxes. Money levied by a government for its support and for public purposes (§ 276).

Terms of a Fraction. The numerator and denominator (§ 93).

Terms of a Proportion. The extremes and means (§ 315).

Unitary Analysis. An analysis that proceeds first to the unit and then to the required number of units (§ 203).

Unit Fraction. A fraction whose numerator is 1 (§ 94).

Units. The standards by which we count or measure (§ 1).

Volume. The number of cubic units a body contains (§ 173).

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ANSWERS

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2. 22.	13. 22.	24. 33.	35. 31.	46. 43.	57. 37.
3. 26.	14. 17.	25. 30.	36. 40.	47. 35.	58. 52.
4. 30.	15. 32.	26. 31.	37. 49.	48. 39.	59. 57.
5. 30.	16. 25.	27. 36.	38. 38.	49. 36.	60. 60.
6. 35.	17. 24.	28. 44.	39. 45.	50. 44.	61. 62.
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8. 28.	19. 28.	30. 35.	41. 52.	52. 36.	63. 79.
9. 23.	20. 35.	31. 40.	42. 64.	53. 45.	64. 82.
10. 17.	21. 37.	32. 41.	43. 50.	54. 37.	65. 79.
11. 19.	22. 33.	33. 42.	44. 36.	55. 44.	

Exercise 6. Page 16

1. 99.	10. 193.	19. 358.	28. 500.	37. 238.	46. 410.
2. 99.	11. 283.	20. 470.	29. 489.	38. 218.	47. 435.
3. 124.	12. $130.	21. 438.	30. 544.	39. 268.	48. 512.
4. 111.	13. 147.	22. 394.	31. 390.	40. 253.	49. 609.
5. 228.	14. 159.	23. 265.	32. 400.	41. 359.	50. 726.
6. $102.	15. 172.	24. 293.	33. 410.	42. 499.	51. 810.
7. 269.	16. 271.	25. 276.	34. 220.	43. 507.	52. 780.
8. 184.	17. 438.	26. 439.	35. 195.	44. 400.	53. 788.
9. 137.	18. 420.	27. 461.	36. 226.	45. 487.	

Exercise 7. Page 18

1. 1388.	9. 23,058.	17. 265,241.	25. 9,436,416.
2. 2210.	10. 16,886.	18. 249,853.	26. $102,354,579.
3. 1652.	11. 99,936.	19. 92,097.	27. $752,856,806.
4. 1662.	12. 116,404.	20. 134,487.	28. $2,055,150,998.
5. 2210.	13. 156,827.	21. 22,027.	29. 76,303,387.
6. 20,000.	14. 142,877.	22. 1157.	
7. 6024.	15. 198,887.	23. $34,593,348.	
8. 14,030.	16. 210,264.	24. 3,026,789.	

Exercise 8. Page 20

1. $13.67.
2. $6.46.
3. $23.30.
4. $14.
5. $22.
6. $21.10.
7. $24.41.
8. $11.49.
9. $16.27.
10. $91.
11. $29.50.
12. $40.39.
13. $30.41.
14. $40.04.
15. $247.91.
16. $3.49.
17. $2.54.
18. $18.74.
19. $15.36.
20. $491.18.
21. $165,423.91.
22. 94,100 square miles.
23. 439,905 square miles.
24. $46,932.31.
25. 3,454,029.
26. 5,952,596.

Exercise 9. Page 24

1. 412.
2. 322.
3. 292.
4. 313.
5. 330.
6. 206.
7. 531.
8. 333.
9. 111.
10. 246.
11. $6.23.
12. $3.21.
13. $8.23.
14. $5.50.
15. $6.51.
16. $13.55.
17. $21.10.
18. $22.35.
19. $31.15.
20. $62.36.
21. $11.54.
22. $80.61.
23. $72.50.
24. $60.
25. $24.42.
26. $154.10.
27. $121.63.
28. $134.55.
29. $361.90.
30. $415.23.
31. 51,113.
32. 11,341.
33. 34,225.
34. 13,463.
35. 11,221.
36. 12,363.
37. 7505.
38. 24,152.
39. 12,920.
40. 52,371.
41. 320.
42. 325.
43. $345.
44. 1411.
45. 240.
46. 1112.
47. 115.
48. 832 miles.
49. 35,100 square miles.
50. 106,110 square miles.
51. 140 square miles.
52. 31,602,024 bushels.

Exercise 10. Page 27

1. 212.
2. 309.
3. 408.
4. 538.
5. 608.
6. 717.
7. 829.
8. 892.
9. 292.
10. 183.
11. 262.
12. 342.
13. 672.
14. 782.
15. 289.
16. 578.
17. 685.
18. 436.
19. 287.
20. 377.
21. 478.
22. 670.
23. 557.
24. 244.
25. 310.
26. 401.
27. 512.
28. 300.
29. 318.
30. 117.
31. 217.
32. 309.
33. 135.
34. 259.
35. 205.
36. 404.
37. 126.
38. 104.
39. 115.
40. 209.
41. 208.
42. 89.
43. 179.
44. 287.
45. 178.
46. 189.
47. 176.
48. 166.
49. 172.
50. 161.
51. 268.
52. 213.
53. 2662.
54. 3209.
55. 5108.
56. 8267.
57. 2093.
58. 4492.
59. 6172.
60. 4091.
61. 7093.
62. 2487.
63. 5078.
64. 3057.
65. 4087.

66. 6082.
67. 6185.
68. 2087.
69. 1287.
70. 4895.
71. 1869.
72. 14,859.
73. 9339.
74. 21,833.
75. 13,389.
76. 17,887.
77. 16,789.
78. 10,083.
79. $11.91.
80. $16.20.
81. $9.09.
82. $30.28.
83. $16.60.
84. $55.18.
85. $11.64.
86. $33.92.
87. $49.95.
88. $47.88.
89. $61.80.
90. $83.65.
91. $85.55.
92. $137.95.
93. $174.85.
94. $104.67.
95. $279.88.
96. $368.95.
97. $137.65.
98. $24.35.
99. $924.50.
100. $1724.50.
101. $3851.50.
102. $4674.75.
103. $6374.25.
104. $2874.50.
105. $4759.50.
106. $939.25.
107. $1924.50.
108. $5499.25.
109. 3119.
110. 3359.
111. 2519.
112. 1159.
113. $754.50.
114. $224.25.

Exercise 11. Page 29

1. $43.
2. $178.45.
3. $157.80.
4. $12.40.
5. $16.07.
6. $18.67.
7. $13.75.
8. $14.60.
9. $68.12.
10. $1183.50.
11. $271.48.
12. $32.10.
13. $935.85.
14. $1448.89.
15. $18.95.
16. $10,493.45.

Exercise 12. Page 32

1. 1943.
2. 14,529.
3. 2768.
4. 28,571.
5. 19,148.
6. 24,110.
7. $2413.75.
8. $61.75.
9. $370.95.
10. $43.50.
11. $555.61.
12. $782.50.
13. $165.
14. $240.90.
15. $30.65.
16. $1553.50.
17. $486.10.
18. 230,362.
19. 386,741 ounces.
20. 2,420,455 ounces.
21. 561,619.
22. 2,732,416,000 bushels.
23. $15,652,223.
24. $1,616,935,103.
25. 229,643,552.
26. $148,594,501.
27. $16,865,439.
28. $363,229,344.
29. $1,192,563.32.
30. 330.
31. 797,000 bales.
32. 6,521,957 tons; 33,284,500 tons.

Exercise 13. Page 38

1. 414.
2. 1227.
3. 1224.
4. 3520.
5. 3048.
6. 5614.
7. 4840.
8. 8181.
9. 874.
10. 1578.
11. 3292.
12. 3575.
13. 2472.
14. 5691.
15. 4896.
16. 6570.
17. 1498.
18. 2052.
19. 6216.
20. 4536.
21. 12,174.
22. 18,222.
23. 32,384.
24. 35,240.
25. 20,454.

26. 32,942.
27. 54,464.
28. 38,763.
29. 5526.
30. 13,488.
31. 33,108.
32. 39,395.
33. 55,416.
34. 62,601.
35. 75,584.
36. 66,303.
37. 59,892.
38. 51,429.
39. 71,104.
40. 89,991.
41. 82,406.
42. 78,621.
43. 142,820.
44. 213,045.
45. 185,556.
46. 282,023.
47. 563,544.
48. 725,625.
49. 154,468.
50. 247,887.
51. 190,904.
52. 419,605.
53. 433,662.
54. 444,003.
55. 386,368.
56. 327,924.
57. 473,274.
58. 629,902.
59. 454,312.
60. 899,991.
61. $336.
62. $2400.
63. 3240.
64. 3934.

Exercise 14. Page 39

1. $3.75.
2. $8.10.
3. $10.60.
4. $13.
5. $13.15.
6. $31.55.
7. $14.88.
8. $22.32.
9. $29.96.
10. $40.25.
11. $28.96.
12. $51.28.
13. $29.25.
14. $65.52.
15. $3164.
16. $1956.
17. $1405.
18. $3432.
19. $1596.
20. $7191.
21. $3468.
22. $45.75.
23. $65.10.
24. $50.48.
25. $128.60.
26. $86.
27. $115.55.
28. $113.40.
29. $225.
30. $288.12.
31. $264.25.
32. $173.04.
33. $662.08.
34. $319.50.
35. $428.67.
36. $12,852.
37. $15,710.
38. $12,378.
39. $4214.
40. $30,380.
41. $26,901.
42. $46,736.
43. $364.50.
44. $813.60.
45. $501.
46. $1226.80.
47. $1607.55.
48. $1243.75.
49. $1973.40.
50. $3256.50.
51. $901.25.
52. $2136.40.
53. $1694.
54. $3402.
55. $3354.48.
56. $8889.12.
57. $84,682.
58. $64,263.
59. $105,604.
60. $158,100.
61. $540,519.
62. $544,581.
63. $508,488.
64. $111.
65. $22.75.
66. $13.50.
67. $25.90.
68. $31.50.
69. $142.
70. $171.
71. $6915.
72. $71.
73. $6.84.
74. $77.46.
75. $2300.
76. $339.75.
77. $112.50.
78. $29.75.
79. $65.95.

Exercise 15. Page 41

1. 1770.
2. 19,080.
3. 14,280.
4. 23,500.
5. 55,800.
6. 15,300.
7. 32,100.
8. $30,100.
9. $284.
10. $439.20.
11. $186.20.
12. $345.60.
13. $546.
14. $504.
15. 128,100.
16. 320,000.
17. 2,940,000.
18. 3,150,000.
19. $3000.
20. $2400.
21. 89,100,000.

Exercise 16. Page 42

1. 55,608.
2. 151,032.
3. 110,628.
4. 192,696.
5. 95,788.
6. 195,642.
7. 184,522.
8. 436,408.
9. 200,129.
10. 273,208.
11. 644,982.
12. 433,519.
13. 338,706.
14. 287,616.
15. 656,197.
16. 366,816.
17. 452,898.
18. 720,802.
19. 268,128.
20. 653,315.
21. 508,776.
22. 651,817.
23. 410,720.
24. 397,467.
25. 627,480.
26. 383,808.
27. 639,878.
28. 859,068.
29. 989,901.
30. 1,553,171.
31. 1,515,192.
32. 1,469,264.
33. 1,389,789.
34. 3,071,880.
35. 4,015,920.
36. 2,980,670.
37. 3,376,360.
38. 2,398,484.
39. 3,044,672.
40. 1,644,032.
41. 5,019,258.
42. 1,542,940.
43. 6,778,982.
44. 3,517,701.
45. 4,480,852.
46. 2,811,468.
47. 5,915,182.
48. 7,411,656.
49. 4,005,980.
50. 7,728,993.
51. 5,485,172.
52. 3,403,944.
53. 3,471,794.
54. 6,195,441.
55. 39,930,966.
56. 23,887,808.
57. 26,611,944.
58. 50,024,883.
59. $22,605.
60. $32,900.
61. $30,660.
62. $26,010.
63. $19,411.
64. $41,088.
65. $75,078.
66. $3,360,000.
67. $5,250,000.
68. $1,920,000.
69. $4,800,000.
70. $13,654.75.
71. $10,174.50.
72. $13,639.92.
73. $25,502.40.
74. $32,181.32.
75. $43,421.84.
76. $13,048.82.
77. $32,484.06.
78. $52,706.70.
79. $734,447.04.
80. $350,513.52.
81. $337,085.60.
82. $411.25.
83. $1940.40.
84. $529.20.
85. $1482.
86. $2386.30.

Exercise 17. Page 44

1. $1050.
2. 42,240.
3. 88,000 pounds.
4. 264 feet.
5. 22,500 feet.
6. $23.80.
7. $195.
8. $1200.
9. $1020.
10. $25.50.
11. $3000.
12. 1960.
13. $4875.
14. $13,875.
15. 15,504.
16. 2100.
17. 126,720.
18. 13,392.
19. 19,456.
20. 4900.
21. 2448.
22. $4200.
23. 86,400.
24. 620,400 pounds.
25. 20,664 pounds.
26. 62,060 pounds.
27. 8640 ounces.
28. 52,095.
29. 28,224 pounds.
30. $1575.
31. $4668.75.
32. $5378.75.
33. $2047.
34. $257.50.
35. $1424.44.
36. 135,000 feet.
37. 16,092 pounds.
38. 16,524 pounds.
39. 5568 pounds.
40. 6,065,400.
41. 62.
42. $1008.
43. $369.60.
44. $31.36.
45. The horses; $136.
46. 17,280,000 miles.

Exercise 18. Page 49

1. 116.
2. 115.
3. 104.
4. 65.
5. 123.
6. 132.
7. 74.
8. 93.
9. 1621.
10. 1374.
11. 1501.
12. 603.
13. 672.
14. 439.
15. 921.
16. 891.
17. $2777\frac{1}{2}$.
18. $2004\frac{2}{3}$.
19. $1053\frac{1}{4}$.
20. $578\frac{3}{5}$.
21. $1045\frac{1}{6}$.
22. $1050\frac{6}{7}$.
23. $1140\frac{3}{8}$.
24. $897\frac{4}{9}$.
25. $23.62.
26. $11.05.
27. $7.18.
28. $6.55.
29. $3.55.
30. $4.21.
31. $8.21.
32. $9.29.
33. 20,251.
34. 5801.
35. 5211.
36. 5720.
37. 8791.
38. 3557.
39. $15,600\frac{1}{2}$.
40. $11,359\frac{1}{3}$.
41. $6557\frac{3}{4}$.
42. $14,004\frac{3}{5}$.
43. $13,389\frac{1}{6}$.
44. $4188\frac{5}{7}$.
45. $5293\frac{1}{8}$.
46. $8467\frac{6}{9}$.
47. $17.60\frac{1}{2}$.
48. $24.14\frac{1}{3}$.
49. $15.30\frac{3}{4}$.
50. $14.47\frac{1}{5}$.
51. $13.66\frac{5}{6}$.
52. $6.00\frac{3}{7}$.
53. $8.12\frac{4}{8}$.
54. $10.40\frac{5}{9}$.

Exercise 19. Page 50

1. $39.
2. 27 feet.
3. 52.
4. 144.
5. 59.
6. 44 feet.
7. 124.
8. 288.
9. $136.
10. 576.
11. 720.
12. $1.65.
13. $1.35.
14. $2.48.

Exercise 20. Page 51

1. 34.
2. 36.
3. 18.
4. 16.
5. 15.
6. 14.
7. 12.
8. 12.
9. 4.
10. 8.
11. $27\frac{3}{10}$.
12. $21\frac{3}{20}$.
13. $10\frac{25}{30}$.
14. $6\frac{35}{40}$.
15. $4\frac{36}{50}$.
16. $4\frac{47}{70}$.
17. $7\frac{52}{70}$.
18. $4\frac{71}{80}$.
19. $3\frac{8}{90}$.
20. $8\frac{57}{90}$.
21. 203.
22. 137.
23. 214.
24. 181.
25. $63\frac{1}{500}$.
26. $454\frac{03}{600}$.
27. $85\frac{221}{700}$.
28. $52\frac{748}{800}$.
29. $109\frac{107}{900}$.
30. 42.
31. 82.
32. 93.
33. $94\frac{700}{6000}$.
34. $61\frac{2820}{7000}$.
35. $92\frac{6623}{8000}$.
36. $95\frac{2542}{9000}$.
37. 24.
38. 17.
39. 27.
40. 37.
41. 165.
42. 17.
43. 175; $250.
44. 17; $1100.
45. 291; 1260.

Exercise 21. Page 53

1. 13 quarts.
2. 15 quarts.
3. 14.
4. 65.
5. 385 pounds.
6. 8120 pounds.
7. 63 pounds.
8. 12 pounds; 3 pounds.
9. Red, 14 pounds; crimson, 15 pounds.
10. 10.

Exercise 22. Page 55

1. 32.	10. 14.	19. 17.	28. 219.	37. 803.	46. 987.
2. 41.	11. 21.	20. 32.	29. 407.	38. 809.	47. 998.
3. 62.	12. 32.	21. 22.	30. 107.	39. 456.	48. 777.
4. 88.	13. 37.	22. 33.	31. 511.	40. 862.	49. 888.
5. 78.	14. 39.	23. 37.	32. 462.	41. 877.	50. 999.
6. 87.	15. 47.	24. 111.	33. 531.	42. 888.	51. 567.
7. 67.	16. 28.	25. 123.	34. 617.	43. 880.	52. 909.
8. 83.	17. 19.	26. 112.	35. 609.	44. 990.	53. 808.
9. 12.	18. 15.	27. 101.	36. 701.	45. 999.	54. 809.

Exercise 23. Page 56

1. 73.	11. 333.	21. 9009.	31. 2754.	40. 640.
2. 92.	12. 426.	22. 750.	32. 1975.	41. 225.
3. 105.	13. 916.	23. 662.	33. 1683.	42. 346.
4. 63.	14. 862.	24. 235.	34. 2929.	43. 696.
5. 204.	15. 999.	25. 775.	35. Cows, $38;	44. 504.
6. 65.	16. 709.	26. 1326.	horses, $95.	45. 625.
7. 48.	17. 809.	27. 1505.	36. 352 miles.	46. 4025.
8. 77.	18. 707.	28. 2002.	37. 720.	47. 329 miles.
9. 129.	19. 2111.	29. 2075.	38. 3075.	48. 3168; 8800.
10. 129.	20. 4706.	30. 1312.	39. 2009.	

Exercise 24. Page 60

1. 84.	6. 8760.	11. 365.	16. 207.	21. 1332.	26. 79.
2. 82.	7. 503.	12. 467.	17. 783.	22. 1875.	27. $111\frac{25}{69}$.
3. 76.	8. 1060.	13. 675.	18. 858.	23. 2397.	28. $112\frac{10}{76}$.
4. 726.	9. 626.	14. 4268.	19. 367.	24. 2086.	29. $372\frac{6}{23}$.
5. 950.	10. 1050.	15. 468.	20. 780.	25. 98.	30. $365\frac{12}{56}$.

31. $283\frac{1}{64}$.	40. $612\frac{6}{412}$.	49. 98.	58. 1350.
32. $259\frac{34}{57}$.	41. $185\frac{25}{653}$.	50. $564\frac{56}{28836}$.	59. 6480.
33. $416\frac{4}{46}$.	42. $617\frac{6}{4222}$.	51. $384\frac{40}{62357}$.	60. $25.
34. $564\frac{15}{47}$.	43. $831\frac{100}{2332}$.	52. $873\frac{45}{59477}$.	61. 5746.
35. $467\frac{6}{59}$.	44. 36.	53. $144.	62. $4938.
36. $682\frac{20}{69}$.	45. 54.	54. $21.	63. $18,246.
37. $617\frac{8}{82}$.	46. 37.	55. $41.	64. 476.
38. $436\frac{8}{2332}$.	47. 75.	56. $81.	65. $780.
39. $890\frac{10}{1124}$.	48. 77.	57. 1411.	66. $90.

Exercise 25. Page 62

1. $8.76.
2. $1.88.
3. $29.35.
4. $625.72.
5. $572.30.
6. $4213.01.
7. $682.02.
8. $617.20.
9. $527.30.
10. $309.87.
11. $481.20.
12. $681.39.
13. $502.75.
14. $491.76.
15. $123.13.
16. $33.87.
17. $879.17.
18. $932.60.
19. $49.50.
20. $79.
21. $77.
22. $43.
23. $70.40.
24. $873.
25. $574.
26. $390.
27. $1198.34.
28. $2.17.
29. $1.57.
30. $9.45.
31. $2.07.
32. $3.10.
33. $3.15.
34. $2.84.
35. $2.67.
36. $4.93.
37. $2.75.
38. $3.20.
39. $3.10.
40. $10.08.
41. $2.50.

Exercise 26. Page 64

1. $9.45.
2. $8.75 ; $1.25.
3. $2.10.
4. $3.50.
5. $1.65.
6. $58.50 ; $120.25.
7. $79.60.
8. $0.21.
9. 12 ; 36.
10. 31.
11. 7425.
12. $71.55.
13. $168.75.
14. $201.60.
15. $12.70.
16. 8538 feet.
17. 36,605 miles.
18. $30.15.
19. 33,645 feet.
20. $5.49.
21. 475 tons.
22. 60 miles.
23. 348 miles.
24. 576 miles.
25. 884 ; 272.
26. $0.93.
27. 2 hours.
28. $28.21.
29. $1005.
30. 90 miles.
31. $40.
32. $438.52 ; $61.48.
33. $4875.
34. 36 ; 3.
35. 33 minutes ; 44 minutes.
36. 440.
37. 240.
38. $0.21.
39. 7 ; 7.
40. 12.
41. $4273.65.
42. $2.85.
43. $2.90.
44. $3.15.
45. $3275.30.
46. $2.35.
47. 3796 ; 60.
48. 212 ; 780.
49. 13,205.
50. 92.
51. 17.
52. 54.
53. 39 ; 13,859.
54. 444.
55. 74.

Exercise 28. Page 75

1. 432 ; 640 ; 1024 ; 3036 ; 8292 ; 4878.
2. 636 ; 444 ; 1002 ; 3027 ; 1104 ; 6777.
3. 830 ; 4225 ; 3010 ; 7200 ; 6305 ; 9270 ; 4005.

4. 53 ; 61 ; 97.
5. 2, 2, 2, 3, 5.
6. 2, 3, 5, 7.
7. 2, 3, 5, 5.
8. 2, 2, 5, 11.
9. 2, 5, 17.
10. 5, 5, 7.
11. 3, 3, 5, 5.
12. 3, 101.
13. 2, 3, 5, 5, 5.
14. 2, 3, 3, 37.
15. 7, 11, 13.
16. 3, 5, 83.
17. 11, 11, 11.
18. 3, 3, 3, 3, 3, 3, 5.
19. 2, 2, 2, 3, 11, 13.
20. 2, 2, 2, 2, 2, 2, 3, 3, 3.
21. 2, 2, 2, 2, 2, 3, 5, 11.
22. 2, 3, 7, 11, 13.
23. 37, 101.
24. 2, 2, 2, 3, 11, 37.

Exercise 29. Page 77

1. 2.
2. 4.
3. 3.
4. 6.
5. 7.
6. 15.
7. 8.
8. 17.
9. 9.
10. 19.
11. 5.
12. 21.
13. 11.
14. 23.
15. 6.
16. 2.
17. 3.
18. 4.
19. 3.
20. 5.
21. 4.
22. 7.
23. 8.
24. 9.
25. 5.
26. 12.
27. 6.
28. 16.
29. 18.
30. 13.
31. 21.
32. 11.
33. 10.
34. 16.
35. 3.
36. 12.
37. 12.
38. 38.
39. 8.
40. 11.
41. 25.
42. 64.
43. 32.
44. 9.
45. 25.

Exercise 30. Page 78

1. 65.
2. 14.
3. 37.
4. 41.
5. 144.
6. 625.
7. 61.
8. 101.
9. 29.
10. 14.
11. 229.
12. 37.
13. 53.
14. 57.
15. 71.
16. 146.
17. 29.
18. 29.

Exercise 31. Page 80

1. 66.
2. 51.
3. 48.
4. 105.
5. 351.
6. 165.
7. 105.
8. 228.
9. 390.
10. 240.
11. 882.
12. 144.
13. 180.
14. 210.
15. 6552.
16. 25,389.
17. 150.
18. 192.
19. 396.
20. 756.
21. 120.
22. 720.
23. 66.
24. 4004.
25. 560.
26. 600.
27. 480.
28. 27,720.
29. 96.
30. 1260.
31. 6930.
32. 1260.
33. 576.
34. 1890.
35. 660.
36. 495.
37. 144.
38. 312.
39. 300.
40. 30 feet.
41. 24 quarts.
42. 50 cents.
43. 96 gallons.
44. 509.
45. 127.

Exercise 32. Page 82

1. 36.
2. 8.
3. 9.
4. 6.
5. 10.
6. 8.
7. 3.
8. 2.
9. 8.
10. 16.
11. 9.
12. 252.
13. 28.
14. 144.
15. $1\frac{1}{2}$.
16. 84.
17. 40.
18. 140.
19. 384.
20. 70.
21. $2.
22. 3.
23. 4.
24. 9.
25. 18.
26. 9.
27. $16.
28. $75; $108.
29. 4.
30. 8.
31. 7.
32. 7.
33. 4.
34. 5.

Exercise 34. Page 87

1. $\frac{3}{4}$.
2. $\frac{3}{4}$.
3. $\frac{2}{4}$.
4. $\frac{2}{4}$.
5. $\frac{3}{5}$.
6. $\frac{1}{5}$.
7. $\frac{4}{5}$.
8. $\frac{2}{5}$.
9. $\frac{3}{8}$.
10. $\frac{4}{8}$.
11. $\frac{6}{8}$.
12. $\frac{4}{8}$.

Exercise 35. Page 88

1. $\frac{1}{4}$.
2. $\frac{3}{7}$.
3. $\frac{9}{11}$.
4. $\frac{4}{7}$.
5. $\frac{6}{11}$.
6. $\frac{1}{3}$.
7. $\frac{1}{3}$.
8. $\frac{1}{2}$.
9. $\frac{2}{3}$.
10. $\frac{8}{25}$.
11. $\frac{1}{3}$.
12. $\frac{1}{3}$.
13. $\frac{1}{4}$.
14. $\frac{3}{4}$.
15. $\frac{5}{9}$.
16. $\frac{3}{4}$.
17. $\frac{1}{4}$.
18. $\frac{1}{3}$.
19. $\frac{16}{39}$.
20. $\frac{6}{13}$.
21. $\frac{1}{2}$.
22. $\frac{1}{3}$.
23. $\frac{2}{5}$.
24. $\frac{11}{48}$.
25. $\frac{2}{3}$.
26. $\frac{9}{32}$.
27. $\frac{1}{7}$.
28. $\frac{2}{7}$.
29. $\frac{37}{40}$.
30. $\frac{11}{12}$.
31. $\frac{11}{12}$.
32. $\frac{12}{13}$.

Exercise 36. Page 89

1. $\frac{30}{60}$.
2. $\frac{32}{60}$.
3. $\frac{27}{60}$.
4. $\frac{22}{60}$.
5. $\frac{40}{80}$.
6. $\frac{25}{80}$.
7. $\frac{28}{80}$.
8. $\frac{18}{80}$.
9. $\frac{72}{144}$.
10. $\frac{12}{144}$.
11. $\frac{30}{144}$.
12. $\frac{14}{144}$.
13. $\frac{78}{156}$.
14. $\frac{8}{156}$.
15. $\frac{10}{156}$.
16. $\frac{14}{156}$.
17. $\frac{128}{256}$.
18. $\frac{48}{256}$.
19. $\frac{40}{256}$.
20. $\frac{28}{256}$.
21. $\frac{144}{288}$.
22. $\frac{24}{288}$.
23. $\frac{60}{288}$.
24. $\frac{42}{288}$.
25. $\frac{160}{320}$.
26. $\frac{60}{320}$.
27. $\frac{50}{320}$.
28. $\frac{45}{320}$.
29. $\frac{50}{100}$.
30. $\frac{30}{100}$.
31. $\frac{16}{100}$.
32. $\frac{18}{100}$.

Exercise 37. Page 90

1. 3.
2. $3\frac{1}{6}$.
3. $3\frac{1}{3}$.
4. 5.
5. 6.
6. $2\frac{1}{4}$.
7. $2\frac{1}{2}$.
8. $4\frac{1}{2}$.
9. 5.
10. 6.
11. $1\frac{1}{2}$.
12. 3.
13. 4.
14. 4.
15. 3.
16. 2.
17. $1\frac{1}{2}$.
18. $1\frac{1}{6}$.
19. $1\frac{3}{4}$.
20. 5.
21. $1\frac{1}{5}$.
22. $1\frac{1}{2}$.
23. $1\frac{4}{5}$.
24. $1\frac{9}{10}$.
25. 7.
26. 1.
27. $1\frac{1}{8}$.
28. $1\frac{1}{4}$.
29. $1\frac{3}{8}$.
30. 7.
31. $1\frac{1}{9}$.
32. $1\frac{1}{6}$.
33. $1\frac{1}{3}$.
34. 2.
35. 4.
36. 2.
37. $2\frac{1}{16}$.
38. 3.
39. 5.
40. 6.
41. $2\frac{1}{2}$.
42. $3\frac{3}{4}$.
43. $4\frac{1}{4}$.
44. $5\frac{1}{2}$.
45. $7\frac{1}{2}$.
46. 1.
47. $2\frac{1}{2}$; 3

Exercise 38. Page 91

1. $\frac{31}{8}$.
2. $\frac{39}{8}$.
3. $\frac{61}{8}$.
4. $\frac{55}{8}$.
5. $\frac{77}{8}$.
6. $\frac{115}{16}$.
7. $\frac{85}{16}$.
8. $\frac{137}{16}$.
9. $\frac{151}{16}$.
10. $\frac{107}{16}$.
11. $\frac{53}{24}$.
12. $\frac{127}{24}$.
13. $\frac{203}{24}$.
14. $\frac{61}{24}$.
15. $\frac{161}{24}$.
16. $\frac{227}{32}$.
17. $\frac{293}{32}$.
18. $\frac{103}{32}$.
19. $\frac{201}{32}$.
20. $\frac{75}{32}$.
21. $\frac{293}{36}$.
22. $\frac{223}{36}$.
23. $\frac{155}{36}$.
24. $\frac{265}{36}$.
25. $\frac{89}{36}$.
26. $\frac{245}{48}$.
27. $\frac{391}{48}$.
28. $\frac{107}{48}$.
29. $\frac{305}{48}$.
30. $\frac{71}{48}$.
31. $\frac{71}{64}$.
32. $\frac{331}{64}$.
33. $\frac{465}{64}$.
34. $\frac{221}{64}$.
35. $\frac{421}{64}$.
36. $\frac{77}{72}$.
37. $\frac{439}{72}$.
38. $\frac{383}{72}$.
39. $\frac{2555}{72}$.
40. $\frac{5225}{72}$.
41. $\frac{7}{8}$; $\frac{24}{8}$; $\frac{31}{8}$.
42. $\frac{5}{16}$; $\frac{80}{16}$; $\frac{85}{16}$.
43. $\frac{7}{32}$; $\frac{96}{32}$; $\frac{103}{32}$
44. 7; 16; 23.

Exercise 39. Page 92

1. $\frac{9}{24}$; $\frac{10}{24}$.
2. $\frac{20}{24}$; $\frac{21}{24}$.
3. $\frac{20}{32}$; $\frac{3}{32}$.
4. $\frac{35}{40}$; $\frac{32}{40}$.
5. $\frac{20}{48}$; $\frac{21}{48}$.
6. $\frac{28}{48}$; $\frac{27}{48}$.
7. $\frac{44}{48}$; $\frac{45}{48}$.
8. $\frac{45}{48}$; $\frac{38}{48}$.
9. $\frac{26}{32}$; $\frac{29}{32}$.
10. $\frac{52}{96}$; $\frac{15}{96}$.
11. $\frac{21}{96}$; $\frac{22}{96}$.
12. $\frac{45}{96}$; $\frac{58}{96}$.
13. $\frac{4}{8}$; $\frac{6}{8}$; $\frac{7}{8}$.
14. $\frac{8}{24}$; $\frac{6}{24}$; $\frac{15}{24}$.
15. $\frac{16}{24}$; $\frac{6}{24}$; $\frac{9}{24}$.
16. $\frac{16}{24}$; $\frac{18}{24}$; $\frac{15}{24}$.
17. $\frac{24}{120}$; $\frac{15}{120}$; $\frac{10}{120}$.
18. $\frac{48}{120}$; $\frac{45}{120}$; $\frac{50}{120}$.
19. $\frac{72}{120}$; $\frac{105}{120}$; $\frac{70}{120}$.
20. $\frac{15}{120}$; $\frac{12}{120}$; $\frac{10}{120}$.
21. $\frac{45}{120}$; $\frac{84}{120}$; $\frac{70}{120}$.
22. $\frac{45}{120}$; $\frac{36}{120}$; $\frac{50}{120}$.
23. $\frac{30}{48}$; $\frac{15}{48}$; $\frac{14}{48}$.
24. $\frac{42}{48}$; $\frac{21}{48}$; $\frac{14}{48}$.
25. $\frac{6}{12}$; $\frac{4}{12}$; $\frac{3}{12}$; $\frac{2}{12}$.
26. $\frac{6}{12}$; $\frac{8}{12}$; $\frac{3}{12}$; $\frac{10}{12}$.
27. $\frac{12}{24}$; $\frac{8}{24}$; $\frac{18}{24}$; $\frac{3}{24}$.
28. $\frac{12}{24}$; $\frac{16}{24}$; $\frac{18}{24}$; $\frac{9}{24}$.
29. $\frac{12}{24}$; $\frac{16}{24}$; $\frac{18}{24}$; $\frac{21}{24}$.
30. $\frac{8}{16}$; $\frac{12}{16}$; $\frac{10}{16}$; $\frac{3}{16}$.
31. $\frac{24}{32}$; $\frac{28}{32}$; $\frac{10}{32}$; $\frac{3}{32}$.
32. $\frac{12}{48}$; $\frac{6}{48}$; $\frac{3}{48}$; $\frac{2}{48}$.
33. $\frac{36}{48}$; $\frac{18}{48}$; $\frac{9}{48}$; $\frac{10}{48}$.
34. $\frac{40}{64}$; $\frac{4}{64}$; $\frac{6}{64}$; $\frac{1}{64}$.
35. $\frac{56}{64}$; $\frac{20}{64}$; $\frac{10}{64}$; $\frac{7}{64}$.
36. $\frac{168}{192}$; $\frac{8}{192}$; $\frac{6}{192}$; $\frac{15}{192}$.

Exercise 40. Page 93

1. $\frac{3}{4}$.
2. $1\frac{1}{4}$.
3. $\frac{5}{8}$.
4. $1\frac{1}{8}$.
5. $1\frac{3}{8}$.
6. $\frac{3}{8}$.
7. $\frac{5}{8}$.
8. $\frac{7}{8}$.
9. $1\frac{1}{8}$.
10. $1\frac{1}{8}$.
11. $1\frac{3}{8}$.
12. $1\frac{5}{8}$.
13. $\frac{5}{16}$.
14. $1\frac{3}{16}$
15. $\frac{11}{16}$.
16. $1\frac{9}{16}$.
17. $\frac{19}{32}$.
18. $\frac{27}{32}$.
19. $\frac{27}{32}$.
20. $1\frac{9}{32}$.
21. $\frac{11}{24}$.
22. $1\frac{5}{24}$.
23. $\frac{2}{3}$.
24. $1\frac{1}{12}$.
25. $\frac{11}{48}$.
26. $\frac{25}{48}$.
27. $\frac{43}{48}$.
28. $1\frac{5}{48}$.
29. $\frac{25}{48}$.
30. $\frac{37}{48}$.
31. $1\frac{2}{7}$.
32. $\frac{31}{48}$.
33. $\frac{47}{48}$.
34. $1\frac{17}{48}$.
35. $\frac{5}{12}$.
36. $1\frac{1}{12}$.
37. $\frac{13}{64}$.
38. $\frac{83}{192}$.
39. $\frac{139}{192}$.
40. $1\frac{59}{64}$.

Exercise 41. Page 94

1. $6\frac{1}{8}$.
2. $10\frac{5}{8}$.
3. $10\frac{7}{8}$.
4. $8\frac{5}{6}$.
5. $11\frac{1}{8}$.
6. $12\frac{7}{12}$.
7. $14\frac{1}{12}$.
8. $15\frac{5}{12}$.
9. $9\frac{11}{24}$.
10. $11\frac{17}{24}$.
11. $18\frac{7}{24}$.
12. $16\frac{18}{24}$.
13. $9\frac{1}{8}$.
14. $10\frac{11}{16}$.
15. $15\frac{25}{48}$.
16. $17\frac{79}{96}$.
17. $70\frac{1}{8}$.
18. $64\frac{2}{15}$.
19. $62\frac{4}{15}$.
20. $81\frac{5}{16}$.
21. $89\frac{19}{24}$.
22. $109\frac{3}{16}$.
23. $92\frac{13}{30}$.
24. $138\frac{13}{16}$.
25. $168\frac{1}{10}$.
26. $142\frac{2}{15}$.
27. $128\frac{2}{3}$.
28. $122\frac{49}{60}$.
29. $113\frac{11}{20}$.
30. $69\frac{49}{60}$.
31. $70\frac{7}{24}$.
32. $154\frac{9}{32}$.
33. $91\frac{7}{24}$.
34. $10\frac{3}{5}$.
35. $12\frac{5}{8}$.
36. $15\frac{1}{16}$.
37. $15\frac{1}{3}$.
38. $13\frac{15}{16}$.
39. $16\frac{3}{5}$.
40. $16\frac{5}{16}$.
41. $12\frac{29}{30}$.
42. $14\frac{27}{32}$.
43. $12\frac{13}{24}$.
44. 14.
45. $15\frac{21}{64}$.
46. $38\frac{5}{12}$.
47. $97\frac{45}{64}$.
48. $59\frac{119}{144}$.

Exercise 42. Page 95

1. $206\frac{7}{8}$ pounds.
2. $98\frac{1}{8}$ yards.
3. $14\frac{5}{6}$ feet.
4. $26\frac{1}{4}$ feet.
5. $12\frac{23}{48}$ inches.
6. $79\frac{1}{8}$ pounds.
7. $57\frac{11}{24}$.
8. $31\frac{11}{16}$ feet.
9. $221\frac{7}{16}$.
10. $5\frac{1}{48}$ inches.
11. $552\frac{5}{24}$ feet.
12. $11\frac{85}{96}$ quarts.

Exercise 43. Page 96

1. $\frac{1}{8}$.
2. $\frac{3}{8}$.
3. $\frac{1}{8}$.
4. $\frac{3}{8}$.
5. $\frac{3}{8}$.
6. $\frac{3}{16}$.
7. $\frac{5}{16}$.
8. $\frac{3}{16}$.
9. $\frac{7}{16}$.
10. $\frac{5}{16}$.
11. $\frac{5}{16}$.
12. $\frac{13}{32}$.
13. $\frac{3}{32}$.
14. $\frac{5}{32}$.
15. $\frac{1}{32}$.
16. $\frac{31}{64}$.
17. $\frac{17}{64}$.
18. $\frac{9}{64}$.
19. $\frac{13}{64}$.
20. $\frac{17}{64}$.
21. $\frac{13}{64}$.
22. $\frac{9}{64}$.
23. $\frac{5}{64}$.
24. $\frac{19}{64}$.
25. $\frac{19}{64}$.
26. $\frac{5}{64}$.
27. $\frac{9}{64}$.
28. $\frac{13}{64}$.
29. $\frac{1}{6}$.
30. $\frac{5}{24}$.
31. $\frac{31}{72}$.
32. $\frac{25}{72}$.
33. $\frac{11}{72}$.
34. $\frac{19}{72}$.
35. $\frac{1}{9}$.
36. $\frac{1}{18}$.
37. $\frac{1}{18}$.
38. $\frac{1}{36}$.
39. $\frac{7}{144}$.
40. $\frac{17}{144}$.
41. $\frac{31}{144}$.
42. $\frac{83}{144}$.
43. $\frac{95}{144}$.
44. $\frac{33}{100}$.
45. $\frac{787}{1000}$.

Exercise 44. Page 97

1. $5\frac{5}{8}$.
2. $6\frac{1}{8}$.
3. $4\frac{1}{8}$.
4. $6\frac{3}{8}$.
5. $2\frac{1}{16}$.
6. $2\frac{1}{16}$.
7. $5\frac{3}{16}$.
8. $2\frac{1}{16}$.
9. $4\frac{1}{16}$.
10. $3\frac{7}{32}$.
11. $3\frac{17}{32}$.
12. $8\frac{7}{8}$.
13. $6\frac{5}{8}$.
14. $7\frac{7}{8}$.
15. $5\frac{7}{8}$.
16. $4\frac{7}{8}$.
17. $14\frac{15}{16}$.
18. $15\frac{15}{16}$.
19. $20\frac{27}{32}$.
20. $22\frac{29}{32}$.
21. $28\frac{31}{32}$.
22. $31\frac{27}{32}$.
23. $4\frac{1}{12}$.
24. $7\frac{11}{12}$.
25. $13\frac{11}{15}$.
26. $15\frac{13}{15}$.
27. $24\frac{1}{4}$.
28. $34\frac{5}{12}$.
29. $37\frac{31}{60}$.
30. $42\frac{23}{60}$.
31. $65\frac{3}{4}$.
32. $61\frac{28}{48}$.
33. $65\frac{31}{32}$.

Exercise 45. Page 98

1. $27\frac{1}{8}$.
2. $57\frac{1}{4}$.
3. $63\frac{1}{10}$.
4. $44\frac{25}{32}$.
5. $61\frac{13}{16}$.
6. $64\frac{19}{24}$.
7. $70\frac{43}{48}$.
8. $44\frac{23}{36}$.
9. $36\frac{55}{64}$.
10. $46\frac{47}{50}$.
11. $\frac{11}{12}$.
12. $\frac{23}{30}$.
13. $\frac{23}{24}$.
14. $\frac{27}{40}$.
15. $\frac{5}{16}$.
16. $\frac{1}{8}$.
17. $\frac{1}{8}$.
18. $2\frac{1}{3}$.
19. $5\frac{5}{6}$.
20. $13\frac{4}{15}$.
21. 20.
22. $\$\frac{1}{4}$.
23. $28\frac{1}{8}$ yards.
24. $154\frac{5}{8}$ acres.
25. $1\frac{1}{6}$ A.M.
26. $125\frac{3}{8}$.
27. $375\frac{17}{32}$.
28. $168\frac{15}{16}$.
29. $29\frac{3}{8}$ yards.
30. $\frac{7}{12}$.
31. $\frac{97}{240}$.
32. $\frac{13}{60}$.
33. $\frac{1}{24}$.
34. $\frac{157}{192}$.
35. $\frac{65}{128}$.
36. $\frac{17}{64}$.
37. $\frac{31}{64}$.
38. $13\frac{25}{48}$.

Exercise 46. Page 100

1. $3\frac{3}{5}$.
2. 6.
3. 10.
4. $13\frac{1}{2}$.
5. $8\frac{1}{4}$.
6. $16\frac{1}{4}$.
7. 28.
8. $6\frac{9}{16}$.
9. $13\frac{1}{8}$.
10. $24\frac{1}{2}$.
11. $67\frac{1}{2}$.
12. 88.
13. 15.
14. $27\frac{1}{2}$.
15. 42.
16. $58\frac{1}{2}$.
17. 77.
18. $112\frac{1}{2}$.
19. 216.
20. $263\frac{1}{2}$.
21. $52\frac{1}{2}$.
22. $81\frac{2}{3}$.
23. $137\frac{1}{2}$.
24. 45.
25. $66\frac{1}{2}$.
26. $115\frac{1}{2}$.
27. 143.
28. $292\frac{1}{2}$.
29. 63.
30. $114\frac{7}{12}$.
31. $\$12\frac{3}{4}$.
32. 110 miles.

Exercise 47. Page 101

1. 12.
2. 18.
3. 14.
4. $28\frac{4}{5}$.
5. 60.
6. $77\frac{1}{2}$.
7. 9.
8. $42\frac{1}{2}$.
9. 84.
10. $56\frac{1}{4}$.
11. 30.
12. $46\frac{2}{3}$.
13. 88.
14. $13\frac{1}{2}$.
15. 25.
16. 42.
17. $56\frac{1}{4}$.
18. $75\frac{5}{8}$.
19. $97\frac{1}{2}$.
20. 225.
21. 15.
22. $23\frac{1}{3}$.
23. $38\frac{1}{2}$.
24. 52.
25. $70\frac{5}{6}$.
26. $15\frac{5}{8}$.
27. $43\frac{3}{4}$.
28. $11\frac{1}{4}$.
29. 14.
30. 45.
31. 45 cents.
32. 84 cents.

Exercise 48. Page 102

1. $\frac{2}{3}$.
2. $\frac{3}{5}$.
3. $\frac{5}{8}$.
4. $\frac{11}{36}$.
5. $\frac{25}{48}$.
6. $\frac{5}{32}$.
7. $\frac{1}{10}$.
8. $\frac{21}{64}$.
9. $\frac{1}{2}$.
10. $\frac{3}{16}$.
11. $\frac{1}{6}$.
12. $\frac{1}{6}$.
13. $\frac{1}{40}$.
14. $\frac{1}{50}$.
15. $\frac{1}{9}$.
16. $\frac{1}{18}$.
17. $\frac{5}{12}$.
18. $\frac{9}{20}$.
19. $\frac{3}{8}$ of a yard.
20. $\frac{5}{12}$ of an acre.
21. $\frac{7}{32}$ of a mile.
22. $\frac{5}{128}$ of an inch.
23. $\frac{3}{5}$.
24. $\frac{2}{7}$.
25. $\frac{4}{5}$.
26. $\frac{1}{4}$.
27. $\frac{5}{16}$.
28. $\frac{2}{9}$.
29. $\frac{13}{24}$.
30. $\frac{3}{8}$.
31. $\frac{7}{27}$.
32. $\frac{1}{4}$.
33. $\frac{11}{15}$.
34. $\frac{1}{4}$.
35. $\frac{2}{5}$.
36. $\frac{1}{4}$.
37. $\frac{1}{3}$.
38. $\frac{9}{160}$.
39. $\frac{9}{40}$.
40. $\frac{105}{512}$.
41. $\frac{7}{32}$ of an inch.
42. $\frac{7}{24}$ of an inch.
43. $\frac{115}{128}$.
44. $\frac{9}{20}$.
45. $\$\frac{1}{2}$; $\$\frac{3}{8}$.
46. $\frac{3}{8}$ of a yard.
47. $\frac{59}{60}$ of a minute.
48. $\$\frac{1}{2}$; $\$\frac{1}{4}$; $\$\frac{3}{8}$.
49. $1\frac{9}{40}$ yards.
50. $\frac{7}{12}$ of a yard.
51. $\frac{3}{8}$ of a yard.
52. $\frac{5}{64}$ of a yard.
53. $\frac{3}{8}$; $\frac{9}{16}$.
54. $\frac{5}{18}$ of a yard; $\frac{5}{8}$ of a yard.
55. $\frac{9}{20}$ of an hour; 27 minutes.
56. $\frac{2}{9}$.
57. $\frac{5}{8}$ of an inch.
58. $\frac{8}{15}$.
59. $\frac{11}{20}$.
60. $\frac{5}{8}$ of a yard.

Exercise 49. Page 105

1. $35\frac{1}{3}$.
2. $78\frac{3}{4}$.
3. $56\frac{1}{4}$.
4. 143.
5. $86\frac{3}{4}$.
6. $76\frac{7}{8}$.
7. $341\frac{1}{4}$.
8. $133\frac{2}{3}$.
9. $76\frac{3}{4}$.
10. $244\frac{7}{12}$.
11. $32\frac{3}{8}$.
12. $51\frac{15}{16}$.
13. $101\frac{3}{4}$.
14. $317\frac{1}{2}$.
15. $471\frac{1}{2}$.
16. $90\frac{3}{16}$.
17. $296\frac{5}{8}$.
18. $775\frac{31}{32}$.
19. $554\frac{1}{4}$.
20. $584\frac{1}{16}$.
21. \$131.
22. $\$148\frac{3}{4}$.
23. \$213.
24. \$222.
25. 1410.
26. 3333.
27. $3661\frac{1}{2}$.
28. 3390.
29. 2680.
30. $4245\frac{1}{2}$.
31. $\$20\frac{1}{4}$.
32. $104\frac{3}{4}$.
33. 60 cents.
34. 1386.
35. $9\frac{5}{8}$.
36. $43\frac{3}{4}$ miles.

Exercise 50. Page 106

1. 285.
2. 322.
3. 378.
4. 651.
5. $193\frac{3}{4}$.
6. $341\frac{1}{4}$.
7. 354.
8. $468\frac{3}{4}$.
9. 360.
10. $533\frac{3}{4}$.
11. 372.
12. $534\frac{5}{8}$.
13. 66.
14. 204.
15. 425.
16. 906.
17. $197\frac{1}{8}$.
18. $258\frac{3}{4}$.
19. $8301\frac{1}{2}$.
20. $4687\frac{1}{8}$.
21. 495.
22. 2660.
23. $1270\frac{1}{2}$.
24. $5432\frac{1}{2}$.
25. 3289.
26. 3573.
27. 3960.
28. 4825.
29. 158,477.
30. 192,054.

Exercise 51. Page 107

1. $8\frac{7}{16}$.
2. $15\frac{5}{6}$.
3. $8\frac{2}{3}$.
4. $43\frac{2}{5}$.
5. $27\frac{5}{8}$.
6. $38\frac{1}{2}$.
7. $49\frac{1}{2}$.
8. $59\frac{1}{2}$.
9. $591\frac{7}{20}$.
10. $83\frac{3}{8}$.
11. $78\frac{1}{4}$.
12. $88\frac{2}{3}$.
13. 250.
14. 250.
15. $152\frac{3}{4}$.
16. $495\frac{1}{32}$.
17. $118\frac{3}{4}$.
18. $270\frac{11}{16}$.
19. $264\frac{17}{56}$.
20. $247\frac{1}{2}$.
21. $2222\frac{2}{9}$.
22. 4020.
23. 2210.
24. $5614\frac{2}{7}$.
25. 9599.
26. $8035\frac{1}{2}$.
27. $8028\frac{3}{8}$.
28. $2345\frac{1}{8}$.
29. $14{,}708\frac{3}{5}$.
30. $17{,}917\frac{9}{10}$.
31. $400.
32. $121\frac{7}{8}$ miles.

Exercise 52. Page 108

1. $133\frac{7}{8}$.
2. $15\frac{3}{4}$.
3. $108; $162.
4. $4\frac{3}{8}; $9\frac{3}{8}$.
5. $258\frac{1}{2}$; 286.
6. $97\frac{1}{6}$; 176.
7. $4\frac{3}{8}$.
8. $16\frac{17}{28}$ pounds.
9. $63\frac{3}{4}$ pounds.
10. $254\frac{1}{2}$ miles.
11. $396\frac{9}{16}$ miles.
12. $1\frac{35}{64}$ inches.
13. $16\frac{5}{16}$ inches.
14. $45.

Exercise 53. Page 109

1. $\frac{5}{48}$.
2. $\frac{3}{32}$.
3. $\frac{3}{64}$.
4. $\frac{1}{32}$.
5. $\frac{3}{32}$.
6. $\frac{5}{96}$.
7. $\frac{3}{80}$.
8. $\frac{3}{128}$.
9. $\frac{1}{72}$.
10. $\frac{1}{144}$.
11. $\frac{1}{150}$.
12. $\frac{1}{150}$.
13. $\frac{1}{128}$.
14. $\frac{1}{320}$.
15. $\frac{1}{144}$.
16. $\frac{1}{168}$.
17. $\frac{27}{17000}$.
18. $\frac{1}{900}$.
19. $\frac{3}{7000}$.
20. $\frac{3}{10000}$.
21. $\frac{11}{10000}$.

Exercise 54. Page 110

1. $\frac{5}{12}$.
2. $\frac{23}{35}$.
3. $\frac{17}{18}$.
4. $1\frac{3}{8}$.
5. $2\frac{1}{6}$.
6. $\frac{1}{8}$.
7. $2\frac{23}{24}$.
8. $4\frac{3}{28}$.
9. $3\frac{5}{8}$.
10. $6\frac{15}{16}$.
11. $4\frac{97}{112}$.
12. $8\frac{11}{16}$.
13. $28\frac{1}{2}$.
14. $16\frac{2}{3}$.
15. $6\frac{2}{3}$.
16. $22\frac{47}{48}$.
17. $17\frac{19}{24}$.
18. $13\frac{13}{16}$.
19. $\$\frac{3}{4}$.
20. $\$1\frac{1}{4}$.
21. $\$1\frac{1}{8}$.
22. $\$1\frac{1}{5}$.
23. $\$4\frac{1}{2}$.

Exercise 55. Page 111

1. 56.
2. 111.
3. 196.
4. 280.
5. 651.
6. 6.
7. 12.
8. 28.
9. 60.
10. $21\frac{7}{8}$.
11. 5.
12. 20.
13. 42.
14. 81.
15. 828.
16. 18.
17. 500.
18. 100.
19. 30.
20. 284.
21. 213.

Exercise 56. Page 112

1. $4\frac{1}{2}$.
2. $6\frac{2}{3}$.
3. $8\frac{3}{4}$.
4. 15.
5. $21\frac{1}{3}$.
6. 8.
7. $9\frac{1}{7}$.
8. $10\frac{4}{5}$.
9. 20.
10. 21.
11. $22\frac{1}{2}$.
12. 20.
13. 20.
14. 35.
15. 60.
16. 80.
17. 64.
18. $22\frac{6}{7}$.
19. $38\frac{2}{5}$.
20. 135.
21. $\frac{8}{9}$.
22. $1\frac{1}{8}$.
23. $\frac{3}{5}$.
24. $1\frac{2}{5}$.
25. $\frac{1}{2}$.
26. $\frac{6}{7}$.
27. $\frac{5}{9}$.
28. $\frac{7}{8}$.
29. $\frac{9}{14}$.
30. $\frac{15}{22}$.
31. $3\frac{1}{9}$.
32. $4\frac{8}{9}$.
33. $10\frac{1}{4}$.
34. $7\frac{1}{6}$.
35. $20\frac{1}{3}$.
36. $13\frac{4}{5}$.
37. 11.
38. $8\frac{3}{14}$.
39. $12\frac{5}{12}$.
40. $18\frac{9}{14}$.

Exercise 57. Page 113

1. $\frac{12}{29}$.
2. $1\frac{11}{19}$.
3. $1\frac{15}{13}$.
4. $1\frac{15}{41}$.
5. $1\frac{1}{7}$.
6. $\frac{8}{11}$.
7. $2\frac{2}{19}$.
8. $7\frac{1}{2}$.
9. $9\frac{3}{10}$.
10. $3\frac{21}{25}$.
11. $4\frac{8}{25}$.
12. $\frac{39}{50}$.
13. $\frac{18}{25}$.
14. $1\frac{1}{3}$.
15. $\frac{3}{10}$.
16. $\frac{9}{80}$.
17. $\frac{3}{10}$.
18. $\frac{1}{14}$.
19. $\frac{5}{62}$.
20. $\frac{9}{320}$.
21. $\frac{3}{160}$.
22. 2.
23. $3\frac{3}{4}$.
24. $1\frac{7}{8}$.
25. $1\frac{1}{8}$.
26. $\frac{1}{2}$.
27. 6.
28. \$11$\frac{1}{2}$.
29. 18.
30. \$33; \$2$\frac{3}{4}$.
31. \$69; \$5$\frac{3}{4}$.

Exercise 58. Page 114

1. \$28.
2. \$72.
3. 108.
4. $21\frac{1}{2}$.
5. $304\frac{1}{4}$.
6. \$56.
7. \$96.
8. 45.
9. $42\frac{3}{4}$.
10. \$60.
11. \$100.
12. 112 feet.
13. 160 feet.
14. \$984.
15. \$1144.

Exercise 59. Page 115

1. $\frac{1}{12}$.
2. $\frac{1}{20}$.
3. $\frac{1}{24}$.
4. $\frac{3}{32}$.
5. $\frac{1}{48}$.
6. $\frac{1}{96}$.
7. $13\frac{1}{3}$.
8. 20.
9. 27.
10. 48.
11. $14\frac{6}{7}$.
12. 96.
13. $\frac{8}{9}$.
14. $1\frac{3}{7}$.
15. $\frac{5}{14}$.
16. $\frac{7}{20}$.
17. $\frac{5}{12}$.
18. $4\frac{1}{5}$.
19. $\frac{1}{3}$.
20. $\frac{1}{3}$.
21. $\frac{2}{3}$.
22. 2.
23. $\frac{1}{6}$.
24. $\frac{1}{6}$.

Exercise 60. Page 116

1. $\frac{1}{3}$.
2. $\frac{1}{3}$.
3. $\frac{3}{7}$.
4. $\frac{1}{3}$.
5. $\frac{1}{5}$.
6. $\frac{1}{3}$.
7. 2.
8. 4.
9. $\frac{1}{5}$.
10. $\frac{1}{8}$.
11. $\frac{1}{16}$.
12. $\frac{1}{32}$.
13. $1\frac{1}{9}$.
14. $1\frac{1}{5}$.
15. $2\frac{4}{5}$.
16. $2\frac{2}{9}$.
17. $2\frac{4}{5}$.
18. $2\frac{7}{10}$.
19. $\frac{2}{3}$.
20. $\frac{34}{43}$.
21. 2.
22. $\frac{1}{2}$.
23. $\frac{1}{3}$.
24. $\frac{2}{3}$.
25. $\frac{1}{10}$.
26. $\frac{1}{5}$.
27. $\frac{1}{2}$.
28. $\frac{1}{4}$.
29. $\frac{3}{4}$.
30. $\frac{1}{8}$.
31. $\frac{1}{3}$.
32. $\frac{2}{3}$.
33. $\frac{1}{16}$.
34. $\frac{3}{8}$.
35. $\frac{5}{8}$.
36. $\frac{7}{8}$.
37. $\frac{1}{5}$.

Exercise 61. Page 117

1. $\$61\frac{1}{4}$.
2. $\$121\frac{7}{8}$.
3. $\$1.06\frac{1}{4}$.
4. $\$18\frac{7}{16}$.
5. $84\frac{3}{8}$ cents.
6. $\$2\frac{7}{8}$.
7. $\$1.93\frac{3}{4}$.
8. $\$1.12\frac{1}{2}$.
9. $\$6.56\frac{1}{4}$.
10. 7.
11. $9\frac{3}{5}$.
12. $8\frac{1}{2}$ miles.
13. 31.
14. $\$3\frac{29}{32}$.
15. $19\frac{1}{4}$.
16. 3 miles.
17. $\frac{41}{1050}$.
18. Greatest, $\frac{7}{45}$; least, $\frac{3}{20}$.
19. $17\frac{7}{8}$ inches.
20. 26.
21. $4\frac{1}{8}$ pounds.
22. $\frac{1}{72}$ of a mile.
23. 10.
24. $3\frac{5}{8}$ inches.
25. $\frac{1}{32}$ of an inch.
26. 5.
27. $\frac{9}{64}$.
28. 32.
29. $\frac{15}{16}$.
30. $\frac{3}{4}$.
31. 800.
32. $35.
33. 36.
34. $22\frac{1}{4}$ miles.
35. $\frac{15}{16}$.
36. 15.
37. 10.
38. $10\frac{1}{2}$.
39. $\frac{5}{32}$.
40. $\frac{9}{16}$.
41. $\frac{1}{300000}$ of an inch.
42. $\frac{7}{10}$.
43. $\frac{2}{3}$.
44. $94\frac{1}{2}$; $\$9\frac{9}{20}$.
45. $\$20\frac{5}{32}$.
46. 35.
47. $71\frac{1}{2}$.
48. 75.
49. $32\frac{1}{7}$.
50. $\$6.63\frac{3}{4}$.
51. $\$5\frac{3}{5}$.
52. 16.
53. $75.
54. 52.
55. $\frac{13}{16}$ of a mile.
56. 18 miles.
57. Greater; $\frac{7}{30}$.
58. Less; $\frac{7}{18}$.
59. Neither.
60. $3\frac{1}{8}$; $6\frac{3}{8}$.
61. $1\frac{23}{25}$ hours; $\frac{6}{17}$ of an hour.
62. 1st, $42; 2d, $21.
63. $27.50.
64. 25 weeks.
65. $484\frac{3}{8}$.
66. 20.
67. 11.
68. $7500.
69. $51.48.
70. 204 pounds.
71. 300 pounds.
72. 50.
73. $\$19\frac{3}{10}$.
74. $36\frac{1}{2}$ miles.
75. 1st, $4100; 2d, $2460.
76. 9 hours.
77. $576; $1080; $540.
78. $47\frac{1}{12}$; $\frac{125}{565}$.
79. $487\frac{1}{2}$ pounds.
80. $2\frac{87}{78}$; $\frac{78}{193}$.
81. 6.
82. Elder, $156\frac{1}{4}$; younger, $93\frac{3}{4}$.
83. 110.
84. $\frac{4}{5}$; $\frac{1}{5}$.
85. $\frac{1}{4}$; $\$1\frac{4}{5}$.
86. $\frac{883}{1000}$.
87. 21.
88. $4\frac{1}{2}$ ounces.
89. $89\frac{1}{2}$ cents.
90. $\frac{39}{85}$; $\frac{46}{85}$.
91. 720.
92. $1.62.
93. $\$41\frac{1}{4}$; $\$57\frac{3}{4}$.
94. 18.
95. 28.

Exercise 64. Page 129

16. 0.4.
17. 0.5.
18. 0.7.
19. 0.62.
20. 0.75.
21. 0.232.
22. 0.348.
23. 0.029.
24. 0.037.
25. 0.006.

26. 0.7231. **33.** 0.300. **40.** 4.9. **47.** 3.007. **54.** 6.2000.
27. 0.4182. **34.** 0.3000. **41.** 1.23. **48.** 4.040. **55.** 9.
28. 0.0328. **35.** 0.30000. **42.** 2.25. **49.** 4.500. **56.** $2.75.
29. 0.0025. **36.** 1.2. **43.** 6.25. **50.** 8. **57.** $71.02.
30. 0.0007. **37.** 1.5. **44.** 4.50. **51.** 7.2255. **58.** $100.08.
31. 0.3. **38.** 2.7. **45.** 4. **52.** 2.8005. **59.** $1003.03.
32. 0.30. **39.** 3.8. **46.** 6.225. **53.** 7.8050.

Exercise 65. Page 131

1. 0.20 ; 0.17.
2. 0.80 ; 0.08.
3. 0.700 ; 0.002.
4. 0.90 ; 0.13.
5. 0.600 ; 0.123.
6. 0.5 ; 0.7.
7. 0.40 ; 0.05.
8. 0.100 ; 0.357.
9. 0.120 ; 0.103.
10. 0.720 ; 0.008.
11. 0.69 ; 0.07.
12. 0.3 ; 0.7.
13. 0.990 ; 0.999.
14. 0.75 ; 0.63.
15. 0.5 ; 0.4.
16. 0.850 ; 0.392.
17. 0.001 ; 0.110.
18. 0.2070 ; 0.2376.
19. 0.412 ; 0.932.
20. 0.6810 ; 0.6881.
21. 0.9 ; 0.7.
22. 0.62 ; 0.57.
23. 0.090 ; 0.655.
24. 0.5840 ; 0.5834.

Exercise 66. Page 132

1. $\frac{1}{5}$. **10.** $\frac{2}{3}$. **19.** $\frac{1}{6}$. **28.** $\frac{1}{3}$. **37.** $187\frac{1}{2}$. **46.** $2\frac{1}{4}$.
2. $\frac{2}{5}$. **11.** $\frac{1}{8}$. **20.** $\frac{2}{3}$. **29.** $\frac{2}{3}$. **38.** $3\frac{3}{8}$. **47.** $3\frac{1}{5}$.
3. $\frac{3}{5}$. **12.** $\frac{5}{8}$. **21.** $\frac{1}{80}$. **30.** $\frac{1}{6}$. **39.** $3\frac{3}{8}$. **48.** $5\frac{3}{4}$.
4. $\frac{4}{5}$. **13.** $\frac{1}{4}$. **22.** $\frac{1}{40}$. **31.** $1\frac{1}{4}$. **40.** $3\frac{3}{8}$. **49.** $2\frac{2}{5}$.
5. $\frac{1}{10}$. **14.** $\frac{1}{2}$. **23.** $\frac{3}{80}$. **32.** $2\frac{1}{2}$. **41.** $\frac{12}{16}$. **50.** $6\frac{4}{5}$.
6. $\frac{3}{20}$. **15.** $\frac{3}{8}$. **24.** $\frac{1}{20}$. **33.** $3\frac{3}{4}$. **42.** $\frac{12}{16}$. **51.** $4\frac{13}{20}$.
7. $\frac{1}{4}$. **16.** $\frac{147}{200}$. **25.** $\frac{1}{16}$. **34.** $6\frac{1}{4}$. **43.** $\frac{4}{16}$. **52.** $5\frac{7}{20}$.
8. $\frac{1}{8}$. **17.** $\frac{7}{8}$. **26.** $\frac{3}{4}$. **35.** $1\frac{7}{8}$. **44.** $\frac{8}{16}$.
9. $\frac{1}{6}$. **18.** $\frac{1}{3}$. **27.** $\frac{3}{16}$. **36.** $18\frac{3}{4}$. **45.** $1\frac{1}{2}$.

Exercise 67. Page 133

1. 0.5.
2. 0.25.
3. 0.125.
4. 0.75.
5. 0.375.
6. 0.625.
7. 0.875.
8. 0.2.
9. 0.4.
10. 0.6.
11. 0.8.
12. 0.0625.
13. 0.1875.
14. 0.3125.
15. 0.4375.
16. 0.9375.
17. 0.03125.
18. 0.09375.
19. $0.333\frac{1}{3}$.
20. $0.666\frac{2}{3}$.
21. $0.166\frac{2}{3}$.
22. $0.833\frac{1}{3}$.
23. 0.12.
24. 0.28.
25. 1.36.
26. $3.053\frac{1}{3}$.
27. 0.1375.
28. 6.2875.
29. 1.38.
30. 6.125.

Exercise 68. Page 135

1. 700¢.
2. 2900¢.
3. 3500¢.
4. 17,500¢.
5. 240¢.
6. 360¢.
7. 970¢.
8. 1280¢.
9. 345¢.
10. 965¢.
11. 805¢.
12. 2963¢.
13. 14,175¢.
14. 24,385¢.
15. 69,205¢.
16. 90,009¢.
17. $3.
18. $4.50.
19. $5.75.
20. $6.06.
21. $7.32.
22. $5.22½.
23. $6.33⅓.
24. $1.66⅔.
25. $26.42.
26. $49.
27. $72.75.
28. $96.54.
29. $0.02.
30. $0.05.
31. $0.55.
32. $0.75.
33. 27,630 m.
34. $5.24.
35. $276; 27,600¢

Exercise 69. Page 136

1. 59.5.
2. 80.73.
3. 85.46.
4. 30.82¼.
5. 28.12.
6. 214.47.
7. 271.28.
8. 637.59.
9. 167.88¾.
10. 151.75⅕.
11. 57.92.
12. 194.04.
13. 99.986.
14. 52.9.
15. $368.
16. $265.
17. $205.
18. $200.
19. 284.74 miles.
20. 251.14 miles.
21. 535.88 miles.
22. 498.64 miles.
23. 1034.52 miles.
24. 222.61.
25. 207.57⅔.
26. Yes.

Exercise 70. Page 138

1. 22.79.
2. 35.92.
3. 53.06.
4. 58.69.
5. 67.13.
6. 20.48.
7. 28.47.
8. 44.62.
9. 14.59.
10. 34.05.
11. 18.67.
12. 24.518.
13. 18.975.
14. 65.727.
15. 24.757.
16. 67.115.
17. 11.933.
18. 39.2235.
19. 7.175.
20. 0.53074.
21. 6.7302.
22. 2.7291.
23. 4.9727.
24. 4.86⅔.
25. 6.73⅓.
26. 5.37½.
27. 162.75 acres.
28. $0.1335.
29. 318.91⅔ barrels.
30. 3.0997 grains.
31. 46.25 feet.
32. 267.96⅔ acres.
33. 9.16 feet.
34. 124.7 miles.
35. 510.8 miles.
36. 67.5 miles.
37. 177.8 miles.

Exercise 71. Page 140

1. 13.4.
2. 39.2.
3. 6.3.
4. 4.8.
5. 16.25.
6. 19.25.
7. 20.
8. 60.
9. 17.
10. 25.5.
11. 35.175.
12. 56.072.
13. 31.425.
14. 10.
15. 10.
16. 40.
17. 33.
18. 32.643.
19. 42.624.
20. 42.9296.
21. 18.5.
22. 0.3.
23. 1.902.
24. 0.2222.
25. 60.
26. 10.
27. 3.125.
28. 4.
29. 210.
30. 100.2001.

31. 1001.1001.
32. 127.525.
33. 702.6975.
34. 49.9125.
35. 83.754.
36. 130.7575.
37. 115.635.
38. 318.32612.
39. 111.111.
40. 57.75 inches; 49 inches.
41. 24.5 inches; 21.245 inches.
42. $5; $9.
43. 472.44; 984.25; 1456.69.
44. 465.9 grains.
45. $24.125.
46. 3.2515.
47. $131.3955.
48. 331.6 miles.
49. 158 miles.
50. 27.6.
51. 468.8 feet.
52. 12.24 acres.
53. $8171.625.

Exercise 72. Page 142

1. 684.
2. 1884.
3. 4680.
4. 496.
5. 4960.
6. 2843.4.
7. 253.6.
8. 3128.4.
9. 31,284.
10. 312,840.
11. 50.
12. 100.
13. 10.
14. 5584.25.
15. 5,394,400.
16. 157,480.
17. $11,325.
18. 156,250 pounds.
19. 70 inches.
20. 125.664 feet.

Exercise 73. Page 143

1. 0.49.
2. 0.81.
3. 1.36.
4. 2.34.
5. 3.75.
6. 5.36.
7. 9.43.
8. 18.24.
9. 83.52.
10. 81.34.
11. 0.1953.
12. 0.5904.
13. 0.2992.
14. 0.8385.
15. 1.5408.
16. 13.3774.
17. 42.9359.
18. 67.47.
19. 37.7798.
20. 99.8001.
21. 0.09906.
22. 0.105.
23. 0.12.
24. 0.46.
25. 2.6256.
26. 21.7761.
27. 81.68325.
28. 0.53801.
29. 0.007416.
30. 23.938992.
31. 57.5 pounds.
32. 15 pounds.
33. 173.75 pounds.
34. 559.375 pounds.
35. 131.9472 inches.
36. 25,132.8 miles.
37. 2,722,196.4 miles.
38. $9.8904.
39. $12.0897.
40. $10.3385.

Exercise 74. Page 145

1. 2.26.
2. 0.62.
3. 0.45.
4. 4.5.
5. 0.045.
6. 0.2937.
7. 1.089.
8. 3.684.
9. 5.873.
10. 6.324.
11. 0.7059.
12. 465.2.
13. 0.07564.
14. 0.09852.
15. 0.00479.
16. $0.23.
17. 41.045 miles.
18. 31.5 miles.
19. 62.5.

Exercise 75. Page 147

1. 2.8.
2. 12.5.
3. 7.2.
4. 7.3.
5. 7.8.
6. 12.1.
7. 0.0422.
8. 0.241.
9. 0.234.
10. 0.0017.
11. 136.
12. 178.
13. 91.
14. 143.
15. 460.
16. 520.
17. 0.17.
18. 0.24.
19. 0.03.
20. 280.
21. 11.
22. 1.85.
23. 76.
24. 340.
25. 0.29.
26. 0.207.
27. 0.333.
28. 0.66.
29. 0.44.
30. 0.29.
31. 20.
32. 0.03125.
33. 2.03.
34. 146.61 feet.
35. 20.
36. 24,500.
37. 80,000.
38. 174.
39. 2200 bushels.
40. 32.
41. 164.
42. $7.20.
43. 720+.
44. 2.78.
45. 1.92.
46. 1.20 ounces.
47. 275.

Exercise 76. Page 149

1. $680; $746.67.
2. $26; $29.60.
3. $121.60.
4. $45.
5. $3.60.
6. $35.52.
7. $83.09.
8. $15,000.
9. 586.4 feet.
10. 31.3 feet.
11. $1497.
12. 1.095.
13. $93\frac{1}{3}$.
14. 0.913.
15. 637.5625.
16. 885.0631.
17. 72.45.
18. 0.05.
19. $50.74.
20. 212.25.

Exercise 78. Page 154

1. 419.
2. 510.
3. 510.
4. 543.
5. 555.
6. 475.
7. 433.
8. 488.
9. 443.
10. 3548.
11. 4621.
12. 5328.
13. $422.49.
14. $235.49.
15. $379.85.
16. $819.82.
17. $3589.76.
18. $4001.

Exercise 79. Page 155

1. 4237.
2. 1704.
3. 3703.
4. 5224.
5. 4674.
6. 920.
7. 23,508.
8. 28,571.
9. 22,208.
10. 49,922.
11. 43,428.
12. 52,400.
13. $24.38.
14. $171.36.
15. $224.45.
16. $271.50.
17. $669.80.
18. $594.
19. $174.35.
20. $267.50.
21. $518.25.
22. $344.75.
23. $337.75.
24. $144.25.

Exercise 80. Page 157

1. $2.50.
2. $82,510.
3. $62.50.
4. $55.
5. $37.
6. $287.50.
7. $250.
8. $6375.
9. $3,211,200.
10. $37,610.
11. $43,077.50.
12. $3600.
13. $16,000.
14. $8125.
15. $16,875.
16. $637.50.

17. $1881.25.
18. $80,000.
19. $61,000.
20. $80,000.
21. $800,000.
22. $350,000.
23. 1200.
24. 1600.
25. 4800.
26. $10,410.
27. 210,500.
28. $10,605.
29. 238,324.
30. $1,053,250.
31. $10,560,000.
32. $105,600.
33. 80,000,000.
34. $30.
35. $2900.
36. $2700.
37. $62.50.
38. $310.
39. $4.375.
40. $12.
41. $1200.
42. $71,320.
43. $900; $1200.
44. 300 pounds; 600 pounds
45. $7687.50; $10,250.
46. $4200; $5600; $2100.
47. 1800 pounds.
48. $400; $200.
49. $1600; 1733\frac{1}{3}$.
50. $600; $800; 866\frac{2}{3}$.
51. The results are equal.
52. The results are equal.

Exercise 81. Page 160

1. 51,300.
2. 61,200.
3. 70,200.
4. $111.
5. $240.
6. 1,846,200.
7. $186.
8. $324.
9. 1827.
10. 178.7.
11. 134.
12. 1.34.
13. 0.134.
14. $176.
15. $43.
16. $97.
17. 5163.
18. 15,900.
19. 66,600.
20. $276.
21. $246.
22. $355.
23. $460.
24. $721.
25. $518.
26. $46.
27. $112.
28. $99.
29. $248.
30. $328.
31. $93.
32. $29.
33. 651.
34. 224,400.
35. 192,400.
36. 19,240.
37. 1924.
38. 3216.
39. 669.
40. 3976.
41. 147.6.
42. 12.4.
43. 12.4.
44. 1.24.
45. 180.
46. 18.
47. 1.8.
48. 0.02.
49. $3600.
50. $4125.
51. $5400.
52. $11,000.
53. $72.
54. 171\frac{7}{8}$.
55. $363.
56. $273.
57. $234.

Exercise 82. Page 161

1. $6521.04.
2. $7883.37.
3. $7512.67.
4. $5432.02.
5. $12,743.28.
6. $42,218.55.
7. $32,092.06.
8. $45,284.82.
9. $43,706.25.
10. $62,734.28.
11. $27,657.63.
12. $325,399.50.

Exercise 83. Page 162

1. $746.30.
2. $28.96.
3. $4.96.
4. $443.10.
5. $624.47.
6. 34,446.4.
7. 365.
8. 493.
9. 250.6.
10. 502.
11. $45.75.
12. $98.25.
13. 496.
14. 640.
15. 720.
16. 405.
17. 495.
18. 39.
19. 3450.
20. 5030.
21. 144.
22. 14; 11.
23. 675.
24. 75.
25. 7.25 hours.
26. 6$\frac{2}{3}$ hours.
27. 1448.

Exercise 84. Page 164

1. $54.
2. $1375.
3. $1400.
4. $46,875.
5. $742.50.
6. $108.
7. $156.25.
8. 1760.
9. $2250.
10. 2700.
11. 4000.
12. 63.
13. 87.
14. $800.
15. $13,000.

Exercise 85. Page 166

1. 655 in.
2. 873 in.
3. 84 ft. ; 1008 in.
4. 16½ ft. ; 198 in.
5. 143 yd. ; 429 ft.
6. 1760 yd. ; 5280 ft.
7. 41 mi. 4224 ft.
8. 25 mi. 3960 ft.
9. 126 mi. 3696 ft.
10. 880 yd.
11. 1600 rd.
12. 12,320 yd.
13. 7200 rd.
14. 17,160 ft.
15. 2640 ft.
16. 3168 ft.
17. 3432 ft.
18. 17.6 yd.
19. 552.96 mi.

Exercise 86. Page 167

1. 2 yd. 2 ft.
2. 2 yd. 2 ft. 4 in.
3. 3 yd. 7 in.
4. 1 yd. 2 ft.
5. 3 yd.
6. 1 rd. 2 yd. 2 ft. 6 in.
7. 2 rd.
8. 2 rd. 4 yd.
9. 6 rd. 2 yd.
10. 2 mi. 60 rd.
11. 3 mi.
12. 4 mi.
13. 12 ft. 6 in.
14. 13 ft. 9 in.
15. 30 ft.
16. 33 ft. 4 in.
17. 50 ft.
18. 60 ft.
19. 80 ft.
20. 81 ft.
21. 20 yd.
22. 21 yd. 2 ft.
23. 166 yd. 2 ft.
24. 250 yd.
25. 2 yd.
26. 2 yd. 1 ft.
27. 1 yd. 14 in.
28. 2 yd. 8 in.

Exercise 87. Page 168

1. 1872 sq. in.
2. 4032 sq. in.
3. 8640 sq. in.
4. 387 sq. ft.
5. 558 sq. ft.
6. 3751 sq. yd.
7. 960 sq. rd.
8. 2400 sq. rd.
9. 5600 sq. rd.
10. 3200 A.
11. 16,000 A.
12. 96,000 A.
13. 10,836 sq. in.
14. 2650 sq. in.
15. 476 sq. ft.
16. 1 sq. rd. 21 sq. yd. 8 sq. ft. 20 sq. in.
17. 1 A. 16 sq. rd.
18. 4 A. 102 sq. rd. 25 sq. yd. 4 sq. ft. 72 sq. in.

Exercise 88. Page 169

1. 3456 cu. in.
2. 43,200 cu. in.
3. 129,600 cu. in.
4. 216,000 cu. in.
5. 57,600 cu. in.
6. 115,200 cu. in.
7. 675 cu. ft.
8. 972 cu. ft.
9. 3375 cu. ft.
10. 3200 cu. ft.
11. 16,000 cu. ft.
12. 64,000 cu. ft.
13. 2 cu. ft.
14. 5 cu. ft.
15. 91 cu. yd.
16. 181 cu. yd. 13 cu. ft.
17. 1 cu. ft. 272 cu. in.
18. 2 cu. ft. 804 cu. in.
19. 43,265 cu. in.
20. 717,984 cu. in.
21. 10 cd. 10 cu. ft.; 47 cu. yd. 21 cu. ft.

Exercise 89. Page 171

1. $1\frac{3}{40}$ T.
2. 0.925.
3. 322,560.
4. $1\frac{81}{144}$.
5. 256 oz.
6. 432 oz.
7. 40 oz.
8. 132 oz.
9. 32,000 lb.
10. 46,000 lb.
11. 57,000 lb.
12. 32,500 lb.
13. 11 lb.
14. 12 lb. 8 oz.
15. 1 T.
16. 5 T.
17. 476 oz.
18. 906 oz.
19. 867 oz.
20. 437 oz.
21. 54,000 lb.
22. 5400 lb.
23. 9 lb.
24. 6700 lb.

Exercise 90. Page 172

1. 108 gi.
2. 184 gi.
3. 138 pt.
4. 174 pt.
5. 356 qt.
6. 272 qt.
7. 257 qt.
8. 125 pt.
9. 3 gal.
10. 12 gal.
11. 1 gal. 3 gi.
12. 10 gal. 2 qt. 1 pt.
13. 75 gal.
14. 59 gal. 1 qt. 1 pt.
15. 23 gal. 1 pt.
16. 9 gal. 1 qt. 1 pt. 2 gi.
17. 99 pt.
18. 223 pt.
19. 253 pt.
20. 636 pt.
21. 99 qt.
22. 149 qt.
23. 3707 qt.
24. 291 qt.
25. 1024 oz.
26. 9504 cu. in.
27. 156 oz.
28. 1080 sq. in.
29. 192,000 lb.
30. $196\frac{5}{8}$ sq. yd.
31. 97,500 lb.
32. 400 sq. rd.
33. 33 lb.
34. 39 lb.
35. 23 lb. 7 oz.
36. 28 lb. 2 oz.
37. 1 T. 187 lb. 8 oz.
38. 1 T. 11 lb.
39. 6 T. 500 lb.
40. 37 T. 1550 lb.
41. 12 oz.
42. $3\frac{1}{2}$ oz.
43. 1158 oz.
44. $466\frac{1}{2}$ oz.
45. 1550 oz.
46. 476 oz.
47. 35,200 oz.
48. 157 oz.
49. 85,500 lb.
50. 131,600 lb.
51. 7200 lb.
52. $6666\frac{2}{3}$ lb.
53. 27 lb.
54. 302 lb.
55. 4800 lb.
56. 10,650 lb.
57. 59,360 lb.
58. 175,000 lb.
59. 1032 lb.
60. 4 ft.
61. 480.
62. $12\frac{3}{8}$.
63. $16\frac{13}{32}$.
64. $42.19.
65. $65.63.
66. $5\frac{1}{4}$.
67. $7\frac{13}{16}$.
68. $16.41.

Exercise 91. Page 175

1. 74 pt.
2. 98 pt.
3. 198 pt.
4. 512 pt.
5. 33 pt.
6. 75 pt.
7. 175 pt.
8. 497 pt.
9. 384 qt.
10. 472 qt.
11. 538 qt.
12. 2150 qt.
13. 188 pk.
14. 226 pk.
15. 371 pk.
16. 1709 pk.
17. 1 bu. 4 qt.
18. 1 bu. 6 qt.
19. 1 bu. 2 pk. 1 qt.
20. 2 bu.
21. 1 bu. 3 pk. 4 qt.
22. 3 bu. 2 qt.
23. 24 bu.
24. 22 bu. 1 pk.
25. 18 bu. 3 pk.
26. 2 bu. 1 pk. 6 qt.
27. 4 bu. 3 pk. 4 qt.
28. 39 bu.
29. 144 pt.
30. 171 pt.
31. 137 pt.
32. 784 pt.
33. 856 pt.
34. 1212 pt.
35. 128 pt.
36. 2240 pt.
37. 1824 pt.
38. 38 pt.
39. 94 pt.
40. 144 pt.
41. 304 qt.
42. 388 qt.
43. 478 qt.
44. 625 qt.
45. 2400 qt.
46. 2800 qt.
47. 88 qt.
48. 296 qt.
49. 288 pk.
50. 358 pk.
51. 18 pk.
52. 36 pk.
53. 84 bu.
54. 129 bu.
55. 193 bu.
56. 239 bu.
57. 11 bu.
58. 61 bu.
59. 31 bu.
60. 99 bu.
61. 300.
62. 720.
63. 43,200 lb.
64. 5.
65. 22,400 lb.

Exercise 92. Page 177

1. 2376 in.
2. 2178 in.
3. 5250 l.
4. 1700 rd.
5. 2300 rd.
6. 2160 ch.
7. 1320 ch.
8. 1900 ch.
9. 3370 ch.
10. 1350 ch.
11. 464 sq. rd.
12. 60 sq. rd.
13. 94 sq. rd.
14. 920 sq. ch.
15. 7500 sq. ch.
16. 23,040 A.
17. 48,000 A.
18. 80,000 A.
19. 403,200 A.
20. 614,400 A.
21. 7 ch.
22. 175 ch.
23. 244 ch.
24. 206 ch.
25. 166 ch.
26. 1 ch.
27. 2080 ch.
28. 2960 ch.
29. 7 A.
30. 2 A.
31. 9.6 A.
32. 1120 A.
33. 3.4 A.
34. 7.8 A.
35. 97.5 A.
36. 4.5 A.
37. 640 A.; 160 A.

Exercise 93. Page 179

1. 21,600 sec.
2. 1080 min.
3. 2160 min.
4. 384 hr.
5. 864 hr.
6. 1164 hr.
7. 364 da.
8. 182 da.
9. 325 da.
10. 96 mo.
11. 192 mo.
12. 384 mo.
13. 6 min.
14. 12 hr.
15. 3 wk.
16. 1 wk.
17. 11 yr.
18. $\frac{1}{2}$ yr.
19. 0.25 of a century.
20. 52 wk. 1 da.
21. 92.
22. 153.
23. 18,262$\frac{1}{2}$.
24. 500.
25. 1928; 1940; 1960; 1972; 2000.
26. Sunday.
27. 55,937 sec.
28. 3329 min.
29. 87,963 min.
30. 696 hr.; 41,760 min.
31. 4 wk. 1 da. 12 hr. 44 min. 3 sec.
32. $71.50.

Exercise 94. Page 181

1. 1020″.
2. 1920″.
3. 2880″.
4. 3360″.
5. 10,800″.
6. 97,200″.
7. 11,100″.
8. 14,880″.
9. 29,108″.
10. 32,762″.
11. 8′.
12. 15′.
13. 1620′.
14. 549′.
15. 1676′.
16. 1° 20′.
17. $\frac{1}{2}$°.
18. 45°.
19. 1800°.

Exercise 95. Page 182

1. 24 ft. 3 in.
2. 84 ft. 1 in.
3. 250 sq. ft. 71 sq. in.
4. 178 sq. ft. 40 sq. in.
5. 258 cu. ft. 626 cu. in.
6. 176 cu. ft. 820 cu. in.
7. 1431 lb. 11 oz.
8. 1642 lb. 3 oz.
9. 139 gal. 3 qt.
10. 209 gal. 3 qt.
11. 166 qt.
12. 241 gal. 5 pt.
13. 237 bu. 3 pk.
14. 94 pk. 1 qt.
15. 139 ft.
16. 284 ft. 6 in.
17. 86 wk. 2 da.
18. 21 hr. 38 sec.
19. 25° 16″.
20. 104 hr. 12 min. 57 sec.
21. 130° 58′ 47″.
22. 10 A.
23. 19 sq. rd.
24. 18 cu. yd. 1 cu. ft.
25. 30 lb. 13 oz.

Exercise 96. Page 184

1. 12 ft. 9 in.
2. 17 ft. 10$\frac{1}{4}$ in.
3. 48 sq. ft. 130 sq. in.
4. 20 sq. ft. 48$\frac{3}{4}$ sq. in.
5. 1 yd. 1 ft.
6. 9 bu. 7 qt.
7. 15 A. 122 sq. rd.
8. 7 hr. 15 min.
9. 47 cu. ft. 1453 cu. in.
10. 74 cu. ft. 797 cu. in.
11. 74 lb. 8 oz.
12. 103 lb. 6 oz.
13. 12 gal. 3 qt.
14. 34 qt. 1 pt.
15. 15 bu. 3 pk.
16. 56 min. 38 sec.

Exercise 97. Page 185

1. 262 ft. 6 in.
2. 748 ft. 9 in.
3. 1069 ft. 9 in.
4. 603 qt. 1 pt.
5. 1568 gal. 3 qt.
6. 3007 bu.
7. 191 sq. ft. 132 sq. in.
8. 57 cu. ft. 1116 cu. in.
9. 216 cu. ft. 1600 cu. in.
10. 988 lb. 12 oz.
11. 774 min. 40 sec.
12. 1110 min.
13. 31 ft.
14. 472 gal. 2 qt.
15. 21 lb.
16. 4 hr. 30 min.
17. 3000 lb.
18. 2191 da. 10 hr. 52 min. 36 sec.
19. 109 ft. 8 in.

Exercise 98. Page 186

1. 2 ft. 5 in.
2. 21 ft. 11 in.
3. 3 lb. 2 oz.
4. 4 yd. 33 in.
5. 3 mi. 4864 ft.
6. 24 min. 21 sec.
7. 7.
8. 9.
9. 4.
10. 11.
11. 9.
12. 20.
13. 8.
14. 32.
15. 1 min. 13 sec.
16. 8; 242.
17. 1$\frac{4}{5}$.
18. 72.
19. 8.
20. 35.
21. 6.
22. 7.
23. 7$\frac{1}{2}$.
24. 144.
25. 112.
26. 17$\frac{1}{2}$.

Exercise 99. Page 188

1. 392 in.
2. 23 ft. 6 in.; $23\frac{1}{2}$ ft.
3. $2.75.
4. 1200.
5. 7.
6. 3.
7. 447.
8. $34\frac{18}{23}$.
9. 3963 mi. 1562 ft.
10. $21\frac{9}{16}$.
11. 30 in.; 2.5 ft.; $\frac{5}{6}$ yd.; $\frac{5}{33}$ rd.
12. 56 in.; 4 ft. 8 in.; $4\frac{2}{3}$ ft.; $4.66\frac{2}{3}$ ft.
13. 62.5 lb.; 62 lb. 8 oz.; $62\frac{1}{2}$ lb.
14. $16{,}008\frac{1}{3}$.
15. $129\frac{1}{31}$.
16. 84 ft.; $27\frac{1}{2}$ fathoms.
17. 5 hr. 52 min.
18. 1 mi. 4764 ft.
19. 25,875 T.
20. 1.85 min.
21. 1.42 min.
22. 6 min. 47 sec.
23. 6 min. 48 sec.

Exercise 101. Page 193

1. 20.
2. 7840.8 T.
3. 960.
4. 12,243.75 lb.
5. 12.5.
6. $11\frac{1}{5}$ T.
7. $216\frac{7}{48}$ lb.; $58{,}845\frac{45}{64}$ lb. 4707 T. $1312\frac{1}{2}$ lb.
8. $93.12.
9. $31.62.
10. 12 da.; $57.
11. $180.50.
12. 12,375 lb.
13. 1056 lb.
14. $1518.
15. 62,475 lb.
16. 216.2 lb.
17. 24.2 T.; 30.25 T.; 36.3 T.
18. 112,800; 56.4.
19. $1541.60.
20. $154.40.
21. 36; 9801; 1,568,160; 125,452,800.
22. $\frac{1}{48}$; $5\frac{43}{64}$; $907\frac{1}{2}$; 72,600.
23. 2268.75 T.
24. 326,700 T.
25. 2.82.
26. 40,837.5 T.
27. 2350 lb.
28. $\frac{1}{3}$; $90\frac{3}{4}$; 14,520; 1,161,600; 43,560 T.

Exercise 102. Page 197

1. $41.74.
2. $118.79.
3. $390.83.
4. $454.09.
5. $51.28
6. $83.03.
7. $402.19.
8. $46.67.
9. $81.25.
10. $8.
11. $76.54.
12. $6.
13. 7500.
14. $50.89.
15. $20.30.
16. $187.78.
17. $202.50.

Exercise 103. Page 198

1. $12.50.
2. $3.90.
3. $2.44
4. $13.50.
5. $27.
6. $6.65.
7. $29.70.
8. $115.20
9. $120.
10. $4.38.
11. $1.50.
12. $363; $242; $242.
13. $163.80.

Exercise 104. Page 200

1. \$25.92.
2. \$9.
3. \$635.45.
4. \$318.75.
5. \$11.20.
6. \$17.28.
7. \$0.07; \$19.25.
8. \$24.
9. \$27.05.
10. \$1.
11. 25¢; \$58.
12. \$26.30.
13. \$24.84.

Exercise 105. Page 202

1. 150.
2. 171.
3. 175 da.
4. 27 da.
5. 2 min. $11\frac{1}{4}$ sec.
6. 1023 lb.
7. 3 hr. $46\frac{2}{3}$ min.
8. 22.5.
9. 3480.
10. 8 ft. $5\frac{1}{3}$ in.
11. 2 hr. 12 min.
12. $2\frac{5}{8}$ mi.
13. \$15.
14. 63.
15. 5 bu.

Exercise 106. Page 204

1. 100.
2. 53.
3. 1280.
4. 960.
5. 1600.
6. \$4809.38.
7. \$3200.
8. \$152.
9. 220.
10. $3\frac{1}{2}$¢.

Exercise 107. Page 205

1. 100,412 T. 974 lb.
2. 5786 T. 783 lb.
3. 5,119,737,264.
4. 19,711,840,608.
5. \$3000.
6. 500 lb.
7. \$241,600.
8. 70.
9. \$138.24.

Exercise 108. Page 206

1. \$0.90.
2. $\frac{2}{5}$¢.
3. \$0.13.
4. \$0.06.
5. \$1.62.
6. \$0.38.
7. \$2.64.
8. 24¢.
9. 10¢.

Exercise 109. Page 209

1. 1 hr. 1 min.
2. 1 min. 1 sec.
3. 2 hr. 1 min.
4. 3 hr. 2 min.
5. 4 hr. 2 sec.
6. 1 hr. 41 min.
7. 2 hr. 10 min.
8. 3 hr. 15 min.
9. 8 hr. 28 min. 1 sec.
10. 11 hr. 12 min. 2 sec.
11. 20 min. 41 sec.
12. 29 min. 42 sec.
13. 1 hr. 54 min. 50 sec.
14. 4 hr. 9 min. 51 sec.
15. 8 hr. 32 min. $24\frac{1}{2}$ sec.
16. 2 hr. 10 min. 42 sec.
17. 2 hr. 49 min. 51 sec.
18. 1 P.M.
19. 1.01.01 P.M.
20. 5 A.M.
21. 6 A.M.
22. 2.30.50 P.M.
23. 1 hr. 20 min. slow.

Exercise 110. Page 210

1. 30° 45′.
2. 61° 45′.
3. 92° 15′.
4. 150° 1′.
5. 165° 2′.
6. 65°.
7. 81° 15′.
8. 106° 32′.
9. 143° 55′.
10. 179° 42′.
11. 30° E.
12. 45° W.
13. 30° W.
14. 26° 15′ W

Exercise 111. Page 212

1. 11 A.M.; 10 A.M.; 1 P.M.; 12 M.
2. 10 A.M.; 11 A.M.; 12 M.; 5 P.M.
3. 2 P.M.; 2 P.M.; 12 M.; 3 P.M.; 8 P.M.
4. 2.30 P.M.; 3.30 P.M.; 4.30 P.M.
5. 11.45 A.M.; 11.45 A.M.; 12.45 P.M.; 11.45 A.M.; 12.45 P.M.
6. 1.45 A.M. the next day; 1.45 A.M. the next day; 2.45 A.M. the next day; 2.45 A.M. the next day; 7.45 A.M. the next day.
7. 2 P.M.
8. 5 P.M. on Dec. 31.
9. 2 P.M.; 1 P.M.; 8 A.M.; 5 A.M.
10. 9 min. 42.72 sec.; standard.
11. 4 min. 6.3 sec.; local.

Exercise 113. Page 216

1. $158.40.
2. 80 yd. 9 in.
3. 54 ft. 10 in.
4. 82 ft. 2 in.
5. $178.75.
6. 62.

Exercise 114. Page 217

1. 5842 sq. ft.
2. 5320 sq. in.
3. 18,450 sq. yd.
4. 220 sq. ft.
5. 19 sq. ft.
6. 430 sq. in.
7. $4\frac{2}{3}$ sq. ft.
8. $101\frac{1}{2}$ sq. ft.
9. 512.
10. $29\frac{3}{40}$.
11. $3334\frac{5}{8}$.
12. 18 rd.

Exercise 115. Page 218

1. 15,072 sq. ft.
2. $503\frac{3}{4}$ sq. ft.
3. 2 sq. rd.
4. 63 sq. yd.
5. $369\frac{1}{2}\frac{9}{4}$ sq. ft.
6. $92\frac{1}{2}$ A.
7. $91\frac{2}{3}$ sq. ft.
8. $136\frac{8}{9}$ sq. ft.
9. $128\frac{5}{8}$ sq. yd.
10. $9936\frac{4}{9}$ sq. ft.
11. 27 ft.
12. 26 ft.
13. 32 ft.
14. 64 rd.

Exercise 116. Page 219

1. 658 sq. ft.
2. $937\frac{1}{2}$ sq. ft.
3. $1443\frac{3}{4}$ sq. ft.
4. 2000 sq. rd.
5. $1093\frac{1}{2}$ sq. in.
6. $10\frac{1}{2}$ sq. ft.
7. 1591 sq. in.
8. $19\frac{5}{6}$ sq. ft.
9. $47\frac{3}{16}$ sq. ft.
10. $16\frac{5}{8}$ sq. ft.
11. 8 ft.
12. 20 ft.
13. 22 ft.
14. 54 in.
15. $10,260.25.
16. 15.57 sq. in.
17. $157\frac{1}{2}$ sq. ft.

Exercise 117. Page 220

1. 45 sq. in.
2. 63 sq. in.
3. 54 sq. in.
4. 84 sq. in.
5. 77 sq. in.
6. 99 sq. in.
7. 252 sq. in.
8. 247 sq. in.
9. 486 sq. in.
10. 936 sq. in.
11. 1134 sq. in.
12. 2214½ sq. in.
13. 60 ft.
14. 325 ft.
15. 11,094 sq. ft.

Exercise 118. Page 222

1. 160 A.
2. 160 A.
3. 160 A.
4. 320 A.
5. 320 A.
6. $16,000.
7. $9600.
8. 10 A.

Exercise 119. Page 223

1. 330 ft.
2. 59.4 ft.
3. 81.4 ft.
4. 9.24 ft.
5. 1.144 in.
6. 13,420 in.
7. 15,620 in.
8. 160.6 in.
9. 396 in.
10. 484 in.
11. 616 in.
12. 1320 in.
13. 387.2 in.
14. 431.2 in.
15. 536.8 ft.
16. 55.44 ft.
17. 84.8232 ft.
18. 135.0888 ft.
19. 100.5312 yd.
20. 14.76552 yd.
21. 1.947792 ft.
22. 2.3562 ft.
23. 7.0686 ft.
24. 3.989832 in.
25. 169.6464 in.
26. 238.7616 in.
27. 383.2752 in.
28. 30.15936 ft.
29. 11.93808 yd.
30. 16.96464 yd.
31. 3.015936 ft.
32. 3.895584 ft.
33. 20 ft.
34. 110 in.
35. 160 in.

Exercise 120. Page 225

1. 78.54 sq. in.
2. 63.6174 sq. in.
3. 50.2656 sq. in.
4. 38.4846 sq. in.
5. 95.0334 sq. in.
6. 314.16 sq. in.
7. 1256.64 sq. in.
8. 2827.44 sq. in.
9. 3.1416 sq. in.
10. 50.2656 sq. in.
11. 28.2744 sq. in.
12. 78.54 sq. in.
13. 314.16 sq. in.
14. 1256.64 sq. in.
15. 0.7854 sq. in.
16. 0.3490⅔ sq. in.
17. 0.031416 sq. in.
18. 0.19635 sq. in.
19. 1.76715 sq. in.
20. 1.3962⅔ sq. in.
21. 19.635 sq. in.
22. 33.18315 sq. in.
23. 16.619064 sq. in.
24. 12.5664 in.
25. 7.0686 sq. in.
26. 10.9956 in.
27. 47.124 in.
28. 10 in.
29. 31.416 ft.
30. 21.46 sq. ft.
31. 220 ft.; 38,013.36 sq. ft.
32. 8.2957875 sq. in.

Exercise 121. Page 226

1. 56. 2. $46\frac{2}{3}$. 3. $58\frac{2}{3}$. 4. 30. 5. 34. 6. $44\frac{1}{3}$. 7. $66\frac{2}{3}$. 8. $69\frac{1}{3}$. 9. $106\frac{2}{3}$. 10. $34.42. 11. 8; 24 in. 12. $26.63.

Exercise 122. Page 228

1. 14. 2. 15. 3. 8. 4. 8. 5. $4.90. 6. $10.50. 7. $3.48; $3.84; $4.19; $4.32; $3.36; $3.58. 8. $8.40. 9. $11.70. 10. $22.40. 11. $1.95.

Exercise 123. Page 230

1. 28. 2. 32. 3. 22. 4. 25. 5. 30. 6. 42. 7. 56. 8. 89. 9. 109. 10. 103. 11. 98. 12. 103. 13. $5.94. 14. $7.26. 15. $7.04. 16. $8.80. 17. $6.60. 18. $6.16. 19. $6.16. 20. $5.06. 21. $5.50. 22. $5.72. 23. $10.78. 24. $13.42. 25. $0.34. 26. $0.38. 27. $0.46. 28. $0.42. 29. $0.68. 30. $0.80. 31. $0.76. 32. $1.16. 33. $1.66. 34. $2.34. 35. $3.12. 36. $4.44. 37. $37.67. 38. $24.30. 39. $26.50.

Exercise 124. Page 231

1. 336. 2. 224. 3. 256. 4. $373\frac{1}{3}$. 5. $426\frac{2}{3}$. 6. $469\frac{1}{3}$. 7. 512. 8. 480. 9. 576. 10. 320. 11. $277\frac{1}{3}$. 12. 400. 13. $453\frac{1}{3}$. 14. 528. 15. 640. 16. $17.55. 17. $35.33.

Exercise 125. Page 232

1. 12. 2. 14. 3. $9.80. 4. $11.52. 5. $13.11. 6. $36.96.

Exercise 126. Page 233

1. 54. 2. 59. 3. 48. 4. 62. 5. 63. 6. 61. 7. 62. 8. 59. 9. 79. 10. $64. 11. $72.60. 12. $127.10. 13. $158.68.

Exercise 127. Page 234

1. \$56.25.
2. \$108.90.
3. \$130.56.
4. \$99.75.
5. \$83.20.
6. \$160.42.
7. \$37.32.
8. $1477\frac{7}{9}$.
9. $2847\frac{2}{9}$.
10. $1726\frac{2}{3}$.
11. 1750.
12. $1013\frac{8}{9}$.
13. $1396\frac{8}{27}$.
14. $1951\frac{2}{27}$.
15. $1710\frac{5}{54}$.
16. \$8020.83.
17. \$936.
18. \$47.92.

Exercise 128. Page 235

1. $10\frac{1}{2}$.
2. $16\frac{2}{3}$.
3. \$15.50.
4. \$3.63.
5. 16 in.; \$0.71.
6. $8\frac{3}{4}$ in.
7. 3.
8. $82\frac{1}{2}¢$.

Exercise 129. Page 236

1. 10,336 cu. in.
2. 9504 cu. in.
3. 8160 cu. in.
4. 9367 cu. in.
5. 5220 cu. ft.
6. 1143 cu. ft.
7. 1302 cu. ft.
8. $1462\frac{1}{2}$ cu. ft.
9. $2016\frac{2}{3}$ cu. ft.
10. 35 cu. ft.
11. $46\frac{1}{4}$ cu. ft.
12. $183\frac{11}{24}$ cu. ft.
13. 4096 cu. in.
14. 7840.
15. 262 sq. in.
16. $36\frac{3}{4}$ sq. ft.
17. $218\frac{29}{72}$.
18. $62\frac{1}{2}$ lb.
19. 2025 lb.
20. 6 ft.
21. 12 ft.
22. 8 sq. in.

Exercise 130. Page 238

1. 248.
2. 585.
3. $408\frac{1}{3}$.
4. 900.
5. 1066.
6. 1248.
7. $18\frac{1}{3}$.
8. $26\frac{2}{3}$.
9. $18\frac{1}{3}$.
10. 50.
11. 468.
12. \$105.60.
13. 36,740.
14. 1152.
15. 1098.
16. 61,424.
17. 210,364.

Exercise 131. Page 240

1. 12.
2. 24.
3. 13.
4. $8\frac{1}{7}$.
5. 10.
6. $10\frac{4}{5}$.
7. 15.
8. $13\frac{5}{7}$.
9. $11\frac{2}{3}$.
10. $61\frac{7}{18}$.
11. $101\frac{0}{21}$.
12. $6\frac{3}{14}$.
13. $71\frac{3}{7}$ lb.
14. $591\frac{1}{21}$ lb.
15. $1666\frac{2}{3}$.
16. $156\frac{4}{27}$.
17. $5555\frac{5}{9}$.

Exercise 132. Page 241

1. $1\frac{1}{4}$.
2. $2\frac{1}{4}$.
3. $2\frac{11}{32}$.
4. $2\frac{1}{2}$.
5. $2\frac{1}{4}$.
6. $3\frac{3}{4}$.
7. 3.
8. $4\frac{15}{32}$.
9. $5\frac{2}{3}$.
10. \$18.75.
11. \$37.69.
12. $17\frac{3}{4}$.
13. \$12.03.
14. \$25.31.
15. \$4.
16. 24 ft.

Exercise 133. Page 243

1. $5\frac{1}{3}$.
2. 7.
3. 8.
4. 12.
5. 12.
6. $13\frac{1}{3}$.
7. $14\frac{1}{4}$.
8. $8\frac{2}{3}$.
9. $9\frac{11}{12}$.
10. 160.
11. 420.
12. $533\frac{1}{3}$.
13. 210.
14. 288.
15. 600.
16. 400.
17. 280.
18. 1600.
19. 960.
20. 240.
21. $666\frac{2}{3}$.
22. $746\frac{2}{3}$.
23. 3450.
24. \$2.30.
25. \$11.52.
26. \$19.44.

Exercise 134. Page 244

1. 288.
2. $403\frac{1}{5}$.
3. 540.
4. 768.
5. 1152.
6. $1209\frac{3}{5}$.
7. 240.
8. 527.
9. $681\frac{29}{40}$.
10. 750.
11. $1062\frac{1}{2}$.
12. $1218\frac{3}{4}$.
13. 1875.
14. $2031\frac{1}{4}$.
15. 2325.
16. 4000.
17. 7000.
18. 9375.
19. 20,000.
20. 27,900.
21. 35,000.
22. \$273.60.
23. \$4896.

Exercise 135. Page 246

1. 351 cu. in.
2. 612 cu. in.
3. 4750 cu. in.
4. $365\frac{1}{2}$ cu. in.
5. 405 cu. in.
6. 35 cu. ft. 1320 cu. in.
7. $2\frac{3}{8}$ ft.
8. 6 ft. 3 in.
9. 18 sq. ft.
10. 19 sq. ft.
11. 13 sq. ft.
12. 15 sq. ft. 36 sq. in.
13. 1357.1712 cu. in.
14. 2827.44 cu. in.
15. 6433.9968 cu. in.
16. 9160.9056 cu. in.
17. 376.992 cu. in.
18. 10.472 cu. ft.

Exercise 136. Page 247

1. 60.
2. 75.
3. $50\frac{5}{8}$.
4. $101\frac{1}{4}$.
5. $7031\frac{1}{4}$.
6. $9445\frac{5}{16}$.
7. $282\frac{6}{7}$.
8. 400.
9. $265\frac{85}{336}$.
10. $2121\frac{3}{7}$.
11. $15{,}910\frac{5}{7}$.
12. 21,120.
13. 288,750.
14. $776{,}242\frac{1}{2}$.
15. $637{,}215\frac{105}{128}$.
16. $424\frac{2}{7}$.
17. $318\frac{3}{14}$.
18. $754\frac{2}{7}$.
19. $433\frac{1}{8}$.
20. $435\frac{15}{256}$.
21. $725\frac{25}{256}$.

Exercise 138. Page 252

1. $\frac{1}{25}$.
2. $\frac{1}{20}$.
3. $\frac{3}{50}$.
4. $\frac{1}{10}$.
5. $\frac{3}{20}$.
6. $\frac{1}{5}$.
7. $\frac{1}{4}$.
8. $\frac{3}{10}$.
9. $\frac{1}{2}$.
10. $\frac{3}{4}$.
11. 1.
12. 2.
13. 5.
14. $1\frac{1}{4}$.
15. $1\frac{1}{2}$.
16. $1\frac{3}{4}$.
17. $1\frac{1}{5}$.
18. $1\frac{2}{5}$.
19. $1\frac{3}{5}$.
20. $1\frac{4}{5}$.
21. $\frac{1}{16}$.
22. $\frac{1}{8}$.
23. $\frac{3}{8}$.
24. $\frac{5}{8}$.
25. $\frac{7}{8}$.
26. $\frac{3}{16}$.
27. $\frac{5}{16}$.
28. $\frac{7}{16}$.
29. $\frac{9}{16}$.
30. $\frac{11}{16}$.
31. $\frac{1}{30}$.
32. $\frac{1}{15}$.
33. $\frac{1}{3}$.
34. $\frac{2}{3}$.
35. $\frac{1}{6}$.
36. $\frac{5}{6}$.
37. $1\frac{1}{8}$.
38. $1\frac{1}{3}$.
39. $1\frac{2}{3}$.
40. $1\frac{7}{8}$.
41. 3; 3.
42. 160; 160.
43. 864; 864.
44. 6330; 6330.
45. 1080; 1080.
46. $\frac{1}{8}$; 8.
47. $\frac{3}{8}$; 270.
48. 70.
49. 22.
50. 11.
51. 123.
52. 92.
53. 198.
54. 352.
55. 507.
56. 972.
57. 1068.
58. 1180.

Exercise 139. Page 253

1. 0.06.
2. 0.10.
3. 0.25.
4. 0.50.
5. 0.75.
6. 0.125.
7. $0.16\frac{2}{3}$.
8. $0.33\frac{1}{3}$.
9. 0.375.
10. $0.66\frac{2}{3}$.
11. 0.005.
12. 0.007.
13. 0.009.
14. 0.0015.
15. 0.0625.
16. 2.
17. 1.5.
18. 1.25.
19. 6.75.
20. 3.75.
21. 1.
22. 10.
23. 0.00005.
24. 0.00065.
25. 0.00125; $\frac{1}{800}$.
26. 0.00875; $\frac{7}{800}$.
27. $3.33\frac{1}{3}$; $\frac{10}{3}$; $3\frac{1}{3}$.
28. \$2; \$2.
29. \$6; \$6.
30. \$45; \$45.
31. \$812.50; \$812.50; \$812.50.
32. \$1150; \$1150.
33. 2; 1.

Exercise 140. Page 254

1. 50%.
2. 70%.
3. 60%.
4. 620%.
5. 750%.
6. 25%.
7. 57%.
8. 34%.
9. 29%.
10. 765%.
11. 12.5%.
12. 27.5%.
13. 37.5%.
14. 75.8%.
15. 963.7%.
16. 25%.
17. 52.5%.
18. 67.5%.
19. 521.25%.
20. 733.75%.
21. $33\frac{1}{3}$%.
22. $33\frac{1}{3}$%.
23. $666\frac{2}{3}$%.
24. $\frac{3}{8}$%.
25. $\frac{1}{80}$%.
26. 50%.
27. 25%.
28. $3\frac{1}{8}$%.
29. 0.7%.
30. 37%; 370%.
31. $3\frac{1}{8}$%.
32. $3\frac{1}{8}$%.

Exercise 141. Page 255

1. 50%.
2. 25%.
3. 75%.
4. 20%.
5. 40%.
6. 60%.
7. 80%.
8. 10%.
9. 70%.
10. 5%.
11. 45%.
12. 4%.
13. 44%.
14. 2%.
15. 74%.
16. $33\frac{1}{3}$%.
17. $66\frac{2}{3}$%.
18. $12\frac{1}{2}$%.
19. $37\frac{1}{2}$%.
20. $62\frac{1}{2}$%.
21. $6\frac{1}{4}$%.
22. $31\frac{1}{4}$%.
23. $3\frac{1}{8}$%.
24. $46\frac{7}{8}$%.
25. $20\frac{5}{6}$%.
26. 225%.
27. 360%.
28. $762\frac{1}{2}$%.
29. $633\frac{1}{3}$%.
30. $7587\frac{1}{2}$%.
31. $33\frac{1}{3}$%.
32. $8\frac{1}{3}$%.
33. $12\frac{1}{2}$%.
34. 75%.
35. 25%.
36. $43\frac{3}{4}$%.
37. $83\frac{1}{3}$%.
38. 10%.

Exercise 142. Page 256

1. $137.
2. $93.
3. $64.
4. $211.
5. $6.66.
6. $7.45.
7. $30.17.
8. $23.10.
9. $5.71.
10. $25.72.
11. $53.38.
12. $59.25.
13. $147.03.
14. $244.50.
15. $440.49.
16. 81.
17. 132.
18. 84.
19. 108.
20. 140.

Exercise 143. Page 257

1. 16.5.
2. 30.1.
3. 8.64.
4. 31.98.
5. 57.66.
6. 0.884.
7. 2.881.
8. 2.067.
9. 0.196.
10. 0.2205.
11. 0.231.
12. 0.0285.
13. 0.1292.
14. 1743.7.
15. 4.4784.
16. 432.
17. $3117\frac{1}{2}$.
18. $1468\frac{1}{5}$.
19. 837.
20. 4836.
21. 1202.
22. $1520\frac{1}{2}$.
23. 3017.
24. 4744.
25. 30,940.
26. $2301.
27. 9390.
28. 18,625.
29. 1185.
30. 972.
31. 1360.
32. $10.50.
33. $350.
34. $2,980,800.
35. $348.
36. 14.625 lb.
37. $3.36.
38. Second; 0.46¢ per day.
39. $155.25.
40. $17.93.

Exercise 144. Page 260

1. 600.
2. 1100.
3. 1200.
4. 1700.
5. 31,996.
6. 146.16.
7. 144.81.
8. $1900.
9. $56,600.
10. 31.
11. 41.
12. 44.44.
13. $33\frac{1}{3}$.
14. 25.
15. 40.
16. 245.
17. 320.
18. 425.
19. 640.
20. 400.
21. 725.
22. 7.29.
23. 5.46.
24. 9.36.
25. 25.
26. 40.
27. $33\frac{1}{3}$.
28. 270.
29. 320.
30. 750.
31. 764.
32. 280.
33. 720.
34. 7.2.
35. 32.
36. 8.8.
37. $7250.
38. $395.
39. $850.
40. 22.05.
41. $275.
42. $6.75.
43. 27,500.

Exercise 145. Page 261

1. 5%.
2. 7%.
3. 4%.
4. 9%.
5. 11%.
6. 16%.
7. 21%.
8. 45%.
9. 33%.
10. 3%.
11. 24%.
12. 22%.
13. 50%.
14. $16\frac{2}{3}$%.
15. $66\frac{2}{3}$%.
16. $37\frac{1}{2}$%.
17. $62\frac{1}{2}$%.
18. $87\frac{1}{2}$%.
19. 25%.
20. $\frac{5}{42}$%.

Exercise 146. Page 262

1. $25.38.
2. $29.76.
3. $43.70.
4. $69.
5. $61.44.
6. $78.26.
7. $61.20.
8. $72.16.
9. $81.60.
10. $231.84.
11. $159.25.
12. $23.97.
13. $44.85.
14. $56.07.
15. $66.06.
16. $60.50.
17. $73.44.
18. $34.11.
19. $42.68.
20. $45.15.
21. $57.97.
22. $40.12.
23. $120.48.
24. $345.46.
25. $383.85.
26. $344.34.
27. $252.28.
28. $433.40.
29. $508.47.
30. $244.75.
31. $727.32.
32. $817.87.
33. $770.99.
34. 25%.
35. $520.
36. $16.40.
37. $39.60.
38. $385.
39. $1911.
40. $41.82.
41. 12¢.
42. $47.25.
43. 3¢.
44. $66.64.
45. $16.68.
46. $245.

Exercise 147. Page 264

1. $169.20.
2. $262.20.
3. $407.40.
4. $285.60.
5. $378.
6. $162.
7. $307.80.
8. $325.92.
9. $393.66.
10. $170.10.
11. $1324.80.
12. $2358.75.
13. $194.40.
14. $51.84.
15. $51.84.
16. 20% and 10% by 15¢.
17. $40.50.
18. $528.
19. $119.88.
20. $144.
21. $480.
22. $594.81.

Exercise 148. Page 266

1. $155.40.
2. $1814.40.
3. $164.45.
4. $53.82.
5. $846.45.
6. $24.37.
7. $1486.75.

Exercise 149. Page 267

1. 17.2%.
2. 20.1%.
3. 20.46%.
4. 21.74%.
5. 28%.
6. 37%.
7. $34\frac{3}{8}$%.
8. 27.1%.
9. 42.625%.
10. 46%.
11. 49.6%.
12. 43%.
13. 46%.
14. $32\frac{1}{2}$%.
15. 37%.
16. $40\frac{15}{16}$%.
17. They are equal.
18. 24%.
19. They are equal.

Exercise 150. Page 269

1. $28.
2. $41.04.
3. $74.75.
4. $84.96.
5. $106.25.
6. $189.88.
7. $207.58.
8. $280.
9. $431.25.
10. $867.30.
11. $194.40.
12. $435.40.
13. $53,521.25.
14. $33,519.50.
15. $94,062.50.
16. $51.
17. $69.70.
18. $56.16.
19. $165.
20. $623.
21. $810.
22. $169.05.
23. $21,800.
24. $18,480.
25. $75.
26. $82.
27. $68.
28. $175.
29. $296.
30. $450.
31. $260.
32. $320.
33. $460.
34. $262.50.
35. $360.
36. $337.50.
37. $41.25.
38. $45.63.
39. $661.22.
40. $634.40.
41. $1603.14.
42. $41.31.
43. $55.44.
44. $88.70.
45. $138.12.
46. $130.
47. $342.04.
48. $352.37.
49. $524.12.
50. $1402.01.
51. $623.70.
52. $16.50.
53. $13.80.
54. $37.04.
55. $14.50.
56. $34.20.
57. $87.87.
58. $378.02.
59. $315.90.
60. $625.50
61. $212.40.
62. $39.25.
63. $138
64. $8.50.
65. $9.12.
66. 15%.
67. 25%.
68. $10.
69. $320.
70. $3.15.
71. $15.40
72. $2.40, loss.
73. $223.20.
74. $150.
75. $360.
76. 50%.
77. $66\frac{2}{3}$%.
78. $800, loss.
79. $1280, loss.

Exercise 151. Page 272

1. $5.50.
2. $10.35.
3. $27.
4. $36.25.
5. $52.50.
6. $41.25.
7. $81.88.
8. $96.60.
9. $187.88.
10. $212.50.
11. $10.
12. $18.75.
13. $31.
14. $13.20.
15. $54.
16. $4794.
17. $6384.
18. $3493.
19. $2737.
20. $5481.
21. $6176.
22. $264.48.
23. $563.50.
24. $678.21.

25. $44.45.
26. 255,000; $18.75.
27. $8787.50; 425,000; $31.25.
28. $786.60.
29. 21¢.
30. $1\frac{1}{3}$¢.
31. 227,500; $19,248.25.
32. 350,000; $42,385.

Exercise 152. Page 274

1. $18.75.
2. $22.50.
3. $30.63.
4. $50.63.
5. $52.50.
6. $71.50.
7. $95.
8. $78.75.
9. $125.
10. $270.
11. $187.50.
12. $223.13.
13. $228.75.
14. $371.25.
15. $431.25.
16. $515.63.
17. $109.20.
18. $78.
19. $60.
20. $45.90.
21. $60.50.
22. $87.50.
23. $134.40.
24. $178.75.
25. $6500.
26. $2400.
27. $3800.
28. $3200.
29. $1975.
30. $1650.
31. 1%.
32. $2\frac{1}{2}$%.
33. $1\frac{1}{2}$%.
34. $1\frac{3}{4}$%.
35. 2%.
36. $1\frac{1}{2}$%.
37. $34.38.
38. $48.75.
39. $84.
40. $126.
41. $375.
42. $756.25.
43. $41.25.
44. $71.50.
45. $52.50.
46. $5500.
47. $1\frac{1}{2}$%.
48. $43.20.
49. $\frac{4}{5}$%.
50. $3000.
51. $4500.
52. $4666.67; $3333.33.

Exercise 153. Page 276

1. $108.75.
2. $169.88.
3. $99.58.
4. $118.62.
5. $194.25.
6. $100.58.
7. $86.28.
8. $307.44.
9. $369.32.
10. $456.63.
11. $3954.
12. $195.98.
13. $6000; $4720.80.

Exercise 154. Page 279

1. $1.39.
2. $4.73.
3. $5.
4. $22.22.
5. $20.28.
6. $25.28.
7. $41.95.
8. $50.
9. $66.66.
10. $63.88.
11. $62.50.
12. $220.82.
13. $3.18.
14. $5.
15. $6.36.
16. $32.72.
17. $45.45.
18. $32.27.
19. $43.18.
20. $34.09.
21. $50.45.
22. $72.72.
23. $118.17.
24. $318.15.
25. $113.63.
26. $240.89.
27. $443.14.
28. $748.56.
29. $49.50.
30. $162.50.
31. $7500.
32. 12 mills.
33. 12 mills.
34. $3760.
35. $42.50.
36. $190.
37. $614.33.

Exercise 155. Page 281

1. $142.50.
2. $218.75.
3. $300.
4. $375.75.
5. $585.
6. $375.
7. $495.
8. $945.
9. $1460.
10. $3787.50.
11. $2490.
12. $12,712.50.
13. $10,276.70.
14. $16,137.50.
15. $45,177.
16. $24.48.
17. $343.75.
18. $7500.
19. $415.80.
20. 15%.
21. 40%.
22. $8275.
23. 35%.
24. $331.
25. $3375.
26. $2750.
27. $6600.
28. $168.50.
29. $271.80.
30. 1750.
31. $750.

Exercise 156. Page 283

1. 12.21 ; 122.1 ; 1221 ; 12,210.
2. 12.21 ; 122.1 ; 1221 ; 12,210.
3. 24.55 ; 245.5 ; 2455 ; 24,550 ; 24,550.
4. 16.22 ; 162.2 ; 1622 ; 16,220 ; 16,220.
5. 50.64 ; 506.4 ; 5064 ; 50,640 ; 50,640.
6. 103.11 ; 1031.1 ; 10,311 ; 103,110 ; 1,031,100.
7. 41.88 ; 418.8 ; 4188 ; 41,880 ; 418,800.
8. 77.15 ; 771.5 ; 7715 ; 77,150 ; 771,500.
9. 324.38 ; 3243.8 ; 32,438 ; 324,380 ; 3,243,800.
10. 18%.
11. 15%.
12. 4%.
13. $4850.
14. $120.
15. $700.
16. $490.
17. $2250.
18. $6000.
19. $7000.
20. $6500.
21. $27\frac{3}{11}$%.
22. $7525.
23. 20%.
24. $650.
25. $1.25.
26. $1.64.
27. $4.
28. 20%.
29. $1039.50.
30. $17,000.
31. $1800.
32. $2.64.
33. $130.
34. $1.92.
35. $1\frac{1}{4}$%.
36. $240.
37. 97.6%.
38. $2375 ; $2625.
39. $17,250.
40. $1.44.
41. 81¢.
42. $654.75.
43. $1470.15.
44. $40.50.
45. $1\frac{1}{4}$%.
46. $280.
47. 25%.
48. $1425.90.
49. $5\frac{1}{2}$%.
50. $1181 ; $1228.24.
51. $65.07.
52. $1.81\frac{9}{11}$% ; $2.72\frac{8}{11}$% ; $95.45\frac{5}{11}$%.

Exercise 157. Page 287

1. $27.50.
2. $42.
3. $55.20.
4. $78.75.
5. $21.20.
6. $63.75.
7. $152.40.
8. $105.
9. $231.25.
10. $78.
11. $107.25.
12. $97.88.
13. $58.45.
14. $63.83.
15. $78.91.
16. $47.31.
17. $367.50.
18. $491.56.

Exercise 158. Page 288

1. $4.51.
2. $87.97.
3. $172.58.
4. $190.46.
5. $115.45.
6. $392.26.
7. $490.77.
8. $305.35.
9. $180.25.
10. $234.11.
11. $635.68.
12. $1392.01.
13. $2759.51.

Exercise 159. Page 290

1. $125.
2. $560.
3. $68.50.
4. $426.
5. $130.
6. $240.
7. $402.
8. $1042.
9. $1080.
10. 3 yr. 6 mo.
11. 2 yr. 3 mo.
12. 2 yr. 4 mo.
13. 2 yr. 6 mo.
14. 5%.
15. 4%.
16. 6%.
17. $16\frac{2}{3}$ yr.
18. 20 yr.
19. $500.
20. $500.

Exercise 160. Page 291

1. 2 yr. 1 mo. 13 da.
2. 3 yr. 3 mo. 20 da.
3. 3 yr. 9 mo. 27 da.
4. 3 yr. 1 mo. 25 da.
5. 2 yr. 4 mo. 16 da.
6. $63.56.
7. $90.73.
8. $138.70.
9. 6%.
10. Jan. 7, 1913.
11. $200.
12. $745.88.

Exercise 161. Page 292

1. $5.04.
2. $5.58.
3. $11.63.
4. $7.60.
5. $11.48.
6. $14.04.
7. $5.63.
8. $7.80.
9. $14.03.
10. $51.73.
11. $96.75.
12. $43.75.
13. $35.55.
14. $143.49.
15. $255.75.
16. $331.25.

Exercise 162. Page 293

1. $10.40.
2. $8.60.
3. $6.27.
4. $10.50.
5. $19.38.
6. $41.25.
7. $1.67.
8. $2.35.
9. $4.76.
10. $16.67
11. $16.63.
12. $20.
13. $2.52.
14. $10.81.
15. $21.66.
16. $38.18.
17. $1.13.
18. $12.19.
19. $18.56.
20. $28.91.

Exercise 163. Page 294

1. 92.
2. 153.
3. 227.
4. 220.
5. 174.
6. 188.
7. 104.
8. 48.
9. 243.
10. 186.
11. 122.
12. 141.
13. 162.
14. 141.
15. 191.
16. 235.

Exercise 164. Page 295

1. $8.92.
2. $10.13.
3. $8.80.
4. $18.75.
5. $7.75.
6. $11.25.
7. $14.79.
8. $7.13.
9. $9.60.
10. $25.20.
11. $3.85.
12. $7.13.
13. $15.87.
14. $18.
15. $2.74.
16. $7.92.
17. $12.31.
18. $7.80.
19. $11.07.
20. $6.03.
21. $8.37.
22. $19.88.
23. $19.67.
24. $12.59.
25. $7.14.
26. $10.79.
27. $9.15.
28. $14.88.

Exercise 165. Page 296

1. $40.49.
2. $49.14.
3. $63.75.
4. $39.48.
5. $51.46.
6. $62.10.
7. $76.97.
8. $58.51.
9. $236.25.
10. $35.27.
11. $56.21.
12. $65.88.

Exercise 166. Page 297

1. $3.08.
2. $3.20.
3. $4.01.
4. $5.99.
5. $14.67.
6. $11.34.
7. $17.67.
8. $4.50.
9. $4.80.
10. $20.10.
11. $19.20.
12. $48.
13. $76.23.
14. $236.50
15. $110.25.
16. $47.25.
17. $94.50.

Exercise 167. Page 298

1. $130.80.
2. $47.70.
3. $130.63.
4. $253.13.
5. $133.62.
6. $135.04.
7. $75.57.
8. $108.75.
9. $79.58.
10. $355.93.
11. $166.71.
12. $473.10.
13. $1467.50
14. $4038.03

Exercise 168. Page 299

1. $281.22.
2. $438.69.
3. $478.07.
4. $790.83.
5. $1107.16.
6. $1379.04.
7. $1591.20.
8. $3040.51.
9. $3042.50.
10. $112.65.
11. $273.36.
12. $610.29.
13. $710.74.
14. $1137.32.
15. $1454.32.
16. $2774.60.
17. $1886.59.
18. $1943.75.

Exercise 169. Page 301

1. Sept. 5, 1911; $353.50.
2. July 4, 1910; $584.25.
3. Feb. 8, 1911; $480.70.
4. June 7, 1910; $333.38.
5. July 14, 1911; $544.29.
6. June 15, 1910; $276.65.
7. $14.40.
8. $10.67.
9. $11.25.
10. $252.08.
11. $1268.75.
12. $289.28.

Exercise 170. Page 303

1. $71.97.
2. $182.70.
3. $487.25.
4. $347.43.
5. $1538.21.
6. $378.44.
7. $534.
8. $406.98.
9. $1.80.

Exercise 171. Page 305

1. $147.33.
2. $142.43.
3. $231.48.
4. $161.04.
5. $26.05.
6. $254.40.

Exercise 172. Page 306

1. $18.83.
2. $2.40.
3. $27.87.
4. $75.50; $37.75; $113.25; $117.03.
5. $23.56.
6. $26.18.
7. $1484.96.
8. $45.44.
9. $261.
10. $432.39.
11. $10.90.
12. $29.90.
13. $277.
14. $1151.16.
15. $775.
16. $240.
17. $350.
18. $450.
19. $750.
20. $840.
21. 4 yr. 11 mo. 6 da.
22. 3 yr. 4 mo.
23. 1 yr. 7 mo. 6 da.
24. 7 yr. 9 mo. 15 da.
25. 5 yr. 7 mo. 9 da.
26. 3%.
27. $4\frac{1}{2}$%.
28. 6%.

Exercise 174. Page 312

1. $\frac{2}{3}$.
2. $\frac{3}{4}$.
3. $\frac{2}{5}$.
4. $\frac{8}{3}$.
5. $\frac{2}{15}$.
6. $\frac{4}{7}$.
7. 4.
8. $\frac{1}{4}$.
9. $\frac{2}{5}$.
10. $\frac{3}{4}$.
11. 8.
12. 3.
13. 3.
14. 20.
15. 6.
16. 35.
17. 18.
18. 14.
19. 12; 16.
20. 20; 25.
21. 100; 160.
22. $10\frac{1}{2}$; 21.
23. $8; $64.
24. $36; $60.
25. $18; $60.
26. 165; 176.
27. 132; 360.
28. $7; $8.
29. $8980.

Exercise 175. Page 313

1. $211\frac{1}{2}$.
2. 129.6.
3. \$1125.
4. 150 lb.; 65 lb.
5. \$1650; \$2100.
6. $187\frac{1}{2}$.
7. 16.
8. \$896; \$1344.
9. \$28; \$56.
10. \$45; \$70.
11. \$2100.
12. 49 ft.
13. $12\frac{4}{7}$ ft.
14. $62\frac{1}{2}$.
15. 14.
16. 99.
17. \$30; \$54.
18. \$2250; \$3150.
19. 4 lb.; 2 lb.

Exercise 176. Page 316

1. 18.
2. 9.
3. 80.
4. 6.
5. 12.
6. 25.
7. $6\frac{2}{3}$.
8. 30.
9. 18.
10. 35.
11. 27.
12. 14.
13. 63.
14. 63.
15. 63.
16. 45.
17. \$11.
18. \$5.
19. 85.
20. 35.
21. 11 ft.
22. 280.
23. 40.
24. 60.
25. 60.
26. 12.
27. 15.
28. 10.
29. 1250.
30. 5.02656.
33. $9\frac{3}{5}$.
34. 144.

35. $401 : 311 = 6817 : 5287$; $5287 : 6817 = 311 : 401$; $401 : 6817 = 311 : 5287$.

Exercise 177. Page 318

1. \$249.60.
2. \$15.12.
3. \$16.50.
4. \$7.75.
5. \$196.
6. \$68.75.
7. $266\frac{1}{4}$ yd.
8. \$20.70.
9. $112\frac{1}{2}$.
10. 39 da.
11. 15.
12. 216 mi.

Exercise 178. Page 319

1. 625. 2. 1.7. 3. 40 ft. 4. 180 ft. 5. 30 ft. 6. 64 ft.

Exercise 179. Page 320

1. $4\frac{1}{4}$ oz.
2. $8\frac{11}{13}$ oz.
3. $5\frac{11}{17}$ lb.
4. $7\frac{1}{32}$ lb.
5. $16\frac{2}{3}$ lb.
6. 60 lb.
7. $37\frac{1}{2}$ lb.
8. 150 lb.
9. 50 lb.
10. $181\frac{9}{11}$ lb.
11. 60 lb.
12. 360 lb.
13. 750 lb.
14. 225 lb.
15. $1031\frac{1}{4}$ lb.
16. $1462\frac{1}{2}$ lb.

Exercise 180. Page 321

1. \$154.70.
2. \$50.
3. 750.
4. 4680.
5. 315 gal.
6. 20 ft.
7. 90%.
8. No.
9. 1431 lb.
10. \$67.50.
11. $10\frac{4}{15}$.
12. 28.405 in.
13. \$121.50.
14. \$40.
15. $13\frac{5}{12}$ T.
16. 3625.
17. 14 da.
18. $38\frac{1}{4}$.
19. $35\frac{3}{4}$.
20. 46,100 cu. ft.

Exercise 181. Page 323

1. 77; 99; 121.
2. 46; 138; 69.
3. \$1176; \$1470.
4. \$936; \$1248; \$1560.
5. \$1350; \$1650; \$1750; \$2000.
6. \$67.50; \$57; \$100.50.
7. \$11.64; \$14.07; \$13.05.
8. \$224; \$288; \$336.
9. \$160; \$200; \$180.
10. \$21.60; \$21.60.
11. \$1235; \$1558.
12. \$9; \$6; \$7; \$10.50.
13. \$600; \$420.
14. \$12; \$12.

Exercise 182. Page 325

1. 38 ft. 6 in.
2. 37 ft.
3. \$5.50.
4. 4 ft.
5. $8\frac{1}{3}$ lb.

Exercise 184. Page 328

1. 289 sq. ft.
2. 361 sq. ft.
3. 529 sq. ft.
4. 8.41 sq. in.
5. 13.69 sq. in.
6. 18.49 sq. in.
7. 0.3844 sq. yd.
8. 1.6129 sq. yd.
9. 24.3049 sq. yd.
10. 0.64 sq. mi.
11. $\frac{49}{64}$ sq. in.
12. $\frac{81}{1024}$ sq. ft.
13. 14.
14. 15.
15. 16.
16. 20.
17. 18.
18. 22.
19. 25.
20. 21.
21. 28.
22. 30.
23. 27.
24. 32.
25. 35.
26. 33.
27. 36.
28. 1.2 in.
29. 1.1 in.
30. 2.4 in.
31. 50 ft.
32. 60 yd.
33. 80 rd.
34. 0.3 mi.
35. 0.9 ft.
36. 0.12 ft.
37. 96 in.
38. 288 in.
39. 19.2 in.
40. 25.6 in.
41. 32.4 ft.
42. 42 ft.
43. 10.
44. 80 rd.
45. 52.
46. $141\frac{1}{64}$ sq. in.
47. 1.7 ft.

Exercise 185. Page 330

1. 29.
2. 31.
3. 23.
4. 37.
5. 41.
6. 39.
7. 57.
8. 61.
9. 43.
10. 91.
11. 99.
12. 79.
13. 83.
14. 73.
15. 22 in.
16. 87.

Exercise 186. Page 332

1. 437.
2. 862.
3. 634.
4. 871.
5. 343.
6. 897.
7. 441.
8. 416.
9. 418.
10. 714.
11. 32.2.
12. 9.07.
13. 0.251.
14. 1.23.
15. 1.62.
16. 1.98.
17. 21.5.
18. 0.063.
19. 0.427.
20. 0.234.
21. 111 ft.
22. 89.7 in.
23. 176.4 in.
24. 7.7 in.; 456.533 cu. in.

Exercise 187. Page 335

1. $\frac{7}{12}$.
2. $\frac{11}{13}$.
3. $\frac{15}{19}$.
4. $\frac{21}{25}$.
5. $\frac{23}{27}$.
6. $\frac{31}{33}$.
7. $\frac{37}{41}$.
8. $\frac{29}{43}$.
9. $\frac{47}{51}$.
10. $\frac{53}{55}$.
11. 1.41.
12. 2.65.
13. 3.32.
14. 3.87.
15. 5.48.
16. 7.07.
17. 11.18.
18. 25.50.
19. 0.75.
20. 0.76.
21. 0.58.
22. 0.63.
23. 0.82.
24. 0.61.
25. 0.94.
26. 0.79.
27. 0.76.
28. 0.77.
29. 0.65.
30. 0.96.
31. 65 ft.
32. 75 ft.
33. 85 ft.
34. 89.157 ft.
35. 38.601 ft.
36. 50.774 ft.
37. 8 ft.
38. 8 in.
39. 24 in.
40. 13.748 ft.
41. 13.416 ft.
42. 19.621 in.
43. 28.284 in.
44. 45.255 in.
45. 63.640 in.
46. 98.995 in.
47. 106.066 in.

Exercise 188. Page 336

1. 17.32 sq. in.
2. 23.53 sq. in.
3. 5.33 sq. in.
4. 42.43 sq. in.
5. 74.83 sq. in.
6. 176.56 sq. in.
7. $68\frac{3}{4}$ sq. ft.
8. $4000.

Exercise 189. Page 337

1. 907.9224 sq. in.
2. 2642.0856 sq. in.
3. 4300.8504 sq. in.
4. 87.25134264 sq. in.
5. 115.75193784 sq. ft.
6. 1134.1176 sq. in.
7. 1661.9064 sq. in.
8. 660.5214 sq. in.
9. 44.65124664 sq. in.
10. 74.20333536 sq. ft.
11. 10 in.
12. 6 in.
13. 8.92 in.
14. 1.60 in.
15. 1.95 ft.
16. 11.28 in.
17. 42.22 ft.
18. 22.36 ft.; 6283.2 sq. ft.
19. 23.1 sq. in.; 2.71 in.

Exercise 190. Page 338

1. 50,653. 3. 205,379. 5. 110.592. 7. 11. 9. 16. 11. 21.
2. 117,649. 4. 421.875. 6. 0.328509. 8. 15. 10. 18. 12. 22.
13. 24 in.

Exercise 191. Page 340

1. 13. 3. 19. 5. 31. 7. 61. 9. 72. 11. 96. 13. 92 in.
2. 17. 4. 24. 6. 48. 8. 51. 10. 95. 12. 98. 14. 59 in.

Exercise 192. Page 343

1. 4.15. 4. 0.91. 7. 3.35. 10. $\frac{11}{12}$. 13. $\frac{15}{16}$. 16. 1.710.
2. 8.63. 5. 2.79. 8. 4.34. 11. $\frac{17}{19}$. 14. 1.260. 17. 0.630.
3. 1.92. 6. 0.50. 9. $\frac{2}{9}$. 12. $\frac{13}{21}$. 15. 1.442. 18. 0.874.

Exercise 193. Page 345

1. 121. 5. 225. 9. 0.486. 13. 1.96 in. 17. 8 ft.
2. 109. 6. 444. 10. 4.22 in. 14. 6 in. 18. 9 ft.
3. 127. 7. 16.9. 11. 9.40 ft. 15. 13:17. 19. 4; 8.
4. 363. 8. 0.097. 12. 4.52 in. 16. 9 in. 20. 2.
21. 8; 125; 1000; 3723.875.

Exercise 195. Page 349

1. 25,000 m.
2. 3700 m.
3. 700 m.
4. 62,500 m.
5. 57,750 m.
6. 23.7 m.
7. 4.75 m.
8. 0.275 m.
9. 7.25 m.
10. 0.685 m.
11. 29.65 m.
12. 487.5 m.
13. 29.375 m.
14. 96.4 m.
15. 250 m.
16. 0.5 m.
17. 2 m.
18. 3 m.
19. 7.62 m.
20. 1.46 m.
21. 0.18 m.
22. 0.99 m.
23. 1 m.
24. 9 m.
25. 246.06 ft.
26. 159.12 ft.
27. 9.02 ft.
28. 11.48 ft.
29. 2.46 ft.
30. 22.47 ft.
31. 147.64 ft.
32. 22.56 ft.
33. 70.46 ft.
34. 3.1 mi.
35. 46.5 mi.
36. 1.55 mi.
37. 2.945 mi.
38. 168.64 mi.
39. 30.318 mi.
40. 524.19 km.
41. 68.87 km.
42. 1.21 km.
43. 5.7885.
44. 194.06.
45. 351.61.

Exercise 196. Page 350

1. 275,000,000 sq. m.
2. 3,450,000 sq. m.
3. 750,000 sq. m.
4. 0.4875 sq. m.
5. 0.9235 sq. m.
6. 6.4275 sq. m.
7. 0.12345 sq. m.
8. 2.25 sq. m.
9. 37.375 sq. m.
10. 1 ha.
11. 4.5 ha.
12. 17.5 ha.
13. 32.55 ha.
14. 10 ha.
15. 600 ha.
16. 90 m.

Exercise 197. Page 351

1. 725.25 cu. m.
2. 2750 cu. m.
3. 4625.75 cu. m.
4. 9125 cu. m.
5. 8.125 cu. m.
6. 9.275 cu. m.
7. 25.325 cu. m.
8. 37.25 cu. m.
9. 71.75.

Exercise 198. Page 352

1. 75,000 l.
2. 27,500 l.
3. 9275 l.
4. 937.5 l.
5. 75 l.
6. 9.75 l.
7. 7.5 l.
8. 6.25 l.
9. 37.5 hl.
10. 48.75 hl.
11. 9.785 hl.
12. 0.2575 hl.
13. 0.975 hl.
14. 0.3875 hl.
15. 2.6 hl.
16. 1.26 hl.
17. 3700 qt.
18. 950 qt.
19. 75 qt.
20. 82.5 qt.
21. 1000.
22. 1260.
23. 11,390.625.
24. 864; 86,400.
25. 612,500.
26. 763,408.8.

Exercise 199. Page 353

1. 1700 g.
2. 900 g.
3. 250 g.
4. 3750 g.
5. 7 g.
6. 9.5 g.
7. 7.5 g.
8. 8.125 g.
9. 7000 mg.
10. 8700 mg.
11. 270 mg.
12. 4250 mg.
13. 15,000,000 mg.
14. 2,700,000 mg.
15. 750 mg.
16. 98 mg.
17. 27,000 kg.
18. 500 kg.
19. 0.25 kg.
20. 2000 kg.
21. 6.6 lb.
22. 165 lb.
23. 0.66 lb.
24. 0.0198 lb.

Exercise 200. Page 354

1. 0.79. 2. 0.7. 3 0.24. 4. 1.03. 5. Yes. 6. 19.3. 7. 15.3153 kg.

Exercise 201. Page 355

1. 15°.
2. $5\frac{5}{9}$°.
3. $23\frac{8}{9}$°.
4. 30°.
5. $42\frac{2}{9}$°.
6. $44\frac{4}{9}$°.
7. $65\frac{5}{9}$°.
8. $93\frac{1}{3}$°.
9. 15° below zero.
10. $12\frac{2}{9}$° below zero.
11. $23\frac{1}{3}$° below zero.
12. $26\frac{1}{9}$° below zero.
13. $31\frac{2}{3}$° below zero.
14. 104°.
15. 167°.
16. 50°.
17. 302°.
18. 392°.
19. 14°.
20. 5°.
21. 13° below zero.

Exercise 202. Page 356

1. 1.8769 sq. m.
2. 5.372 sq. m.
3. 17,671.5 sq. cm.
4. 82,448.1504 sq. mm.
5. 4.5375 sq. m.
6. 12,350 sq. cm.
7. 2 m.
8. 6.28 m.
9. 1262.24 kg.
10. 137.05536 kg.
11. 2835.
12. 113.
13. 0.8125.
14. 0.915 kg.
15. 52.7558 in.
16. 105.132 in.
17. 0.4875.
18. 764.5.
19. 295.16.
20. 141.42 m.
21. 4.472 cm.
22. 12.649 cm.
23. $1.24.
24. $1.50.
25. 13,304.676 l.
26. 663.663.
27. 683.46 cu. in.
28. 7.168 kg.

Exercise 204. Page 362

1. $10,650.
2. $14,093.75.
3. $16,187.50.
4. $35,028.13.
5. $50,006.25.
6. $4659.38.
7. $6030.
8. $6410.63.
9. $12,406.25.
10. $3190.63.
11. $6140.63.
12. $1975.
13. $4837.50.
14. $11,805.
15. $31,218.75.
16. $13,734.38.
17. $16,856.25.
18. $21,196.88.
19. $35,687.50.
20. $27,465.63.
21. $143.75.
22. $393.75.
23. $31.25.
24. 40; $15.

Exercise 205. Page 363

1. $4762.50.
2. $18,750.
3. $8390.63.
4. $7521.25.
5. $5225.
6. $22,312.50.
7. $41,602.50.
8. $5100.
9. $1706.25.
10. $4585.
11. $9956.25.
12. $21.87, gain.
13. $34.37, gain.
14. $53.13, loss.
15. $143.75, loss.
16. $6.25, gain.
17. $25, gain.
18. $40.62, gain.
19. $34.37, gain.
20. $59.37, gain.
21. $153.12, gain.
22. $46.87, gain.
23. $96.88, loss.
24. $156.25, loss.
25. $593.75, gain.
26. $100, gain.
27. $30, gain.
28. $1037.49, gain.

Exercise 206. Page 365

1. $2\frac{1}{2}\%$.
2. $2625.
3. $90,000.
4. $75,500,000.
5. $300.
6. $2\frac{1}{2}\%$.
7. $4\frac{1}{2}\%$.
8. 6%.

Exercise 207. Page 366

1. 4%.
2. 4%.
3. 4%.
4. 5%.
5. 6%.
6. 8%.
7. 4%.
8. 5%.
9. 8%.
10. 6%.
11. $4\frac{4}{13}\%$.
12. $5\frac{5}{11}\%$.
13. $4\frac{6}{11}\%$.
14. 4.04%.
15. 4.98%.
16. 5.37%.
17. 5.26%.
18. 5%.
19. The same.
20. The 5% note; 0.34%.
21. The 8% stock; $\frac{1}{3}\%$.
22. The 6% stock; 0.12%.
23. $120; $119\frac{7}{8}$.
24. $150; $149\frac{7}{8}$.

Exercise 208. Page 367

1. $360.
2. $350.
3. $360.
4. $255.
5. $260.
6. $472.50.
7. $765.
8. $660.
9. $1105.
10. 4.90%.
11. 6.37%.
12. $3\frac{1}{3}\%$.

Exercise 210. Page 371

1. $970.75.
2. $873.53.
3. $518.18.
4. $1087.20.
5. $1572.38.
6. $59.60.

Exercise 211. Page 373

1. $603.75.
2. $725.12.
3. $936.25.
4. $1130.
5. $774.40.
6. $682.67.
7. $626.90.
8. $547.52.
9. $799.40.
10. $324.45.
11. $321.25.
12. $631.27.
13. $1802.84.

Exercise 212. Page 375

3. $158.42. 4. $368.71. 5. $706.49. 6. $813.96. 7. $449.89.

Exercise 213. Page 377

1. $1.25; $248.75.
2. $2.88; $572.12.
3. $6.25; $618.75.
4. $10.42; $1239.58.
5. $18.75; $2231.25.
6. $0.73; $174.27.
7. $11.69; $1263.31.
8. $21.54; $2328.46.
9. $4.13; $271.37.
10. $7.91; $519.59.
11. $1237.50.
12. $714.12.
13. $7.08.

Exercise 214. Page 378

1. $7.13; $249.12.
2. $5.82; $373.87.
3. $8.89; $424.61.
4. $7.46; $551.71.
5. $16.03; $679.22.
6. $14.95; $1266.30.
7. $4.92; $378.09.
8. $17.55; $838.65.
9. $12.25.
10. $12.35.
11. $10.31.
12. $6.19.
13. $6.25; $2493.75.
14. $1653.92.
15. $1660.79.
16. $1649.63.
17. $1625.25.
18. $6.75.
19. On the 4-months note; $0.30.

Exercise 215. Page 381

1. $971.10.
2. $348.42.
3. $718.35.
4. $1214.72.
5. $1656.42.
6. $2140.75.
7. $615.13.
8. $1480.92.
9. $2059.20.

Exercise 216. Page 382

1. $502.53.
2. $1800.
3. $502.51.
4. $1772.73.
5. $2062.31.
6. $2836.14.

Exercise 217. Page 383

1. $609.49. 2. $1917.86. 3. $1505.19. 4. $2343.20. 5. $146.25.
6. Proceeds of the $750 note; $13.25.

Exercise 219. Page 386

1. 3¢.
2. 5¢.
3. 8¢.
4. 8¢.
5. 10¢.
6. 12¢.
7. 12¢.
8. 15¢.
9. 15¢.
10. 18¢.
11. 25¢.
12. 30¢.
13. $249.75.
14. $5.85.
15. $3250.
16. 0.2%.

Exercise 220. Page 387

1. $2502.50.
2. $3753.75.
3. $4854.85.
4. $4809.60.
5. $5611.20.
6. $250.31.
7. $751.50.
8. $641.60.
9. $1283.20.

Exercise 221. Page 388

1. $1748.25.
2. $2145.70.
3. $3595.50.
4. 0.1%.
5. 0.2%.
6. $224.62.

Exercise 222. Page 389

1. $27,549.50.
2. $36,580.30.
3. $37,339.40.
4. $48,202.
5. $26,546.38.
6. $125,231.25.
7. $48,201.75.
8. $74,850.
9. $64,638.
10. $327,336.25.
11. $774,862.38.
12. $1,748,524.73.

Exercise 223. Page 390

1. $1250.
2. $1753.50.
3. $2762.76.
4. $4488.75.
5. $2694.60.
6. $25,801.50.
7. Discount; $\frac{1}{20}\%$.
8. $1.75.
9. $0.30.
10. $1.05.
11. $1319.02.

Exercise 224. Page 391

1. 1680d.
2. 5760d.
3. 1272d.
4. 1708d.
5. 540s.
6. 640s.
7. 10s.
8. 60s.
9. £4 15s.
10. £3 12s.
11. £6 7s.
12. £17 2s.
13. 7.50 fr.
14. 2.75 fr.
15. 6.80 fr.
16. 12.50 fr.
17. 4.50 M.
18. 2.75 M.
19. 16.80 M.
20. 92.75 M.

Exercise 225. Page 393

1. $34.23.
2. $44.10.
3. $58.44.
4. $121.50.
5. $227.95.
6. $383.76.
7. $11.27.
8. $16.59.
9. $34.99.
10. $61.84.
11. $67.31.
12. $91.78.
13. $111.55.
14. $133.28.
15. $571.20.
16. $1046.15.
17. $649.90.
18. $789.28.
19. $1118.52.
20. $1410.10.
21. £684 5s. 4d.
22. 200 fr.
23. 675.21 M.
24. $237.15.
25. $5264.42.
26. $636.
27. Exchange at 19.2; $0.31.

Exercise 227. Page 395

1. 216 cu. in.
2. 247 cu. in.
3. 354 cu. in.
4. 58.32 cu. in.
5. 380.92 cu. in.
6. 768.6875 cu. in.

Exercise 228. Page 396

1. 96 cu. in.
2. 208 cu. in.
3. 320 cu. in.
4. 704 cu. in.
5. 1144 cu. in.
6. $6\frac{7}{8}$ cu. in.
7. $11\frac{21}{64}$ cu. in.
8. 8.99 cu. in.
9. 19.27 cu. in.
10. 42.525 cu. in.
11. 93,537,284 cu. ft.

Exercise 229. Page 397

1. 8 cu. in.
2. 35 cu. in.
3. 60 cu. in.
4. 170 cu. in.
5. 476 cu. in.
6. $5\frac{5}{8}$ cu. in.
7. $12\frac{11}{32}$ cu. in.
8. $28\frac{7}{32}$ cu. in.
9. 63.56 cu. in.
10. 146.575 cu. in.
11. 75.3984 cu. in.

Exercise 230. Page 399

1. 616 sq. in.
2. 3850 sq. in.
3. 5544 sq. in.
4. 13.86 sq. in.
5. 24.64 sq. in.
6. 75.46 sq. in.
7. $2\frac{13}{32}$ sq. in.
8. 1.54 sq. in.
9. 0.9856 sq. in.
10. 157,696 sq. in.
11. 432,586 sq. in.
12. 591,976 sq. in.
13. 153.9384 sq. in.
14. 113.0976 sq. in.
15. 254.4696 sq. in.
16. 50.2656 sq. in.
17. 176.715 sq. in.
18. 122.71875 sq. in.
19. 15.205344 sq. in.
20. 38.4846 sq. in.
21. 12.5664 sq. in.
22. 314.16 sq. in.
23. 3.1416 sq. in.
24. 113.0976 sq. ft.
25. 615.7536 sq. in.
26. 1017.8784 sq. in.
27. 78.54 sq. in.
28. 172.034016 sq. in.
29. 201,142,857$\frac{1}{7}$ sq. mi.
30. $28\frac{2}{7}$.
31. 2,359,727,071,428$\frac{4}{7}$ sq. mi.
32. $6084\frac{4}{7}$.
33. 7 in.

Exercise 231. Page 400

1. 33.5104 cu. in.
2. 523.6 cu. in.
3. 1436.7584 cu. in.
4. 14,137.2 cu. in.
5. 50,965.1296 cu. in.
6. 310,339.8144 cu. in.
7. 57.9059712 cu. ft.
8. 288.6962848 cu. ft.
9. 179.5948 cu. ft.
10. 65.45 cu. in.
11. 220.89375 cu. in.
12. 425.7062 cu. in.
13. 27.5132 oz.

Exercise 232. Page 401

1. 2513.28 sq. ft.
2. 113.0976 cu. in.; 904.7808 cu. in.
3. 5,276,681,625.6 cu. mi.
4. 20.0165.
5. 243.9168.
6. $31\frac{23}{32}$ lb.
7. 9.4248.
8. The same.
9. 0.5236 cu. ft.; 0.7854 cu. ft.; 0.2618 cu. ft.; twice; three times.

Exercise 234. Page 403

1. $160.06.
2. $168.47.
3. $463.73.
4. $182.55.
5. $600.51.
6. $468.28; $0.45.
7. $159.87; $0.15.
8. $361.78; $0.35.

Exercise 235. Page 405

1. \$303.14.
2. \$38.34.
3. $\frac{4}{64}$ in.
4. \$67,031.25.
5. \$4.
6. The second; $3\frac{1}{2}\frac{3}{9}\%$.
7. \$929.19.
8. $5\frac{61}{63}$ in.
9. 520 lb.
10. $62{,}304\frac{11}{16}$ lb.
11. \$4837.50.
12. 922 lb. 4 oz.
13. 27.713 in.
14. 7875 lb.
15. \$89.23.
16. \$76.34.
17. \$82.02.
18. \$73.37.
19. \$72.06.
20. \$65.61.

Exercise 236. Page 408

1. 103.
2. \$46.30.
3. \$5180.70.
4. \$85.51.

Exercise 237. Page 409

1. \$635.60.
2. \$73.01.
3. 70.88%.
4. \$80.
5. $7\frac{1}{2}$ ft.

Exercise 238. Page 410

1. 77.98 lb.
2. 481.0575 lb.
3. 908.855 lb.
4. 2126.4705 sq. in.
5. 54,286.848 lb.
6. 1053 lb.
7. 938.850 lb.
8. 144.31725.
9. 1496.656 lb.

Exercise 239. Page 411

1. $2227\frac{1}{2}$.
2. \$0.83.
3. 40¢.
4. 25%; 20%.
5. 64,800.
6. $5\frac{5}{9}$ ft.
7. 51,397.5.
8. 2420.

Exercise 240. Page 412

1. 5 ft.
2. 14.32 ft.
3. 7.3304 ft.
4. 73.304 sq. ft.
5. 245.4375 sq. ft.
6. 1143.04.

Exercise 241. Page 413

1. 23.562 ft.
2. 5236 ft.
3. 1884.96 ft.
4. 1200.
5. 14.927 in.
6. 36.319 in.
7. 68,706.792 lb.
8. 1847.2608 sq. ft.
9. 17.
10. 750.

Exercise 242. Page 414

1. 110.16 lb.
2. 4.54 lb; 8.071 lb.
3. $8\frac{2}{4}\frac{3}{0}$ H. P.
4. $112\frac{1}{2}$ lb.
5. $121\frac{1}{2}$ lb.
6. 36.

Exercise 243. Page 415

1. 33,958.
2. 24,742.
3. 23,612.
4. 22,148.
5. 27,544.
6. 6488.
7. 6338.
8. 5239.
9. 1873.
10. 1528.
11. 30,605,967.
12. 32,425,336.
13. 9,157,532.
14. 62,929,457.
15. 72,714,908.
16. $396\frac{90}{233}$.
17. $549\frac{15}{225}$.
18. 250.

Exercise 244. Page 416

1. $2\frac{3}{14}$.
2. 50.
3. $\frac{1}{36}$.
4. 1.
5. $\frac{3}{8}$; 0.375.
6. 1.
7. $1\frac{3}{4}$.
8. $1\frac{274}{315}$.
9. $\frac{32}{129}$.
10. $26\frac{121}{224}$.
11. $\frac{187}{320}$.
12. $\frac{1}{8}$.

Exercise 245. Page 417

1. 3.
2. $\frac{1}{8}$.
3. 2.
4. 23.473.
5. 29.499.
6. 14.409.
7. 15.3717.
8. 4.8089.
9. 3.0319.
10. 12.7275.
11. 5.1752.
12. 16.44501936.
13. 655.746168.
14. 1235.914848.
15. 0.0015554726.
16. 17.
17. 2.1.
18. 18.
19. 0.132.
20. 1.089.
21. 8.413.
22. 2.838.
23. 73.611.
24. 1.971.
25. 125.714.

Exercise 246. Page 418

1. 278 ft. 4 in.
2. 246 yd. 22 in.
3. 171 lb.
4. 8 ft. 5 in.
5. 58 yd. 24 in.
6. 17 lb. 8 oz.
7. 921 ft. 8 in.
8. 3240 yd. 20 in.
9. 3257 lb. 13 oz.
10. 27 yd. 2 ft. 3 in.
11. 16 ft. $5\frac{2}{7}$ in.
12. 17 lb. 7 oz.
13. 37 yd. 2 in.
14. 8.
15. 7.
16. 445,605 in.
17. 6736 in.; 561.33⅓ ft.; 34.0202 rd.
18. 26 mo. 4 da.
19. 3 yr. 9 mo. 12 da.

Exercise 247. Page 419

1. $19.50.
2. $143.40.
3. $37.50.
4. $54.79.
5. $17.10.
6. $276.75.
7. $148.

Exercise 248. Page 420

1. 127.279 ft.
2. $20.48.
3. $66.
4. $36.90.
5. $227\frac{25}{64}$.
6. $11.20.
7. 18.
8. $14.04.

Exercise 249. Page 421

1. 6 hr. 31 min. 19 sec.
2. 27 min. 8 sec.
3. 8 hr. 13 min. 32 sec.
4. 54 min. $31\frac{4}{5}$ sec.
5. 80° 14′ 51″ E.
6. 70° 15′ 18″ W.
7. 86° 48′ W.
8. 75° 31′ 15″ W.
9. 2 min. 22.2 sec.
10. 41° 22′ 30″ E.

Exercise 250. Page 422

1. $461.50.
2. 40%, gain.
3. $16\frac{2}{3}\%$.
4. $12.
5. 20%; $62\frac{1}{2}¢$.
6. $78\frac{4}{7}\%$.
7. $208.
8. 50%.
9. $10; $33\frac{1}{3}\%$.
10. $73.10.
11. 10%.
12. 10%.

Exercise 251. Page 423

1. $247.
2. 15%.
3. 3%.
4. $4.75.
5. $186.30.
6. $12\frac{1}{2}\%$.
7. $12,500.
8. $116.37.
9. $935.
10. $3162; $15.81.

Exercise 252. Page 424

1. $2800.
2. $20.10.
3. $16.50.
4. $14,000.
5. $4600.
6. $1\frac{1}{4}\%$.
7. $1072.50.
8. $7500.
9. $825.
10. $7500; $6.90.
11. $8250.

Exercise 253. Page 425

1. $7.50.
2. $2,000,000.
3. 15 mills; $102.60.
4. $43.40.
5. $382.50.
6. $20.25.
7. $525.
8. $107.25.
9. 6 mills.

Exercise 254. Page 426

1. $733.59.
2. $559.55.
3. $361.42.
4. $10.25.
5. 46.199%.
6. $650.
7. $357.20; 24%.
8. 58.96%.
9. The single discount; $4.17.
10. The discount series; $0.30.
11. The same.

Exercise 255. Page 427

1. $78.20.
2. $61.81.
3. $62.43.
4. $33.26.
5. 3 yr. 4 mo. 24 da.
6. $44.92.
7. $740.50.
8. $381.56; Dec. 30.
9. $4.20.
10. $577.50.
11. $416.15.
12. $15.70; $584.30.

Exercise 256. Page 428

1. \$4711.50.
2. \$12,180.
3. $3\frac{3}{5}\%$.
4. 42.
5. $4\frac{1}{2}\%$.
6. \$4.75.
7. \$23,647.50.
8. \$19.50; \$15,034.50.
9. $\frac{8}{11}\%$.
10. 20; \$100.
11. \$100.47, loss.

Exercise 257. Page 429

1. $\frac{5}{6}$.
2. $\frac{5}{12} : \frac{5}{21} = 3\frac{1}{2} : 2$.
3. \$26.67; \$33.33.
4. 16.
5. $1\frac{4}{27}$.
6. 48.
7. 3686.4.
8. $11\frac{2}{3}$ hr.
9. \$25.06; \$54.94.

Exercise 258. Page 430

1. $198\frac{8}{9}\frac{3}{9}$.
2. 9600 kg.
3. 6300.
4. 0.75 m.
5. $1\frac{5}{9}$ m.
6. \$19.20.
7. 1.4 m.
8. 500 m.
9. 236.25 kg.
10. 2870.
11. 149.76 kg.
12. 43,259.832.

Exercise 259. Page 431

1. 346.
2. 413.
3. 2.449.
4. 5.477.
5. 1074.8.
6. 10.61 ft.
7. 21.21 ft.
8. 6.08 ft.
9. 126.
10. 6 in.
11. 7 ft.
12. 35 ft.
13. 14 ft.
14. 35 ft.

Exercise 260. Page 432

1. $21\frac{3}{7}\%$.
2. \$49,280.
3. A, \$72; B, \$81.
4. 2827.44.
5. $23\frac{1}{4}$.
6. \$201.60.
7. 234.68 in.
8. 5 hr.
9. \$4500; \$1200.
10. 15 mi.
11. $1,974,857\frac{1}{7}$.
12. 14.14 ft.
13. 0.7 in.
14. $3764\frac{4}{9}$.
15. \$237.50.
16. 1960.
17. 1320.
18. 71.00016 in.; $71.02857\frac{1}{7}$ in.; 71 in.
19. \$105.12.
20. Sand, 108; soda, 21; lime, 18; alumina, 3.
21. Copper, 962.5; tin, 262.5; zinc, 25.
22. 484.
23. 29.5305903 da.
24. $34\frac{11}{16}$.
25. 468 lb.
26. 11.2.
27. \$145.80.
28. \$3750.
29. 105.
30. \$13.95
31. $4\frac{1}{2}$ hr.
32. \$46.80.
33. 38.1831.
34. $4\frac{1}{2}\%$; $5\frac{1}{2}\%$.
35. $\frac{28}{183}$.
36. $66\frac{5}{7}°$.
37. $29.79\frac{5}{7}$ in.
38. 10 lb.; 50 lb.

39. 201.68.
40. 60 lb.
41. $722.77.
42. 28.97 ft.
43. 5.67 bbl.; 3.54 cu. yd.
44. 1350; 3375.
45. 2.
46. 4.
47. $4\frac{4}{9}$.
48. $4\frac{3}{3}\frac{2}{7}$.
49. 96.
50. $1\frac{2}{3}\frac{3}{7}$.
51. 7.35 mi.

Exercise 261. Page 438

1. 24. 2. 48. 3. 38. 4. 38. 5. 52

Exercise 262. Page 439

1. 165.
2. 275.
3. 282.
4. 172.
5. 234.
6. 315.
7. 286.
8. 374.
9. 510.
10. 261.
11. 273.
12. 630.
13. 24; 108.
14. 23; 135.
15. 59; 224.
16. 31; 96.
17. 40; 135.
18. 79; 430.
19. 59; 610.
20. 57; 435.
21. 92.
22. 126.
23. 180.
24. 185.
25. 475.
26. 182.
27. 159.
28. 98.
29. 350.
30. 2500.
31. 630.
32. 156.
33. $176\frac{11}{12}$ ft.; 579 ft.
34. $305\frac{7}{12}$ ft.; $1608\frac{1}{3}$ ft.
35. 2790 ft.
36. 30,300 ft.
37. Results the same.

Exercise 263. Page 441

1. 32.
2. 512.
3. 3645.
4. 4375.
5. 46,656.
6. 1792.
7. 2.
8. 3.
9. 16.
10. 9.
11. 16.
12. 9.
13. 16,384.
14. 13,122.

Exercise 264. Page 442

1. 254.
2. 364.
3. 511.
4. 255.
5. 160.
6. 780.
7. 129.
8. 242.
9. 63.
10. 1053.
11. 765.
12. 19,531.
13. 252.
14. 605.

Exercise 265 Page 445

1. $97.07.
2. $369.11.
3. $1497.27.
4. $1806.92.
5. $888.80.
6. 7%.
7. $10\frac{1}{2}$ yr.
8. 13 yr.
9. $2098.86.
10. 4%.
11. $2000.
12. $9\frac{1}{2}$ yr.
13. 5%.
14. $4\frac{1}{2}$%.
15. $2\frac{1}{2}$%.

Exercise 266. Page 447

1. $0.\dot{3}$.
2. $0.\dot{6}$.
3. $0.1\dot{6}$.
4. $0.8\dot{3}$.
5. $0.\dot{1}42857\dot{}$.
6. $0.\dot{2}85714\dot{}$.
7. $0.\dot{7}14285\dot{}$.
8. $0.\dot{1}$.
9. $0.\dot{5}$.
10. $0.\dot{7}$.
11. $0.0\dot{9}$.
12. $0.4\dot{5}$.
13. $0.08\dot{3}$.
14. $0.58\dot{3}$.
15. $0.0\dot{6}$.
16. $0.5\dot{3}$.
17. $0.04\dot{5}$.
18. $0.2\dot{2}\dot{7}$.
19. $0.0\dot{3}$.
20. $0.0\dot{1}$.
21. $\frac{1}{3}$.
22. $\frac{2}{3}$.
23. $\frac{5}{9}$.
24. $\frac{7}{9}$.
25. 1.
26. $\frac{20}{99}$.
27. $\frac{7}{33}$.
28. $\frac{10}{33}$.
29. $\frac{10}{11}$.
30. 1.
31. $\frac{242}{999}$.
32. $\frac{127}{999}$.
33. $\frac{161}{999}$.
34. $\frac{80}{333}$.
35. $\frac{863}{999}$.
36. $\frac{36388}{49995}$.
37. $\frac{5}{44}$.
38. $2\frac{89}{666}$.
39. $4\frac{117}{370}$.
40. $7\frac{373}{3000}$.
41. $1\frac{62}{99}$.
42. $2.2\dot{0}$.

Exercise 267. Page 449

1. 14,156.
2. 7427.
3. 12,022,002.
4. 33,220.
5. 2626.
6. 1,101,001.
7. 1331.
8. $82\widehat{10}$, in which $\widehat{10}$ stands for a single figure.
9. $6\widehat{11}4$, in which $\widehat{11}$ stands for a single figure.
10. 10,223.
11. 5607.
12. 467.

Exercise 268. Page 450

1. $64.86.
2. $256.77.
3. $130.24.

Exercise 269. Page 451

1. $59\frac{43}{48}$.
2. $97\frac{39}{49}$.
3. $112\frac{63}{75}$.
4. $79\frac{89}{121}$.
5. $108\frac{14}{81}$.
6. $53\frac{13}{144}$.
7. $66\frac{55}{99}$.
8. $131\frac{47}{64}$.
9. $61\frac{57}{128}$.
10. $67\frac{100}{125}$.
11. $104\frac{23}{88}$.
12. $60\frac{600}{616}$.
13. $176\frac{422}{495}$.
14. $183\frac{421}{504}$.
15. $234\frac{23}{378}$.
16. $65\frac{675}{1152}$.
17. $92\frac{671}{891}$.
18. $246\frac{266}{396}$.
19. $261\frac{105}{252}$.
20. $446\frac{157}{192}$.
21. $156\frac{518}{539}$.

Exercise 270. Page 453

1. 47,100.
2. 29,648.
3. 18,657.
4. 35,532.
5. 120,294.
6. 281,424.
7. 12.
8. 127.
9. 121.
10. $180\frac{14}{15}$.
11. $63\frac{8}{21}$.
12. $215\frac{114}{128}$.

Exercise 271. Page 459

1. 500 yd.
2. $866\frac{2}{3}$ yd.
3. 237.96 yd.
4. 2400 ft.
5. 900 ft.
6. $2083\frac{1}{3}$ ft.
7. 2 mi.
8. 4.5 mi.
9. 12 mi.
10. 20 labors.
11. 1296 square varas.
12. 3,613,040.64 square varas.

17 A. x
1.2
34
17
20.4 V drop

220
20.4
199.6

3
.4
1.2

35
12
70
350
420

220
42
178

$$C = \frac{E}{R}$$

$$E = \overset{32}{C} \times \overset{\text{Ohms}}{R} \quad .48$$

$$7000W \div 220V = 32.A$$

Dist $1.2\,M. \times \overset{\text{Ohms}}{.4} = .48$

$$\begin{array}{r} .4 \\ \hline .48 \end{array}$$

6 copper has .4 Ohms Resistance Per M.

$$C = \frac{E}{R}$$

$$E = \overset{32a}{C} \times \overset{.5}{R} = 16.0$$

$$\begin{array}{r} 32 \\ 220\overline{)7000} \\ 660 \\ \hline 400 \\ 440 \\ \hline 60 \end{array}$$

$$\begin{array}{r} 1.2 \\ .4 \\ \hline .48 \end{array}$$

8 11 6
3 5 7 10
14 9 2